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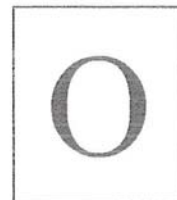
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BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019



SUBJECT : Mathematics

LEVEL : Sec 4E/5N

PAPER : 4048 / 01

DURATION : 2 hours

SETTER : Mr Leong S C

DATE : 26 Aug 2019

CLASS :	NAME :	REG NO :
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READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use
80

*Mathematical Formulae**Compound Interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

1. The first 4 terms of a sequence of numbers T_1, T_2, T_3 and T_4 are given below:

$$T_1 = 1^2 + 2^2 = 5$$

$$T_2 = 2^2 + 3^2 = 13$$

$$T_3 = 3^2 + 4^2 = 25$$

$$T_4 = 4^2 + 5^2 = 41$$

- (a) Write down an expression in terms of n , for the n^{th} term T_n of the sequence.

Answer [1]

- (b) Explain why T_n is always odd.

Answer:

[1]

.....

.....

.....

.....

2. Simplify $\frac{a^2b}{2} \times (2a^0b^{-2})^3$, leaving your answer in positive index form.

Answer [2]

[Turn Over

3. $x = \frac{4}{3}$ is a solution to the equation $18x^2 - kx - 20 = 0$, where k is a constant.

(a) Find the value of k .

Answer $k = \dots\dots\dots$ [1]

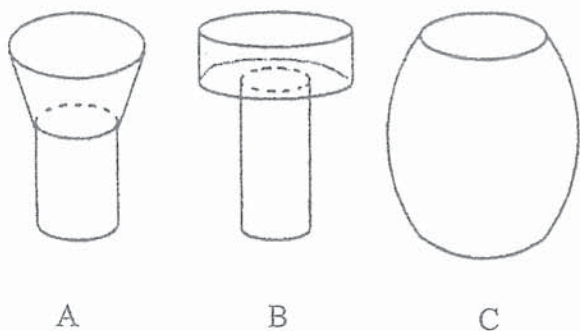
(b) Hence, find the other solution to the equation $18x^2 - kx - 20 = 0$.

Answer $\dots\dots\dots$ [1]

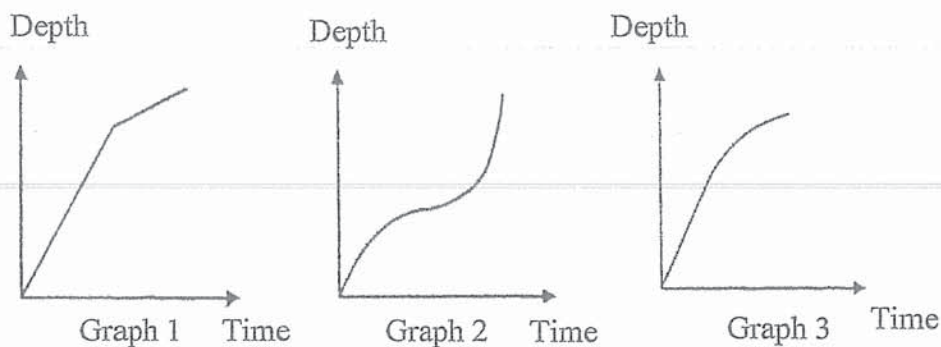
-
4. Given that the size of an exterior angle of a regular polygon is one-fifth its interior angle, find the number of sides of the polygon.

Answer $\dots\dots\dots$ [2]

5. Water is poured at a constant rate into each of the containers shown below.



The graphs show the depth of water in the containers as they are being filled. Match the graphs with the appropriate container and complete the table below.

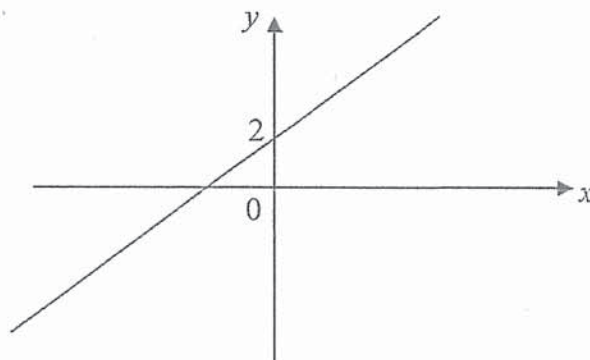


Answer:

[2]

Graph	Container
1	
2	
3	

6. (a) The graph below can be represented by the equation, $y = ax^n + b$. State the value of b , of n and the range of a .



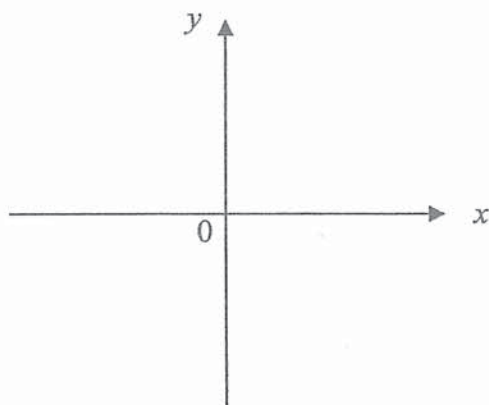
Answer $b = \dots\dots\dots n = \dots\dots\dots$ [1]

$\dots\dots\dots$ [1]

[Turn Over

- (b) On the axes in the answer space, sketch the graph of $y = ax^n$, where $a < 0$ and $n = 2$. [1]

Answer



-
7. Find the largest prime number that satisfy the inequality $3 + y \leq 15 - 2y < y + 10$.

