Calculator Model :

Class	Full Name	Index Number

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PRELIMINARY EXAMINATION 2019

O

4048/01

MATHEMATICS
Paper 1

Secondary 4 Express/ 4A1/ 5 Normal Academic
30 August 2019 **2 hours**

Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any questions it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

DO NOT OPEN THIS PAPER UNTIL YOU ARE TOLD TO DO SO

For Examiner's use
80

Setter: Mr Alvis Mazon Tan

This document consists of **20** printed pages, including this cover page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curve surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta\text{, where } \theta \text{ is in radians}$$

$$\text{Sector Area} = \frac{1}{2}r^2\theta\text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 Write down the following in ascending order.

$$\frac{25}{38} \qquad \sqrt{0.49} \qquad 0.60\tfrac{2}{3} \qquad 0.701$$

Answer $\frac{25}{38}$, $\sqrt{0.49}$, 0.701 , $0.60^{\frac{2}{3}}$ [1]

---

2 (a) Expand and simplify $(2x-1)(2-3x)-3x(2x-5)$.

$(2x-1)(2-3x)-3x(2x-5)$

$= 4x-6x^2-2+3x-6x^2+15x$ [M1]

$= -12x^2+22x-2$

$= -2(6x^2-11x+1)$ [A1]

Answer (a) $-2(6x^2-11x+1)$ [2]

(b) Factorise completely $24ab-4ac+pc-6pb$.

$24ab-4ac+pc-6pb$

$=4a(6b-c)+pc(c-6b)$ [M1]

$=4a(6b-c)-p(6b-c)$

$=(4a-p)(6b-c)$ [A1]

Answer (b) .... [2]

---

3 Calculate $\frac{13.5^3}{6.48-2.57}$, giving your answers corrected to 2 significant figures.

Answer 630 [1]

---

4 If the radius of a sphere increases by 10%, find the percentage increase in its volume.

$$\frac{\frac{4}{3}\pi r^3(1.1)^3 - \frac{4}{3}\pi r^3}{\frac{4}{3}\pi r^3} \times 100\% \quad [M1]$$

$= 33.1\%$ [A1]

Answer 33.1 % [2]

5 On a certain day the exchange rate between the pounds (£) and the Singapore dollars was S\$1.684 = £1.

(a) Calculate the amount of pounds that Renee can buy with S\$1263.

$\frac{\$1263}{\$1.684} = £750$ [A1]

Answer (a) £= 750 [1]

(b) After four weeks, she realized she has too much pounds and she now wants to change £200 back to Singapore dollars.
If the loss by this transaction is S\$6, what is the current exchange rate? Leave your answers corrected to 4 decimal places.

With reference to the original exchange rate

£200 : S\$1.684 × 200

= S\$336.80

S\$336.80 − S\$6 = S\$330.8

New exchange rate:

$\frac{S\$330.8}{£200} = S\$1.654$ [A1]

[M1]

Answer (b) £1 = S\$ 1.654 [2]

6 Integers P and Q, written as products of their prime factors, are

$P = 2^2 \times 3 \times k^2$ and $Q = 2^3 \times 7 \times k$, where k is a prime number.

(a) Express, in terms of k and as a product of its prime factors, the smallest possible integer which is exactly divisible by both P and Q.

$L.C.M = 2^3 \times 3 \times 7 \times k^2$ [B1]

Answer (a) $2^3 \times 3 \times 7 \times k^2$ [1]

(b) Find the smallest integer, n, such that $27\,kn$ is a multiple of P. Give your answer in terms of k if necessary.

$P = 2^2 \times 3 \times k^2$

$27kn = 3^3 kn$

$= 3^2(3k)(2^2 \times k)$ [M1]

$\therefore n = 4k$ [A1]

Answer (b) n = $4k$ [2]

7 Kai Xuan has written down seven numbers.
The mean of these numbers is 8, the median is 7 and the mode is 11.
The smallest number is an even prime number and the largest number is eight times the smallest number.
The second and third numbers are consecutive numbers.

Find the seven numbers.

[M1] $x_1, x_2, x_3, x_4, x_5, x_6, x_7$

$x_1 = 2$, $x_4 = 7$, $x_5 = 11$, $x_6 = 11$, $x_7 = 16$

$$\frac{2+7+11+11+16+x_2+x_3}{7} = 8$$

$\therefore x_2 + x_3 = 9 \Rightarrow x_2 = 4, x_3 = 5.$

[A1] *Answer* = 2 , 4 , 5 , 7 , 11 , 11 , 16 [2]

---

8 Rearrange the formula $v = \frac{-u^2+5}{u^2-a}$ and make u the subject of the formula.

$v(u^2 - a) = -u^2 + 5$

$vu^2 - av = -u^2 + 5$ [M1]

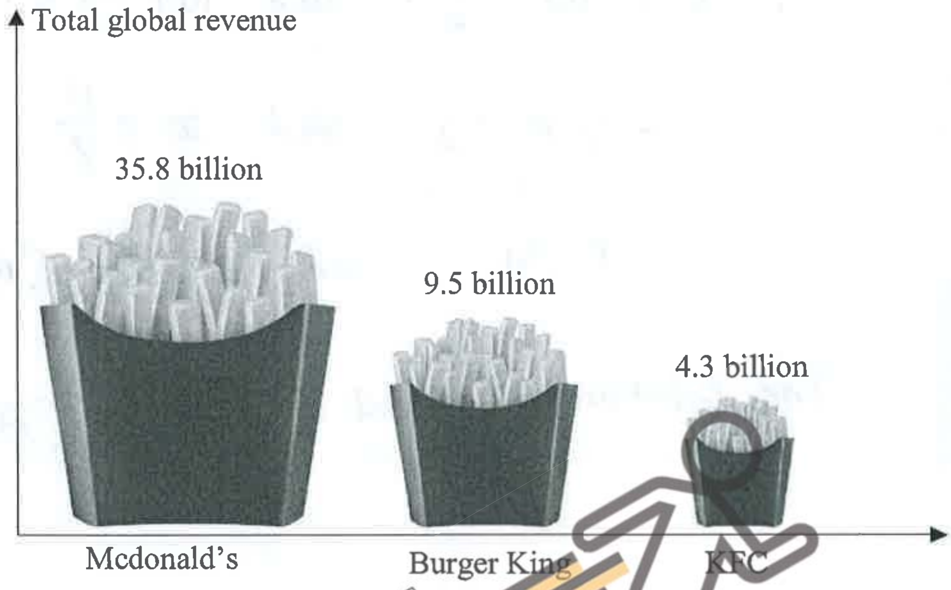
$vu^2 + u^2 = av + 5$

$u^2(v+1) = av + 5$ [M1]

