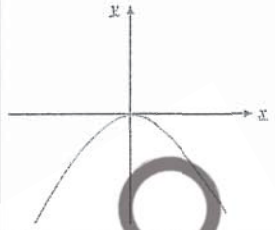
Beatty Secondary School

4E5N Preliminary Examination 2019

1a	$n^2+(n+1)^2$ or $2n^2+2n+1$	B1
1b	For any two consecutive numbers, one would be even and the other one odd. Since the square of an even number is even and the square of an odd number is odd, one of the squares of the two consecutive numbers will be even, and the other will be odd. Hence, their sum will be odd.	B1
2	$\frac{a^2b}{2}\times\left(2a^0b^{-2}\right)^3=\frac{a^2b}{2}\times 8b^{-6}$ $=\frac{4a^2}{b^5}$	M1 A1
3a	$18\left(\frac{4}{3}\right)^2-k\left(\frac{4}{3}\right)-20=0$ $k=9$	[B1]
3b	$18x^2-9x-20=0$ $(3x-4)(6x+5)=0$ $x=\frac{4}{3}$ *or* $x=-\frac{5}{6}$ *(the other solution)*	[B1]
4	$x+5x=180$ $x=30$ $\frac{360}{30}=12$ Number of sides=12	M1 A1
5	Graph 1 – Container B Graph 2 – Container C Graph 3 – Container A	B1 – 1 correct B2 – All correct
6a	$b=2$ $n=1$ Range of a: $a>0$	B1 for both answers correct B1

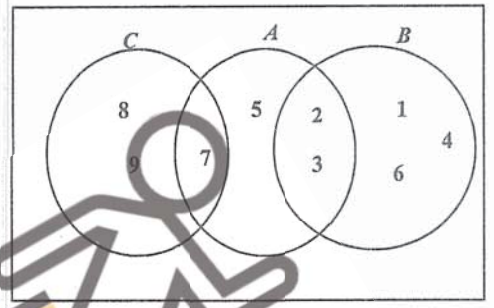
6b		B1
7	$3+y\le 15-2y$ *and* $15-2y<y+10$ $3y\le 12$ $5<3y$ $y\le 4$ $y>\frac{5}{3}$ Largest prime number is 3.	M1 for each correct answer A1
8	$8x+3y=4$ ---------- (1) $4x=y+4$ ---------- (2) *From* (2), $y=4x-4$ ---- (3) *Sub* (3) in (1), $8x+3(4x-4)=4$ $20x=16$ $x=\frac{4}{5}$ $y=4\times\frac{4}{5}-4$ $=-\frac{4}{5}$	 M1 A1 A1
9a	$\lvert q-2p\rvert$ $=\left\lvert\begin{pmatrix}-4\\3\end{pmatrix}-2\begin{pmatrix}6\\5\end{pmatrix}\right\rvert$ $=\left\lvert\begin{pmatrix}-4\\3\end{pmatrix}-\begin{pmatrix}12\\10\end{pmatrix}\right\rvert$ $=\left\lvert\begin{pmatrix}-16\\-7\end{pmatrix}\right\rvert$ $=\left\lvert\sqrt{(-16)^2+(-7)^2}\right\rvert$ $=\lvert\sqrt{305}\rvert$ $=17.464$ units (5 *s.f.*) $=17.5$ units (3 *s.f.*)	 M1 A1

9b	If **r** is parallel to **q**, $\mathbf{r} = k\mathbf{q}$ Now, $\begin{pmatrix} 8 \\ -6 \end{pmatrix} = -2\begin{pmatrix} -4 \\ 3 \end{pmatrix}$, i.e. $\mathbf{r} = -2\mathbf{q}$ or $\mathbf{q} = -\frac{1}{2}\mathbf{r}$ Hence, **r** is parallel to **q**.	B1
10	$\frac{9m^2-12mn+4n^2}{m^2-n^2} \div \frac{3m-2n}{m-n}$ $= \frac{(3m-2n)(3m-2n)}{(m-n)(m+n)} \div \frac{3m-2n}{m-n}$ $= \frac{(3m-2n)(3m-2n)}{(m-n)(m+n)} \times \frac{m-n}{3m-2n}$ $= \frac{3m-2n}{m+n}$	M1 – correct factorisation M1 – for changing divide to multiplication and interchange the numerator with the denominator of the second fraction. A1
11a	Area Scale 1 : 100 Linear Scale 1 : 10 Length of the house = 3 x 10 = 30 m	B1
11b	Volume of the model = $\left(\frac{1}{10}\right)^3 \times 15000000$ $= 15000\ cm^3$	M1 A1
12a	$\frac{3}{8} = \frac{30}{80}$ $80 - 30 - 18 = 32$	M1 A1
12bi	$\frac{30}{80} \times \frac{18}{79} + \frac{18}{80} \times \frac{30}{79}$ $= \frac{27}{158}$	M1 A1
12bii	P(at least 1 red ball) = 1 – P(no red ball) $1 - \left(\frac{62}{80}\right)\left(\frac{61}{79}\right)$ $= \frac{1269}{3160}$ OR	B1
	P(at least 1 red ball) =1 – P(GG) – P(GB) – P(BG) – P(BB) $1 - \left(\frac{30}{80}\right)\left(\frac{29}{79}\right) - \left(\frac{30}{80}\right)\left(\frac{32}{79}\right) - \left(\frac{32}{80}\right)\left(\frac{30}{79}\right) - \left(\frac{32}{80}\right)\left(\frac{31}{79}\right)$ $= \frac{1269}{3160}$ OR P(at least 1 red ball) =P(GR) + P(RG) + P(RR) + P(RB) + P(BR) $\left(\frac{30}{80}\right)\left(\frac{18}{79}\right) + \left(\frac{18}{80}\right)\left(\frac{30}{79}\right) + \left(\frac{18}{80}\right)\left(\frac{17}{79}\right) + \left(\frac{18}{80}\right)\left(\frac{32}{79}\right) + \left(\frac{32}{80}\right)\left(\frac{18}{79}\right)$ $= \frac{1269}{3160}$	
13a	$x = -2$	B1
13b	$y = 3x + c$ *Sub* $(2, -5)$, $-5 = 3(2) + c$ $c = -11$ $y = 3x - 11$	M1 A1
14a	$BC = \sqrt{y^2 - x^2}$ $\tan \angle BDC = \frac{\sqrt{y^2 - x^2}}{x}$	M1 A1
14b	$\cos \angle ABD = -\cos \angle DBC = -\frac{\sqrt{y^2 - x^2}}{y}$	B1
15a	*Total sales* $= (3694 - 1200) \times \frac{100}{8}$ = \$31175	M1 A1
15b	Tap *X*: $2r$ cm^3/min Tap *Y*: r cm^3/min In 5 min, $5(2r) + 5r = 300$ $r = 20$ Rate of Tap *Y* = 20 cm^3/min OR	M1 A1