Answer [3]

8. Solve the simultaneous equations

$$8x + 3y = 4$$

$$4x = y + 4$$

Answer $x = \dots\dots\dots$ [3]

$y = \dots\dots\dots$

9. It is given that $\mathbf{p} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$.

- (a) Find $|\mathbf{q} - 2\mathbf{p}|$.

Answer $\dots\dots\dots$ [2]

- (b) Use vectors to explain whether \mathbf{r} is parallel to \mathbf{q} .

Answer $\dots\dots\dots$ [1]

.....

10. Simplify and express $\frac{9m^2 - 12mn + 4n^2}{m^2 - n^2} \div \frac{3m - 2n}{m - n}$ as a single fraction.

Answer [3]

11. The ground area of a scale model is 1 : 100 of the actual ground area.

- (a) Given that the length of the model is 3 m, calculate the length of the house in metre.

Answerm [1]

- (b) Given that the actual volume of a storeroom in the house is 15 000 000 cm³, calculate the volume of the storeroom in the model in cm³.

Answercm³ [2]

12. A bag contains a total of 80 green, red and blue balls. There are 18 red balls in the bag.

- (a) If a ball is picked randomly, the probability of picking a green ball is $\frac{3}{8}$. How many blue balls are there?

Answer [1]

(b) Two balls are picked randomly from the bag. Find the probability of

- (i) picking a green ball and a red ball,

Answer [2]

- (ii) picking at least one red ball.

Answer [1]

[Turn Over

13. Find the equation of the straight line
 (a) passing through the point $(-2, 5)$ and parallel to the y -axis,

Answer [1]

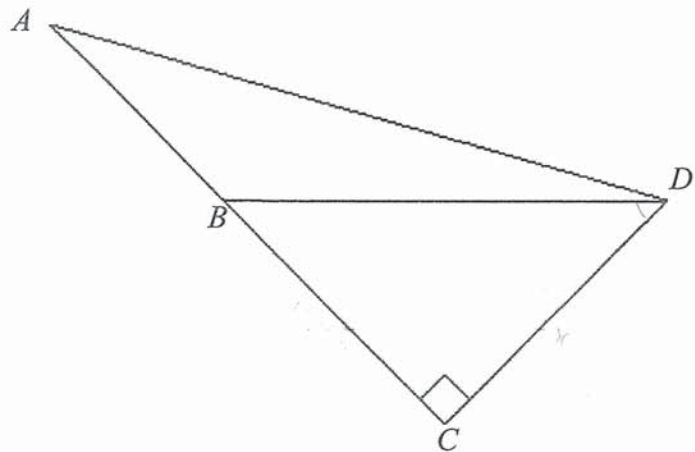
- (b) which passes through the point $(2, -5)$ and its gradient is the same as the line $y - 3x - 5 = 0$.

Answer [2]

14. In the diagram triangle ACD is right-angled, and B is on AC such that $\sin \angle ABD = \frac{x}{y}$.

Find in terms of x and y .

- (a) $\tan \angle BDC$,



Answer [2]

- (b) $\cos \angle ABD$.

Answer [1]

15. (a) An insurance agent is paid a basic monthly salary of \$1200 and a commission of 8% on all the sales if the monthly sales made by the insurance agent exceed \$7500. The insurance agent's salary for a particular month was \$3694. Calculate the total sales made by the insurance agent.

Answer [2]

- (b) Two taps X and Y are turned on at the same time to fill up an empty tank. The rate at which Tap X fills up the tank per minute is twice as fast as that of Tap Y . When used together, both taps can fill the tank with 300 cm^3 of water in 5 minutes. Find the rate of flow of Tap Y in $\text{cm}^3/\text{minute}$.

Answer $\text{cm}^3/\text{minute}$ [2]

[Turn Over

16. The table below shows the frequency of visits to the library by some students in Betta School in a week.

Number of visits	0	1	2	3	4
Number of students	3	5	4	x	2

- (a) If the mode is 3, write down the smallest possible value of x .

Answer $x = \dots\dots\dots$ [1]

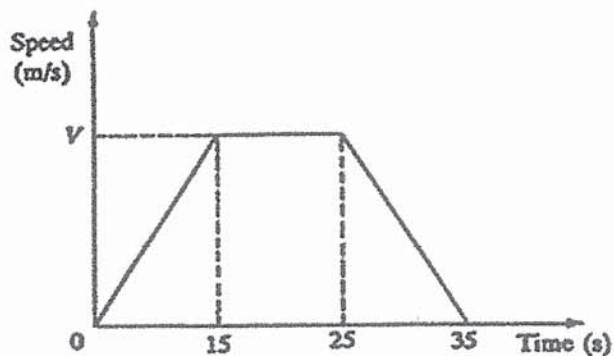
- (b) If the median is 2, write down the largest possible value of x .

Answer $x = \dots\dots\dots$ [1]

- (c) Given that the mean is $2\frac{1}{22}$, calculate the value of x .

Answer $x = \dots\dots\dots$ [2]

17. The diagram shows a speed-time graph of a car. The total distance travelled in 35 s is 540 m.



- (a) Calculate the maximum speed V m/s.

Answer $V = \dots\dots\dots$ m/s [2]

- (b) Calculate the speed of the car at 28 seconds.

Answer 16.8m/s [2]

-
18. The base areas of two geometrically similar pyramids are 98 cm^2 and 242 cm^2 .

- (a) Find the ratio of the height of the larger pyramid to the height of the smaller pyramid.

Answer:..... [1]

- (b) Given the total surface area of the smaller pyramid is 343 cm^2 . Find the total surface area of the larger pyramid.

Answer cm^2 [1]

[Turn Over

- (c) Find the percentage difference in volumes of the two pyramids in terms of the larger pyramid.