$u = \pm\sqrt{\frac{av+5}{(v+1)}}$ [A1]

Answer $u = \pm\sqrt{\frac{av+5}{(v+1)}}$ [3]

9 The graph shows the total revenue, in billion dollars, of three different fast food chain.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer ① The size of the fries and hence the area was used to compare the global revenue. [B1]

② Total area for Mcdonald's appears to be twice that of Burger King but in fact the revenue is more than 3 times of the latter [B1] [2]

10 (a) Solve the inequalities $8 + x < 10 + \frac{3}{2}x \leq 15.5 - 2x$.

$8 + x < 10 + \frac{3}{2}x$ and $10 + \frac{3}{2}x \leq 15.5 - 2x$ [M1]

$-\frac{1}{2}x < 2$ and $x \leq \frac{11}{7}$

$\therefore$ $x > -4$ and $x \leq \frac{11}{7}$ [M1]

The solution is $-4 < x \leq \frac{11}{7}$ [A1]

Answer (a) $x =$ $-4 < x \leq \frac{11}{7}$ [3]

(b) Hence, write down the largest rational number that satisfies

$$8 + x < 10 + \frac{3}{2}x \leq 15.5 - 2x$$

Answer (b) $x =$ $\frac{11}{7}$ [1]

11 The first four terms in a sequence of numbers T_1, T_2, T_3, T_4 are as follow

$$T_1 = \frac{2}{1} - \frac{3}{2}$$

$$T_2 = \frac{3}{2} - \frac{4}{2^2}$$

$$T_3 = \frac{4}{2^2} - \frac{5}{2^3}$$

$$T_4 = \frac{5}{2^3} - \frac{6}{2^4}$$

$$\vdots$$

(a) Write down the n^{th} line and show that it can be expressed as $T_n = \frac{n}{2^n}$.

$$T_n = \frac{n+1}{2^{n-1}} - \frac{n+2}{2^n} \quad [M1]$$

$$= \frac{n+1}{2^n(2^{-1})} - \frac{n+2}{2^n} \quad [M1]$$

$$= \frac{2n+2-n-2}{2^n} = \frac{n}{2^n} \quad [A1]$$

Answer(a) .... [3]

(b) Hence or otherwise, evaluate the following sum and leave your answer as a fraction.

$$T_1 + T_2 + T_3 + \ldots\ldots\ldots\ldots\ldots\ldots T_{11}$$

[M1]

$$T_1 + T_2 + \cdots T_{11} = \left(\frac{2}{1} - \cancel{\frac{3}{2}}\right) + \left(\cancel{\frac{3}{2}} - \cancel{\frac{4}{2^2}}\right) + \left(\cancel{\frac{4}{2^2}} - \frac{5}{2^3}\right)$$

$$\vdots$$

$$\left(\frac{11}{2^9} - \cancel{\frac{12}{2^{10}}}\right) + \left(\cancel{\frac{12}{2^{10}}} - \frac{13}{2^{11}}\right)$$

$$= \frac{2}{1} - \frac{13}{2^{11}}$$

$$= \frac{4083}{2048}$$

[A1]

Answer (b) ...... $\frac{4083}{2048}$ ...... [2]

12 A = {points lying on the line $2x + y = 8$}
B = {points lying on the line $3x - 4y = 12$}
C = {points lying on the line $mx - 4y = c$}

(a) Is (-1,6) ∈ A? Explain your answer clearly.

Sub(-1,6) into "A"

L.H.S = 2(-1)+6 = 4

R.H.S = 8, Since L.H.S ≠ R.H.S, (-1,6) ∉ A [B1] [1]

(b) Find the element p such that $p \in (A \cap B)$.

$2x + y = 8$ — ①

$3x - 4y = 12$ — ②

Sub ① into ②

$3x - 4(8-2x) = 12$ [M1]

$11x = 44$

$x = 4$

$y = 0$

$\therefore p = (4, 0)$ [A1]

Answer (b) p = (4, 0) [2]

(c) Write down a possible value of m and of c such that $B \cap C = \emptyset$.

$B \cap C = \emptyset \Rightarrow$ No Solution.

The line $y = \frac{3}{4}x - 3$ and $mx - 4y = c$ must be parallel but cannot have the same y-intercept.

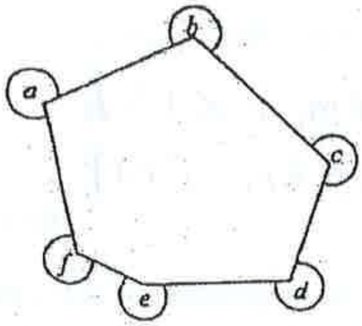
$\therefore m = 3$ [B1]

$c \neq 12$ [B1]

Answer (c) m = 3 [1]

c = $\underbrace{\qquad\qquad}_{\text{Any Value except 12.}}$ [1]

13 The diagram below shows an irregular hexagon .
Calculate the value of $a + b + c + d + e + f$.



Sum of interior ∡ in a hexagon: $(6-2) \times 180° = 720°$

$360° \times 6 = 2160°$

Sum of reflex ∡'s : $a + b + c + d + e + f = 2160° - 720°$ [M1]

$1440°$ [A1]

Answer ......1440......° [2]

14 Jia Lung invested some money in the savings account for 4 years . The rate of compound interest was fixed at 4% per annum compounded anually.
At the end of 4 years, there was \$8436.48 in her account.