	$V_X : V_Y : 2 : 1$ Volume of water in 5 min $= \frac{300}{3}$ $= 100\ cm^3$ Rate $= \frac{100}{5}$ $= 20\ cm^3/min$	 M1 A1
16a	6	B1
16b	$3+5+4-1 = x+2$ $x = 9$	 B1
16c	$\frac{3(0)+5(1)+4(2)+3x+2(4)}{14+x} = 2\frac{1}{22}$ $\frac{21+3x}{14+x} = \frac{45}{22}$ $462+66x = 630+45x$ $21x = 168$ $x = 8$	M1 A1
17a	$Total\ dist = \frac{1}{2}v(10+35)$ $540 = 22.5v$ $v = 24 m/s$	M1 A1
17b	$a = \frac{24}{10}$ $= 2.4\ m/s^2$ Speed $= 24 - 3(2.4)$ $= 16.8$ m/s	M1 A1
18a	$\frac{H_L}{H_S} = \sqrt{\frac{242}{98}}$ $= \frac{11}{7}$	 B1
18b	$\frac{A_S}{A_L} = \left(\frac{7}{11}\right)^2$ $\frac{343}{A_L} = \frac{49}{121}$ $A_L = \frac{121}{49} \times 343$ $A_L = 847\ cm^2$	 B1

18c	$\frac{V_S}{V_L} = \left(\frac{7}{11}\right)^3$ $\frac{V_S}{V_L} = \frac{343}{1331}$ $\%\ difference = \frac{1331-343}{1331} \times 100\%$ $= 74.229\%\ (5\ s.f)$ $= 74.2\%\ (3\ s.f)$	 M1 A1

19a	$s = kv^2$ $36 = kv^2$ $s = k(\frac{v}{2})^2$ $s = 9$	M1 A1
19b	$y = \frac{k}{x}$ $k = 24$ $y = \frac{24}{9}$ $y = 2\frac{2}{3}$	M1 A1
20a	$35 = 5 \times 7$ $3^x \times 5^y \times 7^3$ needs to have a factor of 5 and 7. Therefore, $x = 0$, $y = 1$	 B1 – if 1 correct B2 – If 2 correct
20b	$55q$ $= 5 \times 11 \times q$ $= 5 \times 11 \times 5 \times 7^2 \times 11$ $= 5^2 \times 7^2 \times 11^2$ Since the indices of the prime factors are multiples of 2, $55q$ is a perfect square.	 B1
20c	$p \times q = 3^x \times 5^y \times 7^3 \times 5 \times 7^2 \times 11$ $= 3^x \times 5^{y+1} \times 7^5 \times 11$ Since p x q is a product of prime factors that are odd, p x q is an odd number.	 B1
21a	$(P \cup Q)' \cup (P \cap Q)$ *or* $(P' \cap Q') \cup (P \cap Q)$	B1
21bi	$B \cap C = \varnothing$ *or* $\{\}$	B1
21bii	$(A \cup B)' = \{8,9\}$	B1

21biii		B1 – 5 out of 9 correct B2 – All correct
22a	$\angle QXY = \angle QRS$ (*corr.* $\angle s$, XY // RS) $\angle QYX = \angle QSR$ (*corr.* $\angle s$, XY // RS) $\angle RQS = \angle XQY$ (*common* $\angle s$) ΔQXY *is similar to* ΔQRS. (*AA*)	B1- Either 2 of these reasons A1
22bi	Since ΔQXY is similar ΔQRS, $\frac{Area\ of\ \Delta QRS}{Area\ of\ \Delta QXY} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$ $Area\ of\ \Delta QRS = \frac{25}{9} \times 27 = 75\ cm^2$	 M1, A1
22bii	Consider ΔPQR and ΔPXR: $\frac{Area\ of\ \Delta PQR}{Area\ of\ \Delta PXR} = \frac{\frac{1}{2} \times QR \times h}{\frac{1}{2} \times XR \times h} = \frac{QR}{XR} = \frac{5}{2}$ $Area\ of\ \Delta PQR = \frac{5}{2} \times 46 = 115\ cm^2$	 M1, A1

23a	$EC = \sqrt{4^2 + 2^2}$ $= \sqrt{20}$ Perimeter of sector *CEF* $= 2\sqrt{20} + \sqrt{20}(0.64)$ $\approx 11.8\ cm$	M1 M1 A1
23b	Area of sector *CEF* $= \frac{1}{2}\left(\sqrt{20}\right)^2(0.64)$ $= 6.4\ cm^2$ Area of shaded region $= 4^2 - 2\left(\frac{1}{2} \times 4 \times 2\right) - 6.4$ $= 1.6\ cm^2$	 M1 A1 M1 A1

Answer Key

1(a)	$\frac{t^4}{4v^3}$
1(b)(i)	$\frac{25x}{3y^2}$
1(b)(ii)	$\frac{13-11x}{(3x-1)(2+x)}$
1(c)(i)	$\left(x-\frac{5}{2}\right)^2-\frac{57}{4}$
1(c)(ii)	$x=6.27$ or -1.27
2(a)	$\mathbf{A}=\begin{pmatrix}252 & 168 & 84 & 105\\ 305 & 158 & 115 & 152\\ 316 & 191 & 134 & 167\end{pmatrix}$
2(b)	$\mathbf{P}=\begin{pmatrix}2.10\\ 1.80\\ 2.05\\ 2.55\end{pmatrix}$ $\mathbf{AP}=\begin{pmatrix}1271.55\\ 1548.25\\ 1707.95\end{pmatrix}$
2(c)	The elements represents the total number of cups of ice-cream sold over the 3 days of each flavor respectively.
2(d)	(1484.10 697.95 499.50 742)
3(a)(i)	$r=3.45$ cm
3(a)(ii)	426 cm^2
3(b)(i)	$2x^o$
3(b)(ii)	$x^o=24^o$
4(c)	6.3 cm
5(a)(i)	$\mathbf{b}-2\mathbf{a}$
5(a)(ii)	$\mathbf{b}+3\mathbf{a}$
5(a)(iii)	$\frac{2}{5}(\mathbf{b}+3\mathbf{a})$
5(a)(iv)	$-\frac{4}{5}\mathbf{a}+\frac{2}{5}\mathbf{b}$
5(b)	$\overrightarrow{AB}=\frac{5}{2}\overrightarrow{AX}$ A, X, and B are collinear.
5(c)(i)	$\overrightarrow{AZ}=h\mathbf{b}-\frac{1}{2}h\mathbf{a}$
5(c)(ii)	$\overrightarrow{AZ}=-2\mathbf{a}+k\mathbf{b}$
5(c)(iii)	*(AG)*