Answer [2]

19. (a) s is directly proportional to the square of v . It is known that $s = 36$ for a particular value of v .
Find the value of s when this value of v is halved.

Answer $s =$ [2]

- (b) y is inversely proportional to x and $y = 4$ when $x = 6$. Find the value of y when $x = 9$.

Answer $y =$ [2]

20. Written as a product of its prime factors,

$$p = 3^x \times 5^y \times 7^3 \text{ and } q = 5 \times 7^2 \times 11$$

(a) Find the smallest value of x and of y for which p is a multiple of 35.

Answer $x = \dots\dots\dots y = \dots\dots\dots$ [2]

(b) Explain why $55q$ is a perfect square.

Answer: [1]

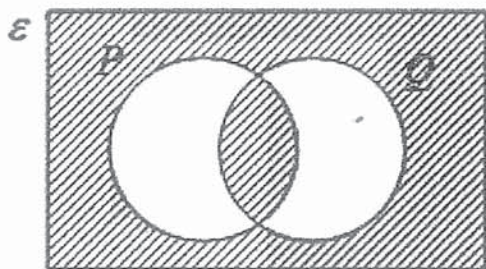
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.....

(c) Show your workings and state, with reason (s), if the product of p and q is an odd or even number.

Answer: [1]

.....
.....
.....
.....

21. (a) Describe the shaded region in set notation.



Answer [1]

- (b) Given that

$$\xi = \{x : x \text{ is a positive integer and } 0 < x < 10\}$$

$$A = \{x : x \text{ is a prime number}\}$$

$$B = \{x : x \text{ is a factor of } 12\}$$

$$C = \{x : \frac{50}{x} < 8\}$$

- (i) Find $B \cap C$.

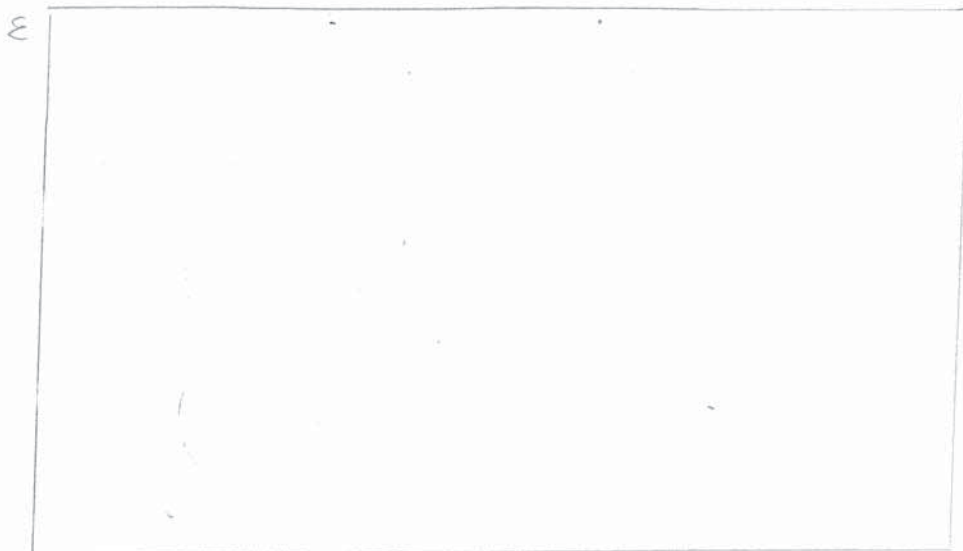
Answer [1]

- (ii) List the elements of $(A \cup B)'$.

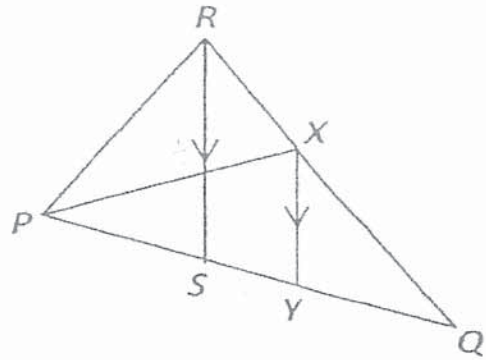
Answer [1]

- (ii) Draw a Venn diagram to illustrate the given information. [2]

Answer:



22. In triangle PQR , X is a point on QR such that $QX : XR$ is $3 : 2$ and RS is parallel to XY .



- (a) Show that triangle QXY is similar to triangle QRS .

Answer:

[2]

.....

.....

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.....

- (b) Given that the area of triangles QXY and PXR are 27 cm^2 and 46 cm^2 respectively. Find the area of

- (i) triangle QRS ,

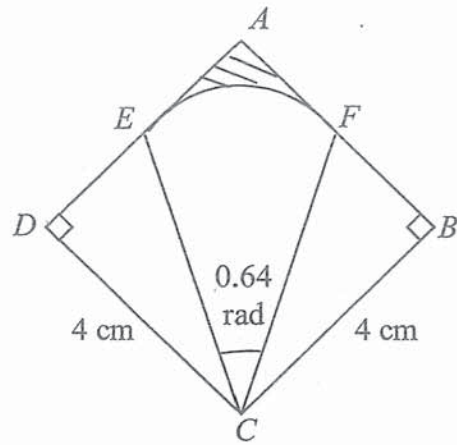
Answer cm^2 [2]

- (ii) triangle PQR .

Answer cm^2 [2]

[Turn Over

23.



The sector CEF with centre at C is inscribed in the square $ABCD$ of side 4 cm .

Given that $AF = FB$, $DE = EA$ and angle $ECF = 0.64$ radian.