How much did Jia Lung invest in the account?

$A = P(1 + \frac{r}{100})^n$

$8436.48 = P(1 + \frac{4}{100})^4$ [M1]

$8436.48 = P(1.04)^4$

$P = \frac{8436.48}{(1.04)^4} = \$7211.54$ # [A1]

Answer \$ ......7211.54...... [3]

15 Akshay jogs at a speed of 10 km/h.
One evening he jogged around his neighborhood for 1 hour 30 minutes.

(a) Calculate the distance that Akshay covered.

$10 km/h \times 1.5 h$
$= 15 km$ [B1]

Answer (a) = 15 km [1]

(b) Given that the scale of the neighbourhood is 1: 25000, find in cm, the map distance that he covered.

1 cm : 25000 cm
1 cm : 0.25 km
$15 km \Rightarrow \frac{15 km}{0.25 km} = 60 cm$ [A1]
[M1]

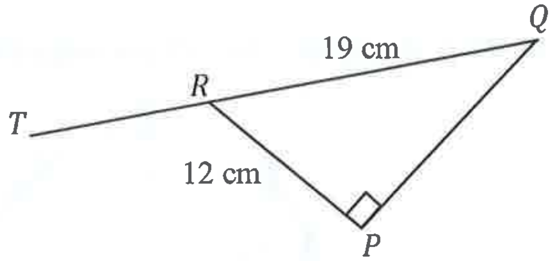
Answer (b) = 60 cm [2]

(c) A reservoir located in his neighbourhood occupies a total area of 1.70 cm^2 on the map. What is the actual area, in m^2, of the reservoir?

$1 cm^2 : 625000000 cm^2$ [M1]
$1 cm^2 : 62500 m^2$
$1.70 cm^2 : 1.70 \times 62500 m^2$
$= 106250 m^2$ [A1]

Answer (c) 10625 m^2 [2]

16 PQR is a right-angled triangle. QRT is a straight line.
$PR = 12$ cm and $QR = 19$ cm.



Find the values of the following, giving your answer to two decimal places where necessary.

(a) $\tan \angle PQR$

$QP = \sqrt{19^2 - 12^2} = \sqrt{217}$

$\tan \angle PQR = \frac{12}{\sqrt{217}}$ [M1]

$= 0.81$ [A1]

Answer (a) 0.81 [2]

(b) $\cos \angle TRP$

$\cos \angle TRP = -\cos(180° - \angle TRP)$

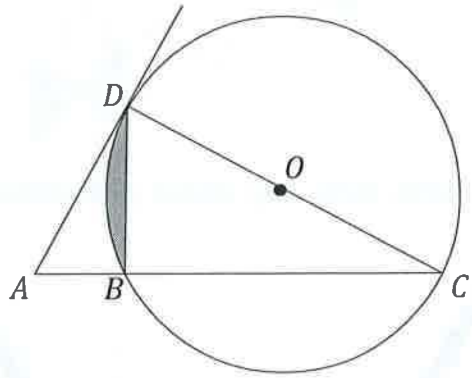
$= -\cos(\angle QRP)$ [M1]

$= -\frac{12}{19}$ [A1]

Answer (b) $-\frac{12}{19}$ [2]

17 In the diagram, O is the centre of the circle BCD with radius 20 cm and CD is the diameter of the circle. The ratio of the length AB to the length AD is 0.5.

A is a point on BC produced such that AD is a tangent to the circle at D.



Calculate the area of the shaded region.

$\angle ADB = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ rad

$\angle BDC = \frac{\pi}{2}$ rad $- \frac{\pi}{6}$ rad $= \frac{\pi}{3}$ rad (tangent at a point) [M1]

$\angle BCD = \pi$ rad $- \frac{\pi}{2}$ rad $- \frac{\pi}{3}$ rad $= \frac{\pi}{6}$ rad [A1]

$\angle BOD = 2 \times \angle BCD$ ($\angle$ at $\odot = 2 \times \angle$ at circumference)

$\angle BOD = 2 \times \frac{\pi}{6}$ rad $= \frac{\pi}{3}$ rad [M1]

Area of shaded region = Area of sector OBD − Area of ΔOBD

$= \frac{1}{2}(20)^2\left(\frac{\pi}{3}\right) - \frac{1}{2}(20)^2 \sin\left(\frac{\pi}{3}\right)$ [M1]

$= 36.2 \text{ cm}^2$.

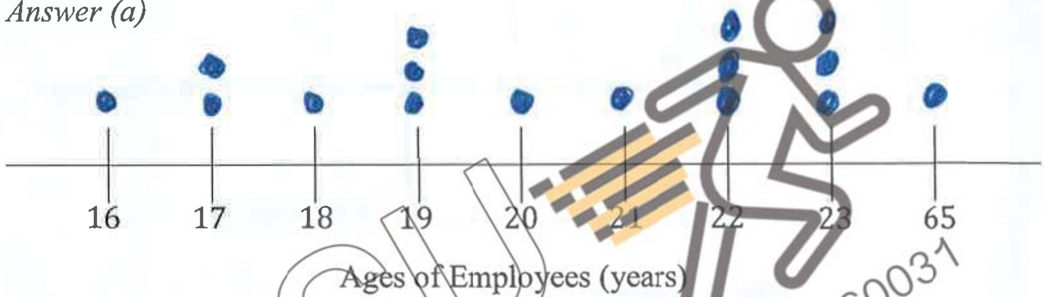
Answer (b) .... cm^2 [5]

18 The table below shows the ages of 16 employees who work part-time at a cafe.

20	21	16	23
22	17	19	65
23	22	17	22
23	19	19	18

(a) Complete the dot diagram to show the distribution of the ages of the employees.