5(d)(i)	$\frac{2}{5}$
5(d)(ii)	$\frac{2}{5}$
6(a)	62.3 km
6(b)	023.2°
6(c)	15.8 km
6(d)	1.7^{o}
7(a)(i)	$(45-20t)$ km
7(a)(ii)	$(25-10t)$ km
7(b)	45.9 km
7(c)	*(AG)*
7(d)	112 minutes
8(a)(i)	2.25 cm
8(a)(ii)	0.75 cm
8(a)(iii)	2.85 cm
8(b)(i)	The cumulative frequency graph of the growth of plants exposed to Bach will be **less steep in the middle/steeper at the upper quartile/wider range/broader range** compared to that of plants exposed to Beethoven. **OR** The middle of the cumulative frequency graph of the growth of plants exposed to Bach is **shifted to the right** compared to that of plants exposed to Beethoven.
8(b)(ii)	The plants grow **better** when exposed to Bach **on average** compared to when exposed to Beethoven as the **median increase in heights** of the plants when exposed to Bach **is higher** than those exposed to Beethoven. The increase in heights of the plants are **more spread** when exposed to Bach than when exposed to Beethoven as the **interquartile range is larger** when exposed to Bach than when exposed to Beethoven.
9(a)	−0.48
9(b)	*(graph)*
9(c)	0.6 or 3.8
9(d)	0.792
9(e)	1.1 or 4.95
10(a)	$h = 6$ cm $r = 3.46$ cm
10(b)	21π
11(a)	$428.65
11(b)	Any value above $3.3146 Assume no cost incurred for cooking. (or any reasonable)

BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019

MARKING SCHEME

SUBJECT :	Mathematics	LEVEL :	Sec 4 Express Sec 5 Normal (Academic)
PAPER :	4048 / 02	DURATION :	2 hours 30 minutes
SETTER :	Mr Teo CK Miss Chong HY	DATE :	29 August 2019

CLASS :	NAME :	REG NO :

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces on the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

Mathematical Formulae

Compound Interest

Total amount = $P(1+\frac{r}{100})^n$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where is in radians

Sector area = $\frac{1}{2}r^2\theta$, where is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 (a) Simplify $\left(\frac{64v^9}{t^{12}}\right)^{-\frac{1}{3}}$. [2]

$$\left(\frac{64v^9}{t^{12}}\right)^{-\frac{1}{3}}$$
$$=\left(\frac{t^{12}}{64v^9}\right)^{\frac{1}{3}} \text{----}M1$$
$$=\frac{t^4}{4v^3} \text{----}A1$$

(b) Express as a single fraction in its simplest form.

(i) $\frac{20x^2}{9} \div \frac{4xy^3}{15y}$. [1]

$$\frac{20x^2}{9} \div \frac{4xy^3}{15y}$$
$$=\frac{20x^2}{9} \times \frac{15y}{4xy^3}$$
$$=\frac{25x}{3y^2} \text{--------}B1$$

(ii) $\frac{4}{3x-1} - \frac{5}{2+x}$. [2]

$$\frac{4}{3x-1} - \frac{5}{2+x}$$
$$=\frac{4(2+x)-5(3x-1)}{(3x-1)(2+x)} \text{-----}M1$$
$$=\frac{13-11x}{(3x-1)(2+x)} \text{---------}A1$$

(c) (i) Express x^2-5x-8 in the form $(x+a)^2+b$. [1]

$$x^2-5x-8$$
$$=\left(x-\frac{5}{2}\right)^2-\left(-\frac{5}{2}\right)^2-8$$
$$=\left(x-\frac{5}{2}\right)^2-\frac{57}{4} \text{----}B1$$

(ii) **Hence**, solve the equation $x^2-5x-8=0$, giving your answers correct to two decimal places. [3]

$$x^2-5x-8=0$$
$$\left(x-\frac{5}{2}\right)^2-\frac{57}{4}=0$$
$$\left(x-\frac{5}{2}\right)^2=\frac{57}{4} \text{----}M1 \text{ (ECF their (ci))}$$
$$x-\frac{5}{2}=\pm\sqrt{\frac{57}{4}}$$
$$x=6.27 \text{ or } -1.27 \text{----}A2$$

2 An ice-cream shop sells 4 different flavours of ice-cream. The table below shows the number of cups of ice-cream sold by the shop over 3 days.

	Chocolate	Vanilla	Strawberry	Durian
Friday	252	168	84	105
Saturday	305	158	115	152
Sunday	316	191	134	167

(a) Represent the above information using a 3×4 matrix A. [1]

$$A=\begin{pmatrix}252 & 168 & 84 & 105\\ 305 & 158 & 115 & 152\\ 316 & 191 & 134 & 167\end{pmatrix}$$ --- B1

(b) The selling price of each cup of ice-cream is shown in the table below.

	Chocolate	Vanilla	Strawberry	Durian
Selling Price	\$2.10	\$1.80	\$2.05	\$2.55

Write down a matrix **P** such that the product **AP** represents the total sales of all the ice-cream for each day respectively. Evaluate **AP**. [2]

$$P=\begin{pmatrix}2.10\\ 1.80\\ 2.05\\ 2.55\end{pmatrix}$$ --- B1

$$AP=\begin{pmatrix}252 & 168 & 84 & 105\\ 305 & 158 & 115 & 152\\ 316 & 191 & 134 & 167\end{pmatrix}\begin{pmatrix}2.10\\ 1.80\\ 2.05\\ 2.55\end{pmatrix}$$

$$=\begin{pmatrix}1271.55\\ 1548.25\\ 1707.95\end{pmatrix}$$ --- B1

(for **P** given as row matrix, B0)

(c) Evaluate $\begin{pmatrix}1 & 1 & 1\end{pmatrix}A$ and state what the elements of the product represents. [2]

$$\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}252 & 168 & 84 & 105\\ 305 & 158 & 115 & 152\\ 316 & 191 & 134 & 167\end{pmatrix}=\begin{pmatrix}873 & 517 & 333 & 424\end{pmatrix}$$ --- B1

The elements represents the total number of cups of ice-cream sold over the 3 days of each flavor respectively. --- B1

(d) The cost price of each cup of ice-cream is shown in the table below.