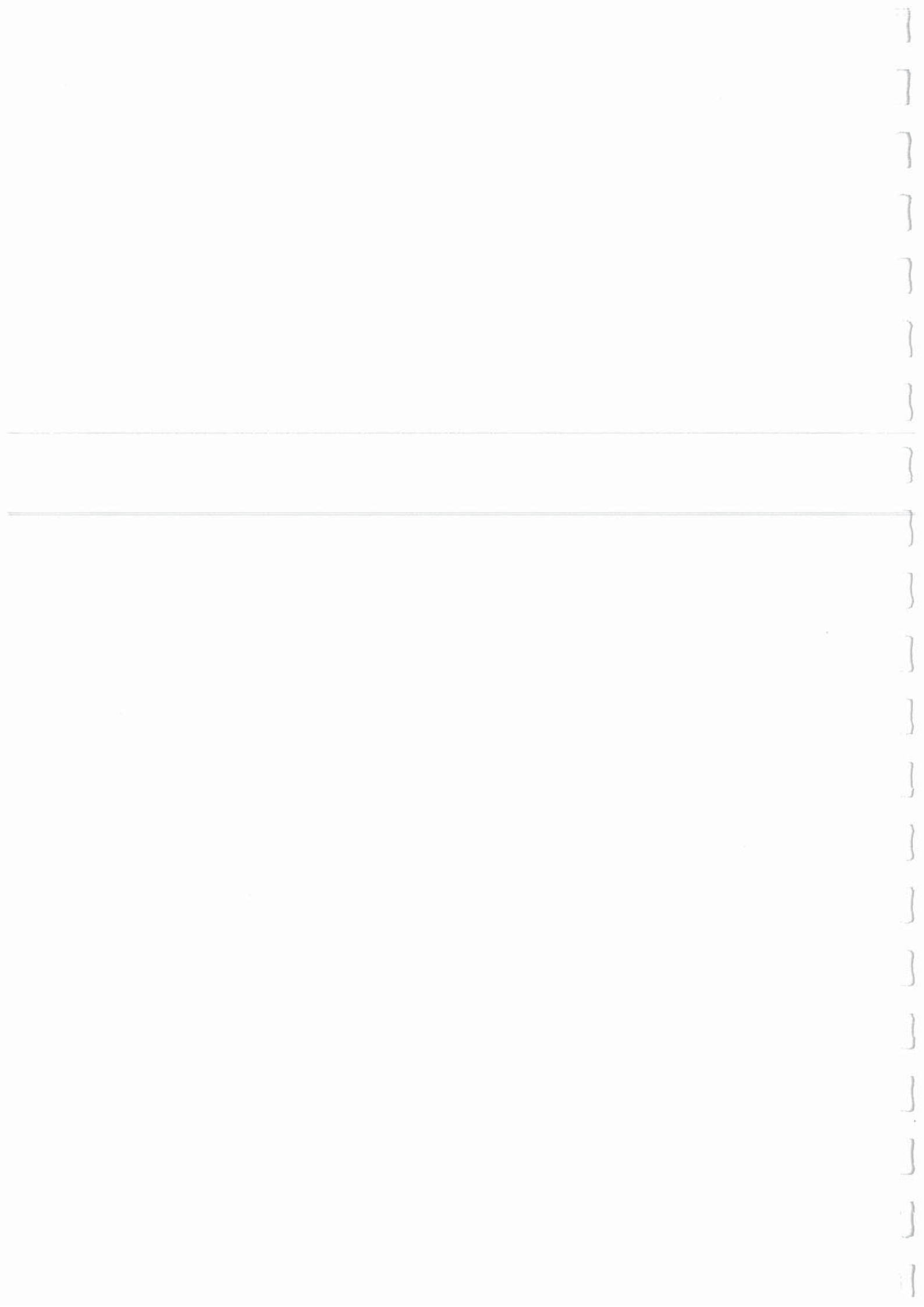
(a) Calculate the perimeter of the sector CEF .

Answer cm [3]

(b) Find the area of the shaded region.

Answer cm^2 [4]

End of Paper



*Mathematical Formulae**Compound Interest*

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 (a) Simplify $\left(\frac{64v^9}{t^{12}}\right)^{-\frac{1}{3}}$. [2]

(b) Express as a single fraction in its simplest form.

(i) $\frac{20x^2}{9} \div \frac{4xy^3}{15y}$. [1]

(ii) $\frac{4}{3x-1} - \frac{5}{2+x}$. [2]

(c) (i) Express $x^2 - 5x - 8$ in the form $(x+a)^2 + b$. [1]

(ii) Hence, solve the equation $x^2 - 5x - 8 = 0$, giving your answers correct to two decimal places. [3]

- 2 An ice-cream shop sells 4 different flavours of ice-cream. The table below shows the number of cups of ice-cream sold by the shop over 3 days.

	Chocolate	Vanilla	Strawberry	Durian
Friday	252	168	84	105
Saturday	305	158	115	152
Sunday	316	191	134	167

- (a) Represent the above information using a 3×4 matrix A .

[1]

- (b) The selling price of each cup of ice-cream is shown in the table below.

	Chocolate	Vanilla	Strawberry	Durian
Selling Price	\$2.10	\$1.80	\$2.05	\$2.55

Write down a matrix P such that the product AP represents the total sales of the ice-cream for each day respectively. Evaluate AP .

[2]

- (c) Evaluate $(1 \ 1 \ 1)A$ and state what the elements of the product represents. [2]

- (d) The cost price of each cup of ice-cream is shown in the table below.