Answer (a) [1]



(b) Find the median of the distribution of ages.

$\frac{20+21}{2} = 20.5$ [B1]

Answer (b) Median = 20.5 years [1]

(c) Calculate the mean age of the employees.

Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{366}{16} = 22.9$ [B1]

Answer (c) Mean = 22.9 years [1]

(d) Pranav made the following statement:

"The mean is the most accurate way to determine the average age of the employees" Validate if Pranav statement is true.

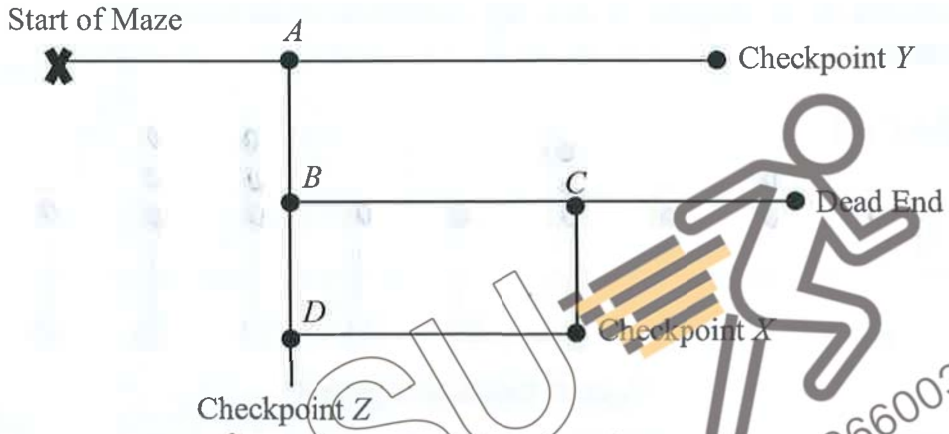
① Not Valid [B1]

② Mean is affected by extremal points in the data set. [B1]

[2]

19 Allyson participates in a game show. In order to win a prize, she has to navigate through a maze.
The prize is located at checkpoint X. There are no prizes awarded at checkpoint Y and Z.
The diagram below shows four junctions A, B, C and D in the maze.
Once Allyson runs pass a junction, she is not able to make a turnaround.

The probability that Allyson goes straight, without changing direction, at every junction is $\frac{3}{7}$.



(a) Find the probability that Allyson hits the dead end.

P(Allyson hits the dead end)

$= \left(\frac{4}{7} \times \frac{4}{7} \times \frac{3}{7}\right)$ [M1]

$= \frac{48}{343}$ [A1]

Answer (a) $\frac{48}{343}$ [2]

(b) Find the probability that Allyson wins a prize.

P(Allyson wins a prize)

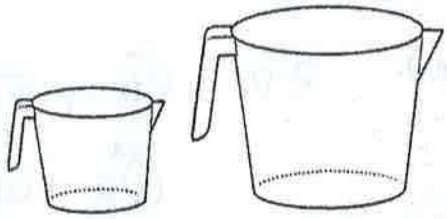
$= \left(\frac{4}{7} \times \frac{4}{7} \times \frac{4}{7}\right) + \left(\frac{4}{7} \times \frac{3}{7} \times \frac{4}{7}\right)$ [M2]

$= \frac{64}{343} + \frac{48}{343}$

$= \frac{112}{343}$ [A1]

Answer (b) $\frac{112}{343}$ [3]

~~20~~ Two similar jugs have base area of 45 cm^2 and 125 cm^2.



(a) Find the ratio of the height of the smaller jug to the ratio of the height of the larger jug.

$$\left(\frac{h_{small}}{h_{large}}\right)^2 = \frac{45}{125} \Rightarrow \frac{h_{small}}{h_{large}} = \sqrt{\frac{45}{125}} = \frac{3}{5} \text{ [B1]}$$

Answer (a) ....3.......... : ....5........ [1] ↕ space

(b) The curved surface area ~~of the bottom~~ of the smaller jug is 63 cm^2.
Find the curved surface area ~~of the bottom~~ of the larger jug.

$$\left(\frac{3}{5}\right)^2 = \frac{63}{\text{Curved S.A of larger jug}} \text{ [M1]}$$

$$\therefore \text{curved surface area of larger jug} = \frac{63 \times 25}{9} = 175 \text{ cm}^2 \text{ [A1]}$$

Answer (b) ....175........ cm^2 [2]

(c) The capacity of the larger jug is 2.5 litres. Find the capacity of the smaller jug. Give your answer in cubic centimetres.

$$\left(\frac{3}{5}\right)^3 = \frac{V_{small}}{2.5\,l} \text{ [M1]}$$

$$V_{small} = \frac{27}{125} \times 2.5\,l$$

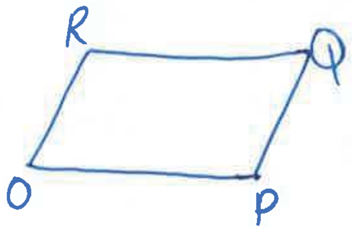
$$= 0.54\,l$$

$$= 540 \text{ cm}^3 \text{ [A1]}$$

Answer (c) ....540........ cm^3 [2]

21 $OPQR$ is a parallelogram such that $\overrightarrow{PQ} = \binom{2}{4}$ and P is the point (3,2).

(a) Express $\overrightarrow{RP}$ as a column vector.