	Chocolate	Vanilla	Strawberry	Durian
Cost Price	\$0.40	\$0.45	\$0.55	\$0.80

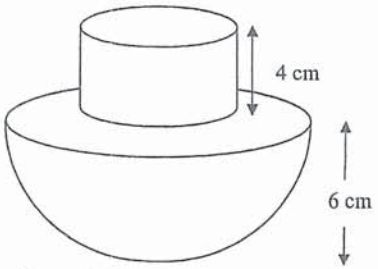
Write down two matrices such that the elements of their product represent the total profit received by the shop over the 3 days for the sale of each flavor of ice-cream respectively.
Evaluate this product. [2]

$$\begin{pmatrix}873 & 517 & 333 & 424\end{pmatrix}\begin{pmatrix}1.70 & 0 & 0 & 0\\ 0 & 1.35 & 0 & 0\\ 0 & 0 & 1.50 & 0\\ 0 & 0 & 0 & 1.75\end{pmatrix}$$ --- M1 for correct 4×4 matrix

$$=\begin{pmatrix}1484.10 & 697.95 & 499.50 & 742\end{pmatrix}$$ --- A1

3 (a) The diagram shows a toy which is made up of a cylindrical part and a hemispherical part.

The height of the cylindrical part is 4 cm and the radius of the hemispherical part is 6 cm.



(i) Find the radius of the cylindrical part, given that the volume of the cylindrical part is 150 cm^3. [2]

$\pi \times r^2 \times 4 = 150 ----M1$

$r^2 = \dfrac{150}{\pi \times 4}$

$r = 3.4549...$

$r = 3.45$ cm (3sf) $----A1$

(ii) Calculate total surface area of the toy. [3]

SA of hemisphere $= 2 \times \pi \times 6^2 + \pi \times 6^2 = 108\pi ----$ M1 (ECF their (i))

Total SA $= 108\pi + 2 \times \pi \times 3.4549... \times 4 ----$ M1 (ECF their (i))

$= 426.178... = 426$ cm^2 $----A1$

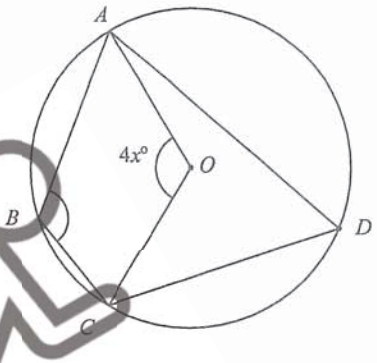
OR

SA of hemisphere $= 3\pi(6)^2 = 108\pi$ --- M1

SA of cylinder $= \pi(3.4549)^2 + 2\pi(3.4549)(4) = 39.575\pi$ --- M1

TSA $= 108\pi + 39.575\pi - \pi(3.4549)^2 = 426$ cm^2 --- A1

(b) In the diagram, A, B, C and D lie on a circle with centre O. Angle $AOC = 4x°$.



(i) Find angle ADC, giving your answer in terms of $x°$. [2]

State a reason for your answer.

angle ADC=2*x*° (angle at centre = 2 angle at circumference) $----B2$

(ii) Given further that angle $ABC = 5x° + 12°$, find the value of x. [2]

$5x° + 12° = 180° - 2x° ----M1$

$7x° = 168°$

$x° = 24° ----A1$

OR

$2(5x + 12) = 360 - 4x$ --- M1

$10x + 24 = 360 - 4x$

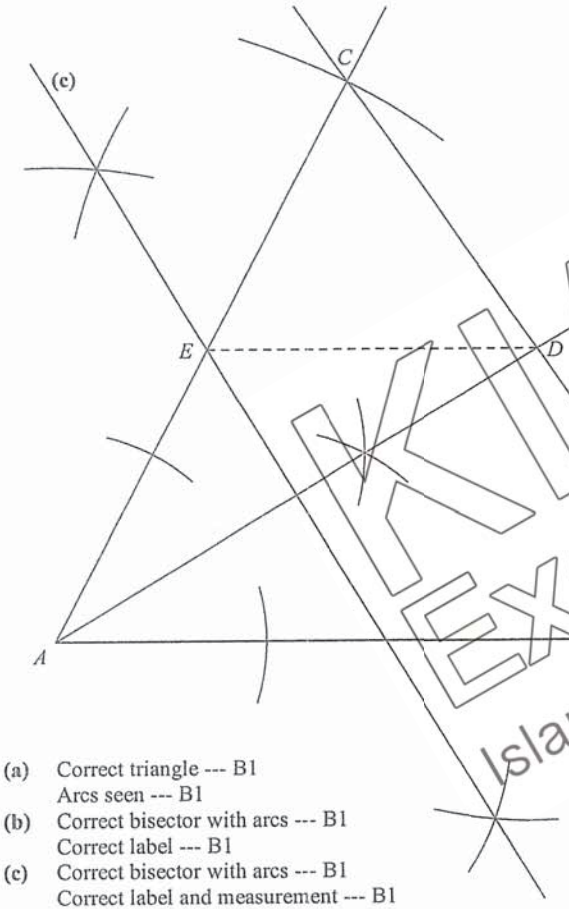
$14x = 336$

$x = 24$ --- A1

4 (a) Construct a triangle ABC such that $AB = 13\,\text{cm}$, $AC = 12\,\text{cm}$, angle $ABC = 55°$, and angle ACB is acute.
The line AB has been drawn for you. [2]

(b) Construct the angle bisector of angle BAC such that it intersects the line BC, and label the point of intersection D. [2]

(c) Construct the perpendicular bisector of AD such that it intersects the line AC, and label the point of intersection E. Measure and write down the value of DE. [2]