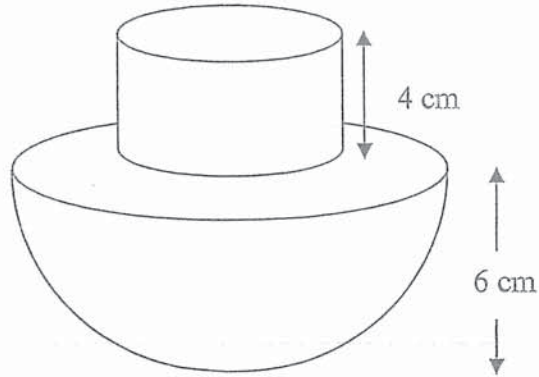
	Chocolate	Vanilla	Strawberry	Durian
Cost Price	\$0.40	\$0.45	\$0.55	\$0.80

Write down two matrices such that the elements of their product represent the total profit received by the shop over the 3 days for the sale of each flavor of ice-cream respectively.

Evaluate this product.

[2]

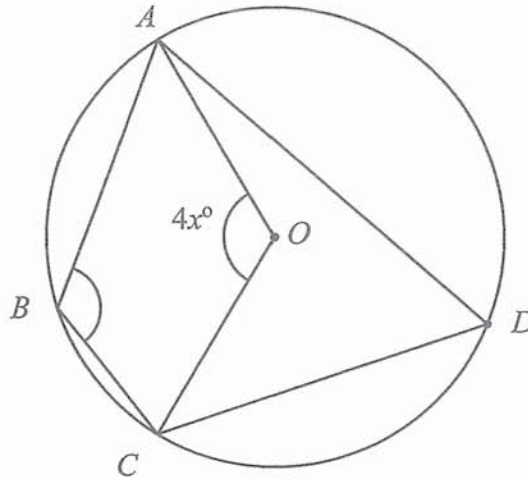
- 3 (a) The diagram shows a toy which is made up of a cylindrical part and a hemispherical part.
The height of the cylindrical part is 4 cm and the radius of the hemispherical part is 6 cm.



- (i) Find the radius of the cylindrical part, given that the volume of the cylindrical part is 150 cm^3 . [2]

- (ii) Calculate total surface area of the toy. [3]

- (b) In the diagram, A , B , C and D lie on a circle with centre O . Angle $AOC = 4x^\circ$.



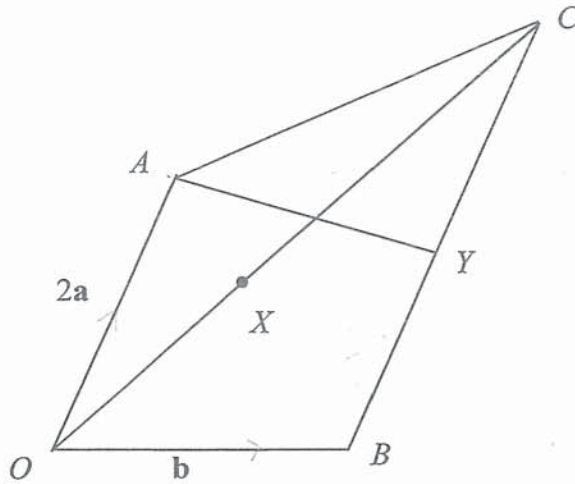
- (i) Find angle ADC , giving your answer in terms of x° . [2]

State a reason for your answer.

- (ii) Given further that angle $ABC = 5x^\circ + 12^\circ$, find the value of x . [2]

- 4 (a) Construct a triangle ABC such that $AB = 13$ cm, $AC = 12$ cm, angle $ABC = 55^\circ$, and angle ACB is acute.
The line AB has been drawn for you. [2]
- (b) Construct the angle bisector of angle BAC such that it intersects the line BC , and label the point of intersection D . [2]
- (c) Construct the perpendicular bisector of AD such that it intersects the line AC , and label the point of intersection E . Measure and write down the value of DE . [2]





In the diagram, $\overline{OA} = 2\mathbf{a}$, $\overline{OB} = \mathbf{b}$. BC is parallel to OA and $BC = \frac{3}{2}OA$. X is a point on OC such that $OX = \frac{2}{3}XC$. Y is the midpoint of BC .

(a) Express in terms of \mathbf{a} and/or \mathbf{b} , as simply as possible,

(i) \overline{AB} , [1]

(ii) \overline{OC} , [1]

(iii) \overline{OX} , [1]

(iv) \overline{AX} . [1]

(b) What can you deduce about the points A , X and B ?
Justify your answer. [2]

(c) (i) AY produced meets OB produced at a point Z . Given that $\overline{AZ} = h\overline{AY}$, express \overline{AZ} in terms of a , b and h . [3]

(ii) Given also that $\overline{OZ} = k\overline{OB}$, express \overline{AZ} in terms of a , b , and k . [1]

(iii) Hence, show that $h = 4$ and $k = 4$. [2]

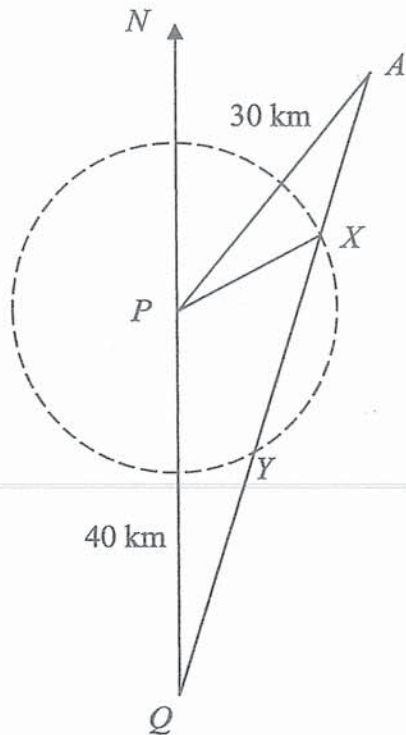
(d) Find the value of

(i) $\frac{\text{area of } \triangle OAX}{\text{area of } \triangle OAC}$, [1]

(ii) $\frac{\text{area of } \triangle OBX}{\text{area of } \triangle ABC}$. [1]

- 6 In the diagram, P and Q are the bases of two lighthouses such that P is located 40 km due north of Q .