$\overrightarrow{RP} = \overrightarrow{RO} + \overrightarrow{OP}$

$= \overrightarrow{QP} + \overrightarrow{OP}$

$= -\overrightarrow{PQ} + \overrightarrow{OP}$ [M1]

$= \binom{-2}{-4} + \binom{3}{2}$

$= \binom{1}{-2}$ [A1]

Answer (a) $\binom{1}{-2}$ .... [2]

(b) The point J lies on $\overrightarrow{RP}$ produced such that $\overrightarrow{PJ} = m\overrightarrow{RP}$

Show that $\overrightarrow{OJ} = \binom{3+m}{2-2m}$

$\overrightarrow{PJ} = m\overrightarrow{RP}$

$\overrightarrow{OJ} - \overrightarrow{OP} = m(\overrightarrow{RP})$ [M1]

$\overrightarrow{OJ} = m(\overrightarrow{RP}) + \overrightarrow{OP}$

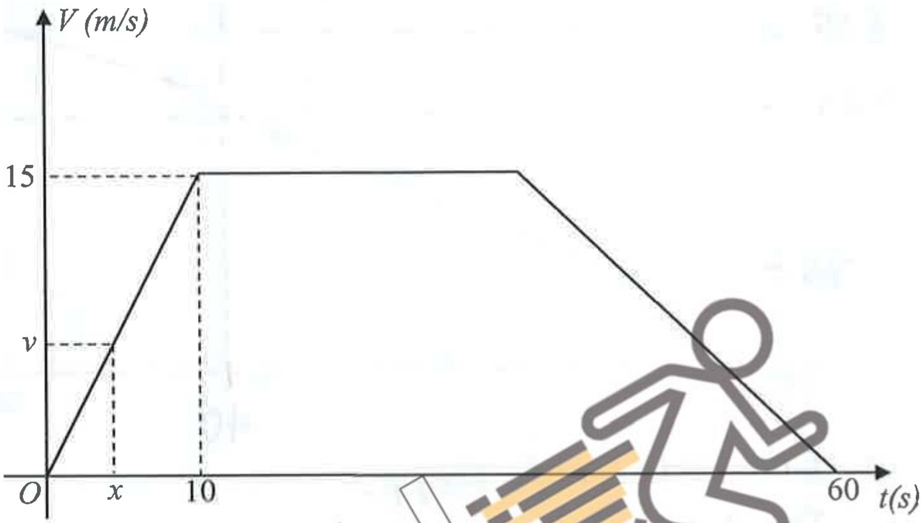
$= m\binom{1}{-2} + \binom{3}{2}$

$= \binom{m}{-2m} + \binom{3}{2}$

$= \binom{3+m}{2-2m}$ (shown) [A1]

[2]

22 The diagram shows the speed time graph of a car. The car starts from rest and accelerates uniformly to a speed of 15 m/s in 10 seconds.
The car then travels at a constant speed for some time before it decelerates uniformly at 0.75 m/s^2 until it comes to rest.
The whole journey takes one minute.



(a) Given that the speed of the car after x seconds is v m/s, express v in terms of x.

By similar Δs

$$\frac{15}{10} = \frac{v}{x}$$

$v = 1.5x$ [B1]

Answer v = $1.5x$ .... [1]

(b) For how long does the car travel at the maximum speed?

Let the time where he ends at constant speed be t.

$$\therefore \frac{(0-15)\,m/s}{(60-t)\,s} = -0.75\,m/s^2 \quad [M1]$$

$\therefore t = 40s$

Answer 30 .... [2]

Duration: $40s - 10s = 30s$ [A1]

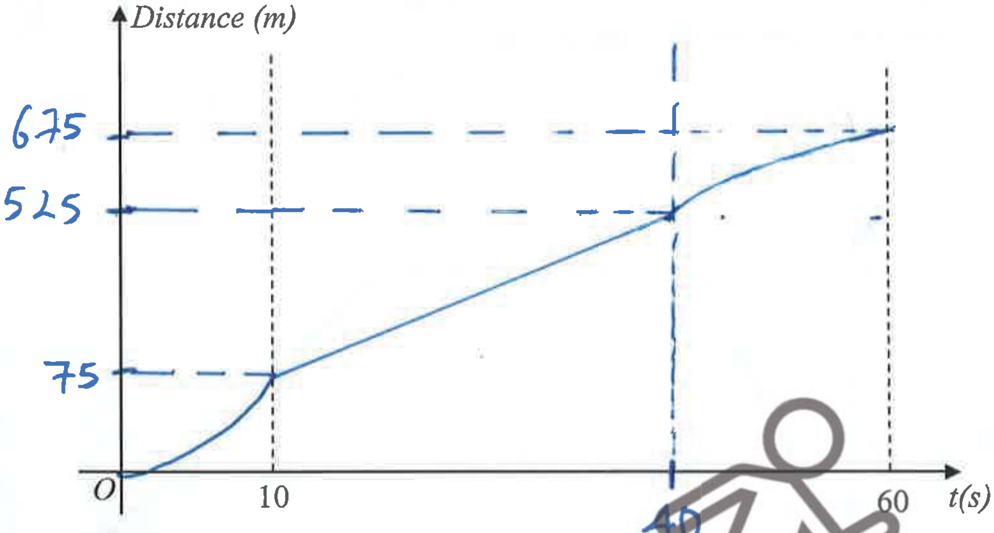
(c) Calculate the total distance travelled by the car during this 1 minute journey.

$\frac{1}{2}(60+30) \times 15\,m/s$ [M1]

$= 675m$ [A1]

Answer $675m$ .... m [2]

(d) Hence, sketch the distance time graph for the whole journey, indicating all relevant values in your sketch.



[2]

[B1] : Correct shape.
[B1] : Correct values.

~ End of Paper ~

Calculator Model :

Class	Full Name	Index Number

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I believe, therefore I am

Marking Scheme

PRELIMINARY EXAMINATION 2019

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4048/02

MATHEMATICS
Paper 2

Secondary 4 Express / Secondary 5 Normal (Academic)
3 September 2019 **2 hours 30 minutes**

Candidates answer on the Question Paper.
Additional Materials: Graph Paper

READ THESE INSTRUCTIONS FIRST

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For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is **100**.

DO NOT OPEN THIS PAPER UNTIL YOU ARE TOLD TO DO SO

For Examiner's Use
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Setter: Mrs Jane Cheng

This document consists of **21** printed pages, including this cover page.

Answer **all** the questions.