(a) Correct triangle --- B1
Arcs seen --- B1

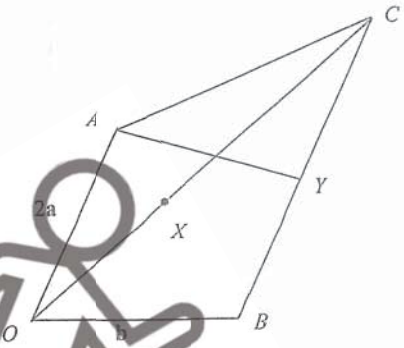
(b) Correct bisector with arcs --- B1
Correct label --- B1

(c) Correct bisector with arcs --- B1
Correct label and measurement --- B1

No question labels = minus 1 overall

(c) 6.3 cm (±0.1)
(actual = 6.24 cm

5



In the diagram, $\overrightarrow{OA} = 2\mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$. BC is parallel to OA and $BC = \frac{3}{2}OA$. X is a point on OC such that $OX = \frac{2}{3}XC$. Y is the midpoint of BC.

(a) Express in terms of $\mathbf{a}$ and/or $\mathbf{b}$, as simply as possible,

(i) $\overrightarrow{AB}$, [1]

$\overrightarrow{AB} = \mathbf{b} - 2\mathbf{a}$ ------B1

(ii) $\overrightarrow{OC}$ [1]

$\overrightarrow{OC} = \mathbf{b} + 3\mathbf{a}$ ------B1

(iii) $\overrightarrow{OX}$, [1]

$\overrightarrow{OX} = \frac{2}{5}(\mathbf{b} + 3\mathbf{a})$ oe------B1

(iv) $\overrightarrow{AX}$. [1]

$\overrightarrow{AX} = \overrightarrow{AO} + \overrightarrow{OX}$

$= -2\mathbf{a} + \frac{2}{5}\mathbf{b} + \frac{6}{5}\mathbf{a}$

$= -\frac{4}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$ -------M1

(b) Write down two facts about the points A, X and B. [2]

$\overrightarrow{AB} = \frac{5}{2}\left(\frac{2}{5}\mathbf{b} - \frac{4}{5}\mathbf{a}\right)$

$= \frac{5}{2}\overrightarrow{AX}$ ------B1

A, X, and B are collinear. --- B1

(c) (i) AY produced meets OB produced at a point Z. Given that $\overrightarrow{AZ} = h\overrightarrow{AY}$, express $\overrightarrow{AZ}$ in terms of a, b and h. [3]

$\overrightarrow{AZ} = h\overrightarrow{AY}$

$\overrightarrow{AY} = \overrightarrow{AB} + \overrightarrow{BY}$ ------$M1$

$= \mathbf{b} - \frac{1}{2}\mathbf{a}$ -------$M1$

$\overrightarrow{AZ} = \mathrm{h}\mathbf{b} - \frac{1}{2}\mathrm{h}\mathbf{a}$ ----$A1$

(ii) Given also that $\overrightarrow{OZ} = k\overrightarrow{OB}$, express $\overrightarrow{AZ}$ in terms of a, b,and k. [1]

$\overrightarrow{AZ} = -2\mathbf{a} + k\mathbf{b}$ ----B1

(iii) Hence, show that $h = 4$ and $k = 4$. [2]

$\mathrm{b}\mathbf{h} - \frac{1}{2}\mathrm{h}\mathbf{a} = 2\mathbf{a} + k\mathbf{b}$

Comparing

$h = 4, k = 4$ ---------A1, A1

(d) Find the value of

(i) $\frac{\text{area of } \Delta OAX}{\text{area of } \Delta OAC}$, [1]

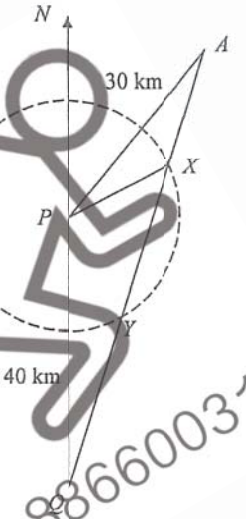
$\frac{\text{area of } \Delta OAX}{\text{area of } \Delta OAC} = \frac{2}{5}$ ----$B1$

(ii) $\frac{\text{area of } \Delta OBX}{\text{area of } \Delta ABC}$. [1]

$\frac{\text{area of } \Delta OBX}{\text{area of } \Delta ABC} = \frac{3}{5} \times \frac{2}{3} = \frac{2}{5}$ ----$B1$

6 In the diagram, P and Q are the bases of two lighthouses such that P is located 40 km due north of Q.

A is a boat 30 km from P and on a bearing of 055° from P.



(a) Find the distance AQ. [3]

$AQ^2 = 30^2 + 40^2 - 2(30)(40)\cos 125^\circ$ ----$M1M1$

$= 2500 - 2400\cos 125^\circ$

$AQ = 62.26221525...$

$AQ = 62.3$ km(3 sf) -------$A1$

(b) Find the bearing of A from Q. [2]

$\frac{\sin \angle PQA}{30} = \frac{\sin \angle APQ}{62.26221525}$

$\sin \angle PQA = \frac{30 \times \sin 125^\circ}{62.26221525}$

$\angle PQA = 23.2469296^\circ$

Bearing of A from $Q = 023.2^\circ$.

Light from P can be seen within 20 km radius of P, beyond which the light becomes too faint to be seen. When the boat moves from A to Q, X and Y are the positions on the boat's journey which are 20 km from P.

(c) Calculate

(i) the shortest distance of the boat from P. [2]

$\angle PQX = \angle PQA = 23.2469296^o$

$\sin \angle PQX = \frac{h}{40}$ ------$M1$

$h = 15.78778476$ km

$h = 15.8$ km -----$A1$

(ii) the **smallest** angle of elevation of the lighthouse P from the boat as it travels from X to Y, given that the height of lighthouse P is 600m. [2]

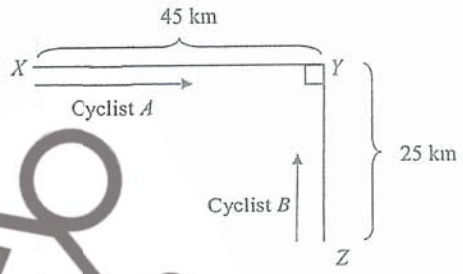
smallest angle of elevation

$= \tan^{-1}\left(\frac{600}{20000}\right)$ -----$M1$

$= 1.7183...$

$= 1.7^o$ -----$A1$

7 The diagram shows two roads XY and YZ meeting at point Y. The roads are perpendicular to each other. $YX = 45$ km and $YZ = 25$ km.