A is a boat 30 km from P and on a bearing of 055° from P .



- (a) Find the distance AQ .

[3]

- (b) Find the bearing of A from Q .

[2]

Light from P can be seen within 20 km radius of P , beyond which the light becomes too faint to be seen. When the boat moves from A to Q , X and Y are the positions on the boat's journey which are 20 km from P .

(c) Calculate

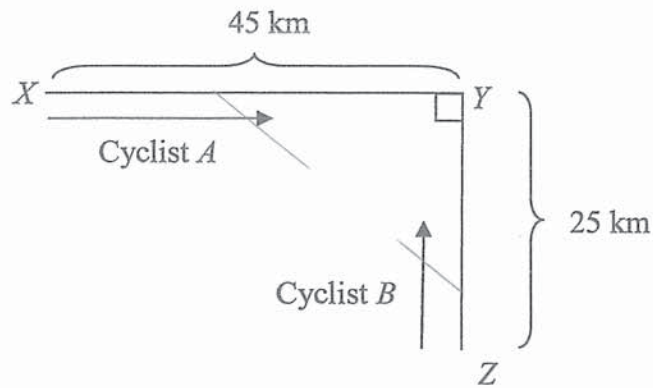
(i) the shortest distance of the boat from P .

[2]

(ii) the **smallest** angle of elevation of the lighthouse P from the boat as it travels from X to Y , given that the height of lighthouse P is 600m.

[2]

- 7 The diagram shows two roads XY and YZ meeting at point Y . The roads are perpendicular to each other. $YX = 45$ km and $YZ = 25$ km.



Cyclist A is travelling from point X towards point Y at a constant speed of 20 km/h.
Cyclist B is travelling from point Z towards point Y at a constant speed of 10 km/h.

- (a) Write down an expression, in terms of t , for the distance in kilometres

(i) between cyclist A and point Y , after t hours, [1]

(ii) between cyclist B and point Y , after t hours. [1]

- (b) Form an expression, in terms of t , for the shortest distance, d , between the two cyclists and show that it reduces to $\sqrt{500t^2 - 2300t + 2650}$.

Hence find the shortest distance between the two cyclists after 15 minutes. [4]

- (c) The two cyclists are 10 km apart at a certain instant, t hours.
Form a quadratic equation in terms of t and show that it reduces to
 $10t^2 - 46t + 51 = 0$.

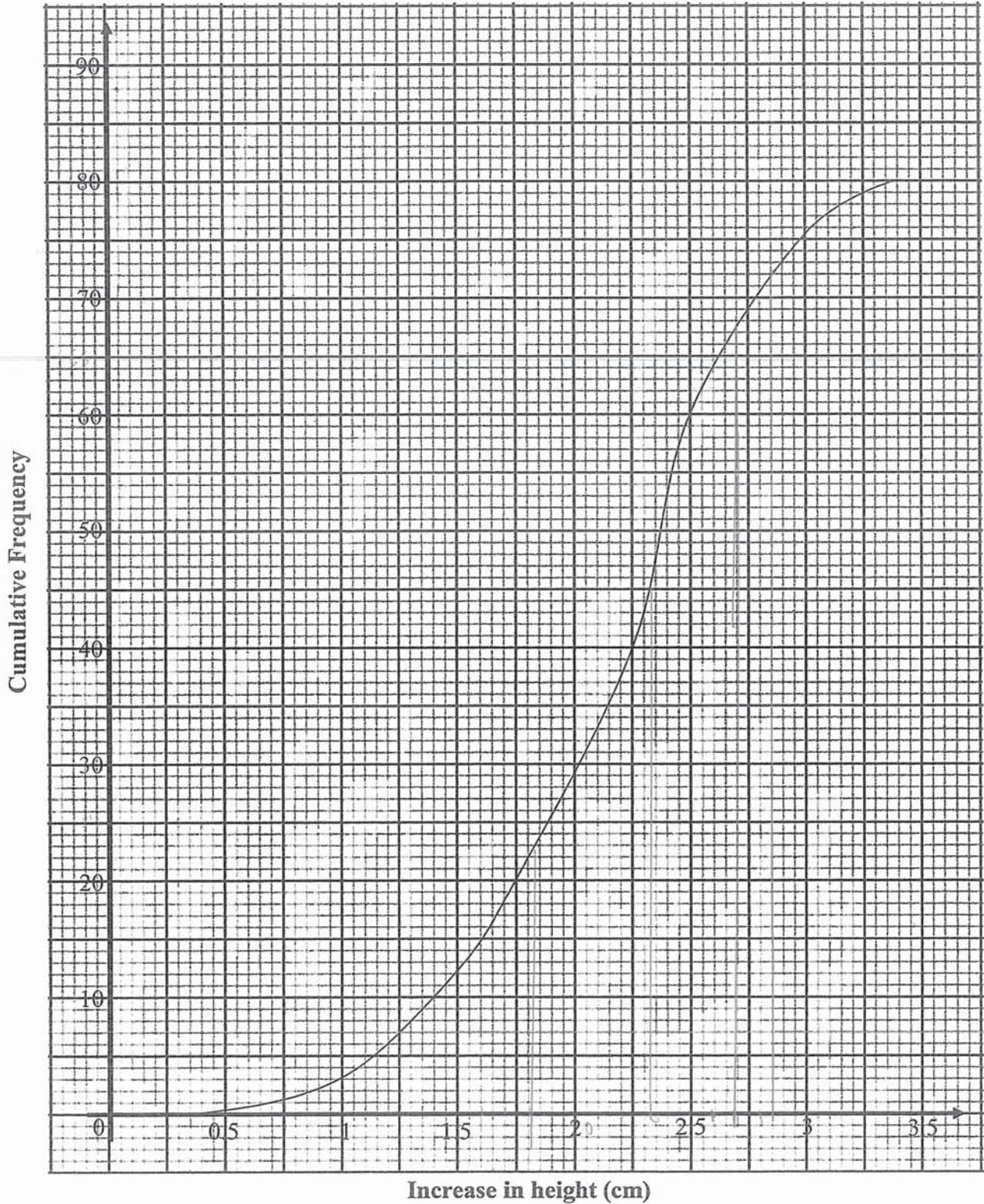
[2]