1 **(a)** Solve the inequality $\frac{x+2}{3} \geq \frac{4-x}{7}$. [2]

$7(x+2) \geqslant 3(4-x)$

$7x + 14 \geqslant 12 - 3x$ (M1)

$10x \geqslant -2$

$x = -\frac{1}{5}$ (A1)

(b) Express as a single fraction in its simplest form $\frac{2x}{(3x-5)^2} + \frac{x}{5-3x}$. [2]

$\frac{2x}{(3x-5)^2} - \frac{x}{3x-5}$

$= \frac{2x - x(3x-5)}{(3x-5)^2}$ (M1)

$= \frac{7x - 3x^2}{(3x-5)^2} = \frac{x(7-3x)}{(3x-5)^2}$ (A1)

(c) Simplify $\left(\frac{27a^6}{b^{12}}\right)^{-\frac{1}{3}}$. [2]

$\left(\frac{27a^6}{b^{12}}\right)^{-\frac{1}{3}}$

$= \left(\frac{b^{12}}{27a^6}\right)^{\frac{1}{3}}$ (M1)

$= \frac{b^4}{3a^2}$ (A1)

(d) Simplify $\frac{24p^3q^2}{5r^3} \div \frac{8p^4r}{15q^3}$. [2]

$$= \frac{24p^3q^2}{5r^3} \times \frac{15q^3}{8p^4r} \quad \text{(M1)}$$

$$= \frac{9q^{2+3}p^{3-4}}{r^{3+1}}$$

$$= \frac{9q^5}{pr^4} \quad \text{(A1)}$$

(e) Solve the equation $\frac{15}{x+2} = 2x + 3$ [3]

$$15 = (x+2)(2x+3)$$

$$15 = 2x^2 + 7x + 6$$

$$2x^2 + 7x + 6 - 15 = 0$$

$$2x^2 + 7x - 9 = 0 \quad \text{(M1)}$$

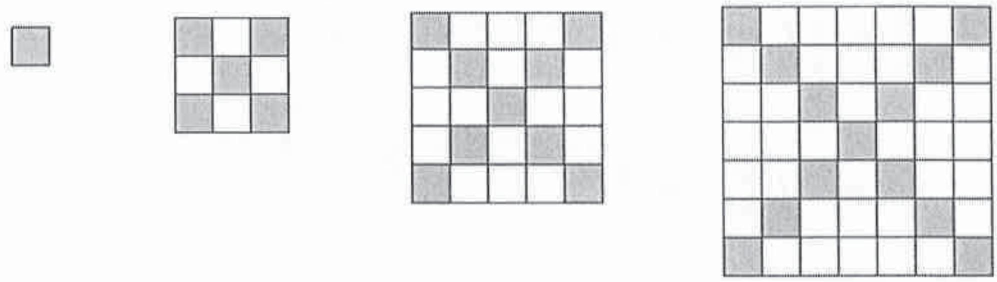
$$(2x+9)(x-1) = 0$$

$2x + 9 = 0$ or $x - 1 = 0$

$x = -4\frac{1}{2}$ $\quad$ $x = 1$

(A1) $\quad$ (A1)

2 A series of diagrams of shaded and unshaded small squares is shown below.
The shaded squares are those which lie on the diagonals of the diagram.



(a) Copy and complete the table below. [2]

Diagram, n	1	2	3	4	5
Number of shaded squares, S	1	5	9	13	17
Number of unshaded squares, U	0	4	16	36	64
Total number of squares, T	1	9	25	$7^2=49$	$9^2=81$

$(2n-1)^2$

(b) By observing the number patterns, without drawing further diagrams,
(i) write down the total number of squares in diagram 12, [1]

$n=12 \quad (2n-1)^2=(2\times12-1)^2=529$ (A1)

(ii) find an expression, in terms of n, for the total number of squares, T. [1]

$T=(2n-1)^2$
$=4n^2-4n+1$ (A1)

(c) **(i)** Find an expression, in terms of n, for the number of shaded squares, S. [1]

$S=4n-3$ (A1)

(ii) Write down the number of the diagram that has 41 shaded squares. [1]

$4n-3=41$
$4n=44$
$n=11$ (A1)

(d) Hence, or otherwise, find an expression, in terms of n, for the number of unshaded squares, U. [2]

$U=T-S$
$=(2n-1)^2-(4n-3)$ (M1)
$=4n^2-4n+1-4n+3$
$=4n^2-8n+4$
$=4(n^2-2n+1)$ (A1)

3 P is the point $(-5, 12)$ and Q is the point $(5, -4)$

(a) Find the length of PQ. [2]

$$l = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$= \sqrt{(12+4)^2 + (-5-5)^2}$$

$$= \sqrt{356}$$

$$= 18.867$$

$$= \underline{18.9 \text{ units}} \quad (3 \text{ s.f})$$

(b) Find the equation of the line PQ. [2]

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{12+4}{-5-5}$$

$$= -\frac{8}{5}$$

$$y = mx + c$$

$$-4 = -\frac{8}{5}(5) + c$$

$$c = 4 \quad \text{M1}$$

Equation :

$$y = -\frac{8}{5}x + 4 \quad \text{A1}$$

OR

$$5y = -8x + 20$$

(c) The equation of the line l_1 is $8x + 5y + 10 = 0$.

(i) Show how you can decide whether the line l_1 does or does not intersect the line PQ? [2]

$l_1 \quad 5y = -8x - 10 \qquad m = -8/5$ (M1)

$y = -8/5 x - 2$

Both equations have the same gradient, they are parallel to each other.

Hence, the line l_1 does not intersect with PQ.