Cyclist A is travelling from point X towards point Y at a constant speed of 20 km/h.
Cyclist B is travelling from point Z towards point Y at a constant speed of 10 km/h.

(a) Write down an expression, in terms of t, for the distance in kilometres

(i) between cyclist A and point Y, after t hours, [1]

$(45 - 20t)$ km

(ii) between cyclist B and point Y, after t hours. [1]

$(25 - 10t)$ km

(b) Form an expression, in terms of t, for the shortest distance, d, between the two cyclists and show that it reduces to $\sqrt{10t^2 - 46t + 53}$.
Hence find the shortest distance between the two cyclists after 15 minutes. [4]

$d^2 = (45 - 20t)^2 + (25 - 10t)^2$ ----------$M1$

$d = \sqrt{2025 - 1800t + 400t^2 + 625 - 500t + 100t^2}$

$= \sqrt{500t^2 - 2300t + 2650}$ ----------$A1$

At $t = 0.25$ hour,

$d = \sqrt{500(0.25)^2 - 2300(0.25) + 2650}$ -------$M1$

$= 45.893...$

$= 45.9$ km -------$A1$

(c) The two cyclists are 10 km apart at a certain instant, t hours.
Form a quadratic equation in terms of t and show that it reduces to $10t^2 - 46t + 51 = 0$. [2]

$10 = \sqrt{500t^2 - 2300t + 2650}$ ---------M1

$100 = 500t^2 - 2300t + 2650$

$500t^2 - 2300t + 2550 = 0$

$20t^2 - 92t + 102 = 0$

$10t^2 - 46t + 51 = 0$ -----B1

(d) Given that $t < 2$, find the time, in minutes, when the two cyclists are 10 km apart.
Correct your answer to 3 significant figures. [3]

$10t^2 - 46t + 51 = 0$

$t = \dfrac{-(-46) \pm \sqrt{(-46)^2 - 4(10)(51)}}{2(10)}$ ----M1

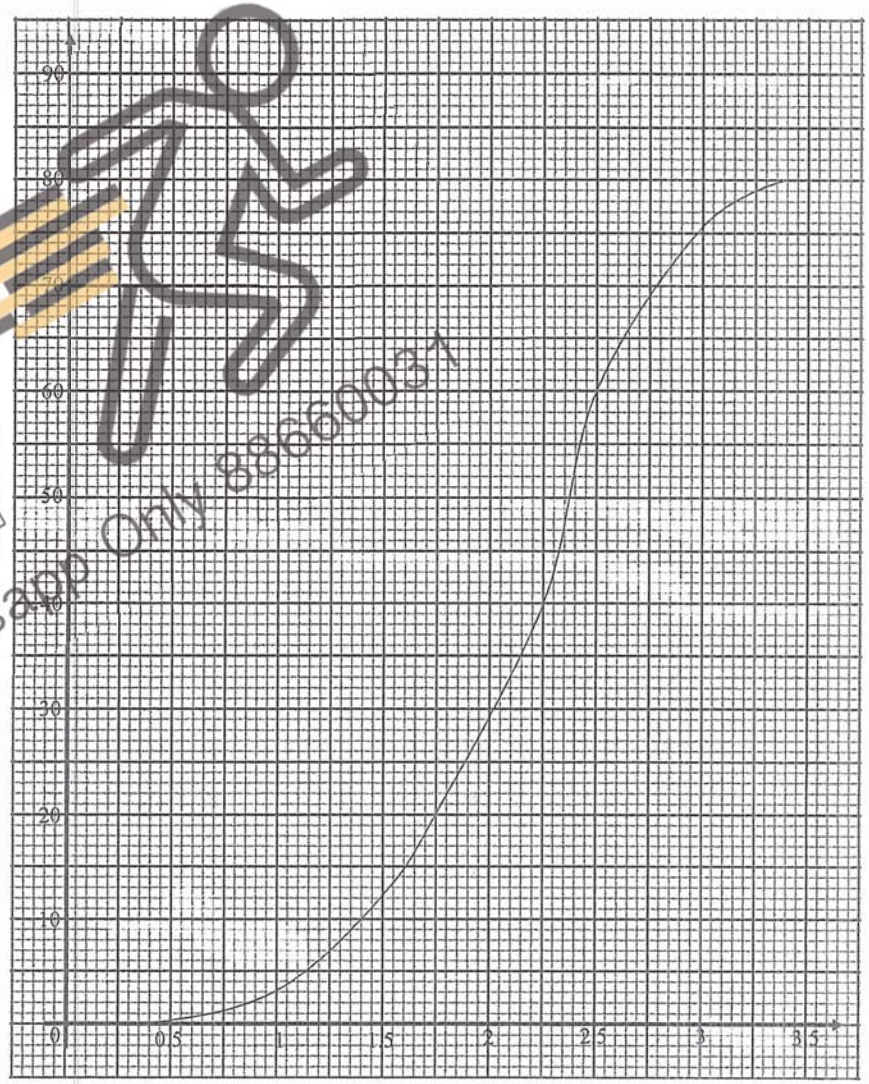
$= \dfrac{46 \pm \sqrt{76}}{20}$

$= 2.73588...$ *or* $1.8641...$ ------A1

Hence $t = 1.8641...$ = 112 minutes (3 sf). ----------A1

8 A scientist wanted to test the effect of different music on the growth of plants.
80 *Rosa chinensis* plants were exposed to Beethoven's Ninth Symphony, and their growth were observed over a period of 10 days.
The cumulative frequency curve below shows the increase in the height of the plants at the end of 10 days.

Increase in heights of plants



(a) Find

(i) the median increase in the height of the plants, [1]

2.25 cm --- B1

(ii) the interquartile range, [2]

$2.5 - 1.75 = 0.75$ cm --- M1, A1

(iii) the 90th percentile. [2]

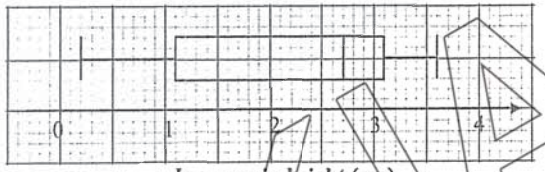
90^{th} percent $= \frac{90}{100} \times 80$ --- M1

$= 72$

90^{th} percentile $= 2.85$ cm --- A1

(all answers ±0.025)

(b) Another group of 80 *Rosa chinensis* plants were exposed to Bach's Goldberg Variations, and their growth were also observed over a period of 10 days.
The box-and-whisker plot below shows the increase in the heights of the plants after 10 days.