- (d) Given that $t < 2$, find the time, in minutes, when the two cyclists are 10 km apart.
Correct your answer to 3 significant figures.

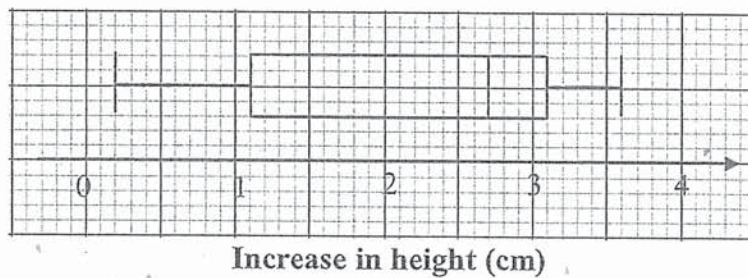
[3]

- 8 A scientist wanted to test the effect of different music on the growth of plants. 80 *Rosa chinensis* plants were exposed to Beethoven's Ninth Symphony, and their growth was observed over a period of 10 days. The cumulative frequency curve below shows the increase in the height of the plants at the end of 10 days.

Increase in heights of plants



- (a) Find
- (i) the median increase in the height of the plants, [1]
- (ii) the interquartile range, [2]
- (iii) the 90th percentile. [2]
- (b) Another group of 80 *Rosa chinensis* plants were exposed to Bach's Goldberg Variations, and their growth were also observed over a period of 10 days. The box-and-whisker plot below shows the increase in the heights of the plants after 10 days.



- (i) Describe how the cumulative frequency graph of the growth of the 80 plants exposed to Bach's Goldberg Variations will differ from that of those exposed to Beethoven's Ninth Symphony. [1]
- (ii) Make two comparisons between the growths of the plants under the two conditions. [2]

- 9 The variables x and y are connected by the equation $y = \frac{1}{2x^2} + 0.1x^2 - 3$. The table below shows some values of x and the corresponding values of y , correct to 2 decimal places.

x	0.5	1	1.5	2	3	4	5	6
y	-0.98	-2.40	-2.55	-2.48	-2.04	-1.37	p	0.61

- (a) Find the value of p . [1]

$$-0.48$$

- (b) In the space provided on the next page, using a scale of 2 cm to represent 1 unit on the horizontal axis and 2 cm to represent 0.5 units on the vertical axis, draw the graph of

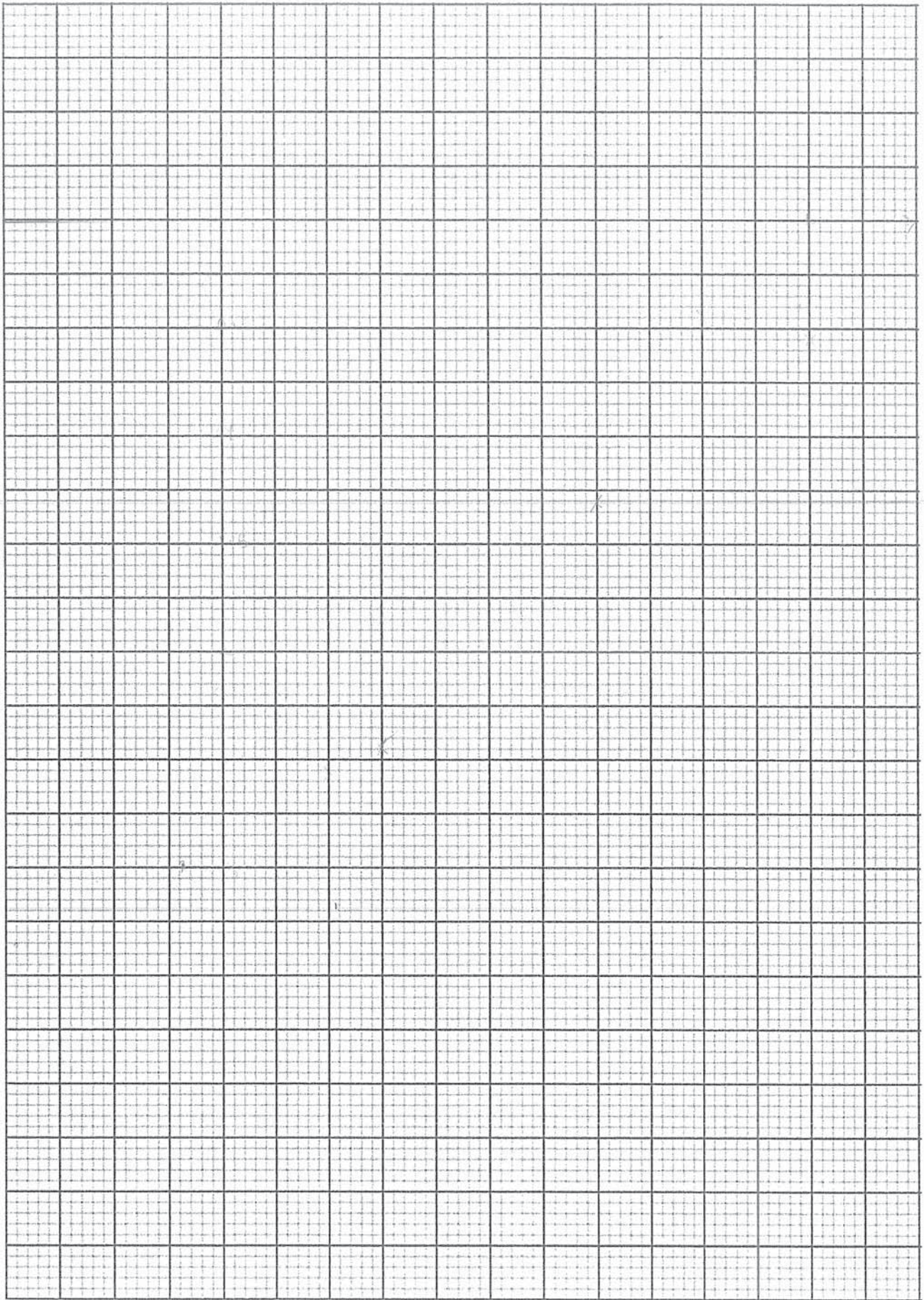
$$y = \frac{1}{2x^2} + 0.1x^2 - 3 \text{ for } 0.5 \leq x \leq 6. \quad [3]$$