(A1)

(ii) The equation of line l_2 is $3y = 4x - 39$.
Find the coordinates of the point of intersection of the line l_1 and the line l_2. [3]

$l_1 \quad 8x + 5y = -10$ ①

$l_2 \quad 4x - 3y = 39$ ② (M1)

eqn ② × 2 $\quad 8x - 6y = 78$ ③

eqn ① − ③ $\quad 8x + 6y = -88$

$11y = -88$

$\underline{y = -8}$ (A1)

sub $y = -8$ into eqn ①

$8x + 5(-8) = -10$

$8x = -10 + 40$

$8x = 30$

$x = 3\,3/4$

$x = 3.75$ (A1)

3 Mrs Tan is a Korean Language teacher.
She conducts classes for basic and advaced students on weekdays and weekends.
Each student has a 15-week block of lessons with one lesson per week.
The matrix K shows the number of students she teaches each week in one 15-week block.

$$\begin{array}{cc} & \begin{array}{cc}\text{Basic} & \text{Advanced}\end{array} \\ \mathbf{K} = & \begin{pmatrix} 12 & 3 \\ 5 & 8 \end{pmatrix} \end{array} \begin{array}{l} \text{Weekday} \\ \text{Weekend} \end{array}$$

(a) Evaluate the matrix **P** = 15**K**. [1]

$$P = 15\begin{pmatrix} 12 & 3 \\ 5 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 180 & 45 \\ 75 & 120 \end{pmatrix} \quad \text{A1}$$

(b) Mrs Tan charges $20 for each basic lesson and $32 for each advanced lesson.
Represent the lesson charges in a 2 × 1 matrix **L**. [1]

$$L = \begin{pmatrix} 20 \\ 32 \end{pmatrix} \quad \text{A1}$$

(c) Evaluate the matrix **T** = **PL**. [2]

$$T = \begin{pmatrix} 180 & 45 \\ 75 & 120 \end{pmatrix}\begin{pmatrix} 20 \\ 32 \end{pmatrix} \quad \text{M1}$$

$$= \begin{pmatrix} 5040 \\ 5340 \end{pmatrix} \quad \text{A1}$$

(d) State what the elements of **T** represent. [1]

The elements of T represent the total amount of money Mrs Tan collects for a 15-week block of lessons on weekdays and weekends respectively.

(e) Mrs Tan wants to attract more students, so in the next 15-week block she reduces her prices by 10%.
For this block of lessons, on weekdays she has 15 basic students and 5 advanced students.
On weekends she has 7 basic students and 6 advanced students.

Calculate the total amount of money she earns for this 15-week block of lessons. [3]

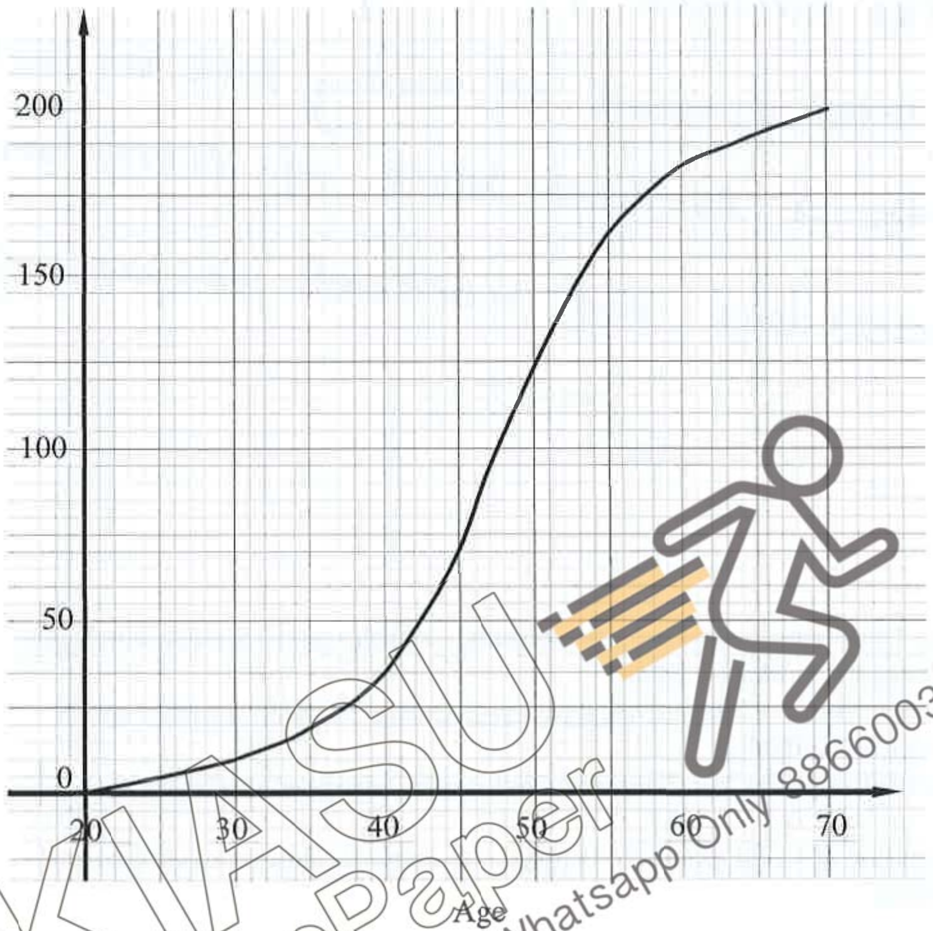
$0.90 \times [(15+7) \times 20 + (5+6) \times 32)] \times 15$

(M1) (M1)

$= \$ \underline{10692}$

5 The cumulative frequency graph shows the distribution of the age groups of the *Fitness First* club.

(a)



(i) Complete the grouped frequency table for the ages of the members. [1]

Age (x)	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$	$50 \leq x < 60$	$60 \leq x < 70$
Frequency	10	25	90	60	15

(ii) Calculate the mean age of each member. [1]

$\bar{x} = (10 \times 25 + 25 \times 35 + 90 \times 45 + 60 \times 55 + 15 \times 65) \div 200$

$= \underline{47.25}$ years old A1

(iii) Calculate the standard deviation. [2]

$SD = \sqrt{\frac{\Sigma fx^2}{N} - \left(\frac{\Sigma fx}{N}\right)^2} = \sqrt{\frac{464000}{200} - (47.25)^2}$ M1

$= 9.3508$

$= \underline{\underline{9.35}}$ (3 s.f) A1

(iv) Find the percentage of members whose age is 30 years old and above but less than 60 years old. [1]

No. of members = 25 + 90 + 60 = 175

$\frac{175}{200} \times 100\% = 87.5\%$ A1

(v) A magazine article stated that citizens aged 50 and above are less active than those aged below 40. [2]
Comment on whether the data from the *Fitness First* club supports this claim.