Increase in height (cm)

(i) Describe how the cumulative frequency graph of the growth of the 80 plants exposed to Bach's Goldberg Variations will differ from that of those exposed to Beethoven's Ninth Symphony. [1]

The cumulative frequency graph of the growth of plants exposed to Bach will be **less steep in the middle/steeper at the upper quartile/wider range/broader range** compared to that of plants exposed to Beethoven. --- B1

OR

The middle of the cumulative frequency graph of the growth of plants exposed to Bach is **shifted to the right** compared to that of plants exposed to Beethoven.- B1

(ii) Make two comparisons between the growths of the plants under the two conditions. [2]

The plants grow **better** when exposed to Bach **on average** compared to when exposed to Beethoven as the **median increase in heights** of the plants when exposed to Bach **is higher** than those exposed to Beethoven. --- B1
The increase in heights of the plants are **more spread** when exposed to Bach than when exposed to Beethoven as the **interquartile range is larger** when exposed to Bach than when exposed to Beethoven. --- B1

9 The variables x and y are connected by the equation $y = \frac{1}{2x^2} + 0.1x^2 - 3$. The table below shows some values of x and the corresponding values of y, correct to 2 decimal places.

x	0.5	1	1.5	2	3	4	5	6
y	−0.98	−2.40	−2.55	−2.48	−2.04	−1.37	p	0.61

(a) Find the value of p. [1]

$p = -0.48$ --- B1

(b) In the space provided on the next page, using a scale of 2 cm to represent 1 unit on the horizontal axis and 2 cm to represent 0.5 units on the vertical axis, draw the graph of $y = \frac{1}{2x^2} + 0.1x^2 - 3$ for $0.5 \le x \le 6$. [3]

Refer to attached

(minus 1 if the graph turns to the left or is vertical at $x = 0.5$)

(c) Using your graph, write down the solution(s) to the equation $\frac{1}{2x^2} + 0.1x^2 - 1.5 = 0$. [2]

$x = 0.6$ or 3.8 ± 0.1 --- B2

(d) By drawing a tangent, find the gradient of the curve at $x = 4$. [2]

Tangent drawn. --- M1

$\text{gradient} = \frac{-0.25 - (-2.15)}{5.4 - 3}$

$= 0.792$ $(0.7 - 0.85)$ $(\text{actual} = 0.784375)$ --- A1

(e) By drawing a suitable straight line on the same axes, solve the equation $\frac{1}{2x^2} + 0.1x^2 - 0.5x = 0$ [3]

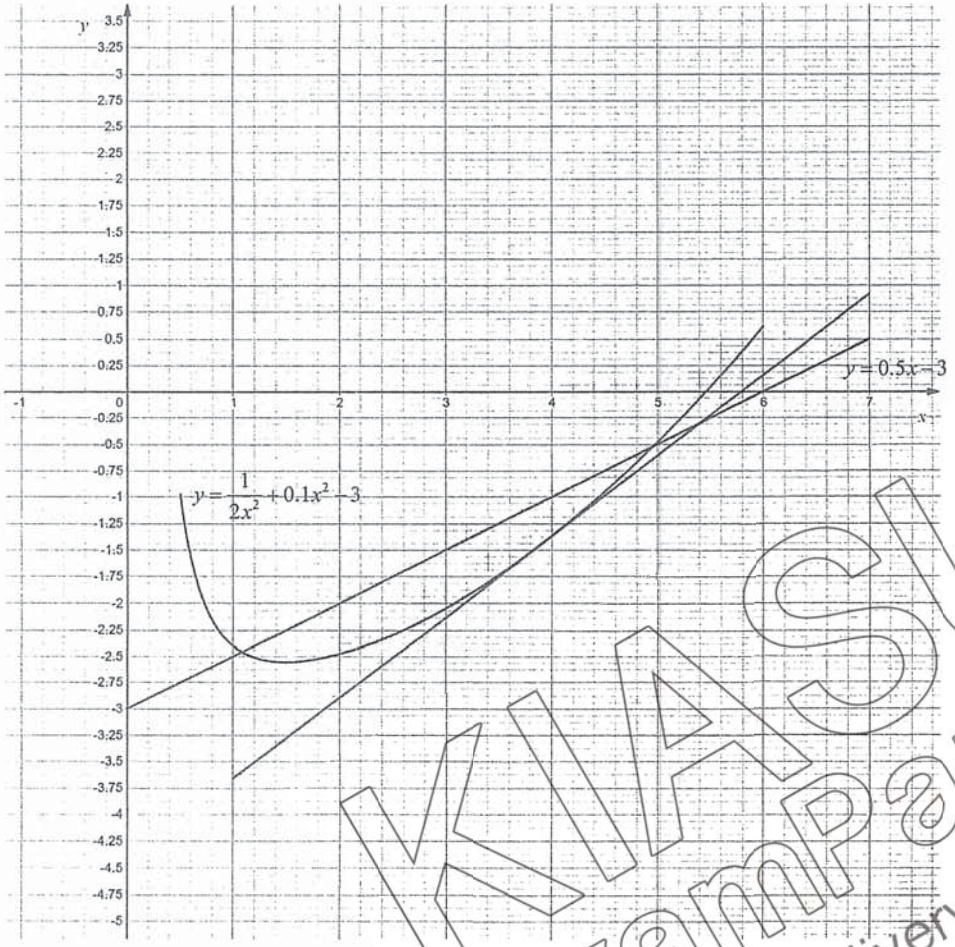
$\frac{1}{2x^2} + 0.1x^2 - 0.5x = 0$

$\frac{1}{2x^2} + 0.1x^2 - 3 = 0.5x - 3$

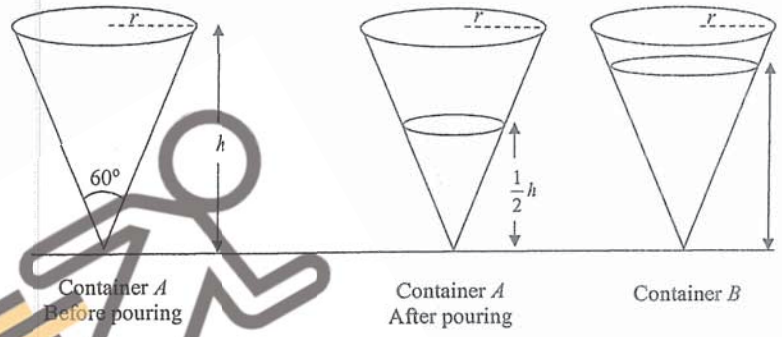
$\therefore$ Draw $y = 0.5x - 3$ --- M1

Correctly drawn graph --- B1

$x = 1.1$ or 4.95 ± 0.1 --- B1



10 A right conical container of capacity 24π cm^3 and vertical angle 60° is completely filled with water. The height of the container is h cm and the base radius is r cm.