- (c) Using your graph, write down the solution(s) to the equation $\frac{1}{2x^2} + 0.1x^2 - 1.5 = 0$. [2]

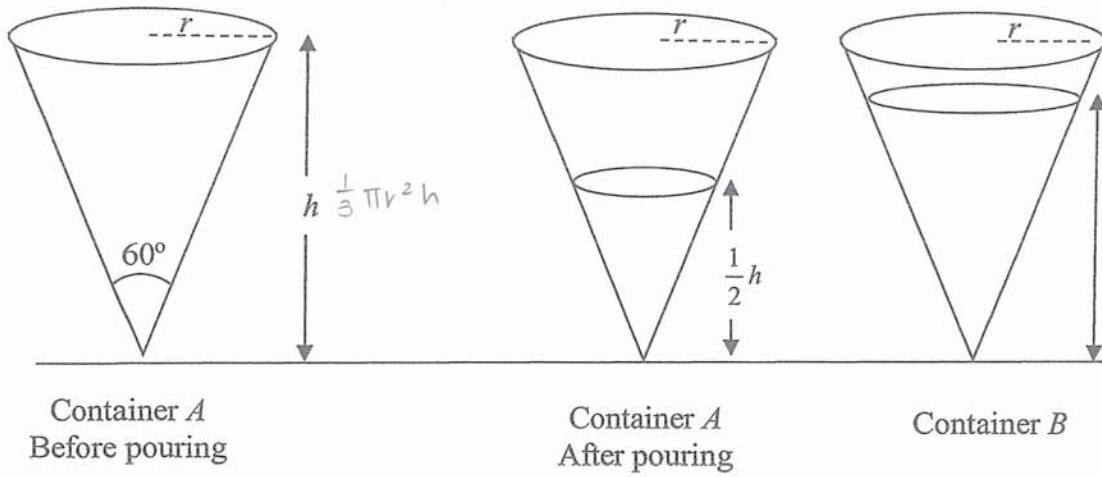
- (d) By drawing a tangent, find the gradient of the curve at $x = 4$. [2]

- (e) By drawing a suitable straight line on the same axes, solve the equation

$$\frac{1}{2x^2} + 0.1x^2 - 0.5x = 0. \quad [3]$$



- 10 A right conical container of capacity $24\pi \text{ cm}^3$ and vertical angle 60° is completely filled with water. The height of the container is $h \text{ cm}$ and the base radius is $r \text{ cm}$.



- (a) Find the value of r and of h .

[5]

The water in the container is poured into another identical container B so that the depth of water in container A is $\frac{1}{2}h$ cm.

(b) Find the volume of water in container B in terms of π .

[2]

- 11 A class of students plans to sell breakfast sets during the National Day carnival in order to raise funds for a charity.

Each breakfast set consists of 2 scrambled eggs, 2 slices of toast, 2 sausages, and a cup of coffee.

The students estimate that they will sell 250 breakfast sets.

Item	Description	Unit cost
Eggs	Pasar Fresh Eggs (10 per pack)	\$1.80
	Pasar Fresh Eggs (30 per pack)	\$4.35
Bread	Sunshine Enriched Soft White Bread (14 slices)	\$1.80
	FairPrice Wholemeal Bread (12 slices)	\$1.55
Sausages	Tierney's Chicken Hot Dog (10 per pack)	\$5.25
	FairPrice Sausages (6 per pack) (\$0.35 off per 2 packs)	\$3.20
Coffee	Nescafe 3 in 1 Instant Coffee (35 per pack) (Buy 5 get 1 free)	\$6.15
	Gold Roast 3 in 1 Coffeemix (25 per pack)	\$3.95

- (a) Find the lowest possible total cost of the ingredients for the breakfast sets. [5]

- (b) The school provides \$200 in funding for the students, and up to 30% of the sales can be used to cover for their expenses, while the remaining goes to the charity. The students also wish to raise at least \$600 for charity. Find the minimum they must charge for each breakfast set (to the nearest ten cents). Justify your answer, showing all necessary workings clearly. State an assumption you have made in your calculations.

[4]