(a) Find the value of r and of h. [5]

$\frac{1}{3}\pi r^2 h = 24\pi$ ------ $M1$

$r^2 h = 72$

$\tan 30^o = \frac{r}{h}$ ------ $M1$

$h = \frac{r}{\tan 30^o}$

Hence, $r^3 = 72\tan 30^o$ ------- $M1$

$r = \sqrt[3]{72\tan 30^o} = 3.4610... = 3.46$ cm ---- $A1$

$h = 6$ cm -------- $A1$

OR

$r^2 + h^2 = (2r)^2$

$r^2 = \frac{h^2}{3}$ --- (1)

$\frac{1}{3}\pi r^2 h = 24\pi$ --- (2)

sub (1) into (2):

$\frac{1}{3}\pi\left(\frac{h^2}{3}\right)h = 24\pi$

$h^3 = 216$

$h = 6$ cm

$r = \sqrt{\frac{6^2}{3}}$

$= 3.46$ cm

The water in the container is poured into another identical container B so that the depth of water in container A is $\frac{1}{2}h$ cm.

(b) Find the volume of water in container B in terms of π. [2]

$$\frac{V_A}{\text{original volume}} = \left(\frac{1}{2}\right)^3 = \frac{1}{8} \text{----} M1$$

$$\text{Volume of water in container } B = \frac{7}{8} \times 24\pi = 21\pi \text{ cm}^3 \text{----} A1$$

OR

$$\text{volume remaining } = \frac{1}{3}\pi\left(\frac{\sqrt{12}}{2}\right)^2\left(\frac{6}{2}\right) = 3\pi$$

$$\text{volume in } B = 24\pi - 3\pi = 21\pi \text{ cm}^3$$

11 A class of students plan to sell breakfast sets during the National Day carnival in order to raise funds for Food Bank Singapore.

Each breakfast set consists of 2 scrambled eggs, 2 slices of toast, 2 sausages, and a cup of coffee.

The students estimate that they will sell 250 breakfast sets.

Item	Description	Unit cost
Eggs	Pasar Fresh Eggs (10 per pack) Pasar Fresh Eggs (30 per pack)	\$1.80 \$4.35
Bread	Sunshine Enriched Soft White Bread (14 slices) FairPrice Wholemeal Bread (12 slices)	\$1.80 \$1.55
Sausages	Tierney's Chicken Hot Dog (10 per pack) FairPrice Sausages (6 per pack) (\$0.35 off per 2 packs)	\$5.25 \$3.20
Coffee	Nescafe 3 in 1 Instant Coffee (35 per pack) (Buy 5 get 1 free) Gold Roast 3 in 1 Coffeemix (25 per pack)	\$6.15 \$3.95

(a) Find the lowest possible total cost of the ingredients for the breakfast sets. [5]

	Cost		
Eggs	10 per pack $(50)(1.80)$ $= 90$	30 per pack $(17)(4.35) = 73.95$	Mix $(16)(4.35)+(2)(1.80) = 73.20$
Bread	Sunshine $(36)(1.80)$ $= 64.80$	FairPrice $(42)(1.55) = 65.10$	Mix $(35)(1.80)+(1)(1.55) = 64.55$ $(28)(1.80)+(9)(1.55) = 64.35$ $(34)(1.80)+(2)(1.55) = 64.30$ $(10)(1.80)+(30)(1.55) = 64.50$
Sausages	Tierney's $(50)(5.25)$ $= 262.50$	FairPrice $(84)(3.20)-(42)(0.35)$ $= 254.10$	Mix $(82)(3.20)-(41)(0.35)+(1)(5.25)$ $= 253.30$ $(80)(3.20)-(40)(0.35)+(2)(5.25)$ $= 252.5$
Coffee	Nescafe $(7)(6.15)$ $= 43.05$	Gold Roast $(10)(3.95) = 39.50$	Mix $(5)(6.15)+(2)(3.95) = 38.65$

B1 for each correct choice for each item.

total cost = $73.20 + 64.30 + 252.50 + 38.65 = \$428.65$ --- A1

(not lowest cost per item used: M1 for each correct item, no A1)

(insufficient quantities purchased: M0)

(b) The school provides \$200 in funding for the students, and up to 30% of the sales can be used to cover for their expenses, while the remaining goes to the charity.
The students also wish to raise at least \$600 for charity,
Find the minimum they must charge for each breakfast set (to the nearest ten cents).
Justify your answer, showing all necessary workings clearly.
State an assumption you have made in your calculations. [4]

$$\text{min charge to raise \$600} = \frac{(428.65-200)+600}{250} = \$3.3146 \quad \text{--- M1 (ECF their (a))}$$

$$\left(\frac{30}{100}\right)(3.3146)(250) = 248.595 > 431.60-200 \quad \text{--- M1 (ECF their (a))}$$

$\therefore$ min charge = \$3.40 --- A1

Assume no cost incurred for cooking. (or any reasonable) --- B1

OR

$$\text{min charge to raise \$600} = \frac{(428.65-200)+600}{250} = \$3.3146 \quad \text{--- M1 (ECF their (a))}$$

$$\text{min charge to cover expenses} = \left[\frac{(428.65-200)}{30}\times 100\right] \div 250 = \$3.048 \quad \text{--- M1 (ECF their (a))}$$

$\therefore$ min charge = \$3.40 --- A1

Assume no cost incurred for cooking. (or any reasonable) --- B1

Alternative assumptions:
1) No accidents that result in wastage during the event
2) No free samples were given out
3) All 250 sets were sold