Accept any other reasonable answer.

percentage of members ≥ 50: $\frac{75}{100} \times 100\% = 37.5\%$
percentage of members < 40: $\frac{35}{100} \times 100\% = 17.5\%$ M1

It is not true that citizens ≥ 50 yrs old are less active A1

(b) The table below gives information about the ages of the members in the *Any Time Fitness* club.

	Members aged under 50	Members aged 50 or over	
Male	50	34	84
Female	36	30	66
	86	64	

(i) One of these members is selected at random. [1]
Find, as a fraction in its lowest terms, the probability that he or she is under 50.

$P(\text{member} < 50) = \frac{86}{150} = \frac{43}{75}$ A1

(ii) Two of the members are selected at random. [2]
Find the probability that
(a) both members are female,

No. of female members = 66

$P(\text{Both are female}) = \frac{66}{150} \times \frac{65}{149} = \frac{143}{745}$ M1 A1

[2]

(b) they are both aged 50 or over, but only one is a male member.

No. of members ≥ 50 yrs old: male : 34, female : 30

P(both ≥ 50 yrs old, only one is a male member)

$= \frac{34}{150} \times \frac{30}{149} + \frac{30}{150} \times \frac{34}{149} = \frac{68}{745}$ M1 A1

6 A litre of *95-octane* unleaded petrol cost $\$x$ in *January 2019*.

(a) Mr Ang paid \$ 85.50 for his petrol. Write down in terms of x, the amount of petrol bought. [1]

Mr Ang $= \dfrac{85.5}{x}$ litre A1

Mr Bala paid \$100 for his *98-octane* unleaded petrol which cost 25 cents more per *litre*.

(b) Write down in terms of x, the amount of petrol bought by Mr Bala. [1]

Mr Bala $= \dfrac{100}{x+0.25}$ litre A1

(c) If Mr Ang received 2 *litres* less petrol than Mr Bala, write down an equation to represent this information and show that it can reduce to $16x^2 - 112x + 171 = 0$. [3]

$$\frac{100}{x+0.25} - \frac{85.5}{x} = 2$$ M1

$$100x - 85.5(x + 0.25) = 2x(x + 0.25)$$ M1

$$\frac{29}{2}x - 21\frac{3}{8} = 2x^2 + \frac{1}{2}x$$

$$2x^2 + \left(\frac{1}{2} - \frac{29}{2}x\right) + 21\frac{3}{8} = 0$$

$$2x^2 - \frac{28}{2}x + \frac{171}{8} = 0$$

$$16x^2 - 112x + 171 = 0 \text{ shown}$$

A1

(d) Solve the equation $16x^2 - 112x + 171 = 0$. [2]

$a = 16 \quad b = -112 \quad c = 171$

$$x = \frac{-(-112) \pm \sqrt{(-112)^2 - 4(16)(171)}}{2 \times 16} \quad \text{M1}$$

$x = 4.75$ (2 d.p) or $x = 2.25$

A1

(e) The price of the 98-octane unleaded petrol in *January 2019* was a reduction of 7% on the price in *December 2018*. [2]
Find the price of the 98-octane unleaded petrol in December 2018 if it cost less than \$3 for a litre of 95-octane unleaded petrol in January 2019.

95-octane = x = \$2.25 (Jan 2019)

98-octane = 2.25 + 0.25 = \$2.50 per litre in Jan 2019

\$2.50 —— 93% of Dec 2018 price M1

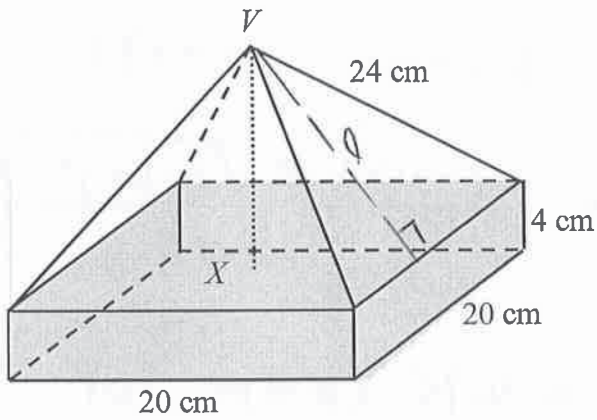
$$\frac{2.50}{93} \times 100\% = 2.68817$$

$= \$2.69$ (2 d.p)

A1

Price of 98-octane in Dec 2018 was \$2.69

7 The diagram shows a container consisting of a square bottom with rectangular sides, each 20 cm by 4 cm, and a regular pyramid on top with perpendicular height given by VX. Water is poured into the container till the brim of the cuboid.



(a) Find the height VX of the pyramid. [2]

Slant height $l = \sqrt{24^2 - 10^2} = \sqrt{476}$ cm (M1)

$VX^2 = (\sqrt{476})^2 - 10^2$

$VX = \sqrt{376}$

$= 19.3907$

$= \underline{19.4 \text{ cm}}$ (3 s.f) (A1)

(b) Calculate the total surface area of the container. [2]

Total surface Area

$= 4(20 \times 4) + 20 \times 20 + 4 \times \frac{1}{2} \times 20 \times \sqrt{476}$ (M1)

$= 320 + 400 + 872.697$

$= 1592.697$

$= \underline{1590 \text{ cm}^2}$ (3 s.f)

(c) Find the volume of water in the cuboid. [1]

Volume of water in the cuboid

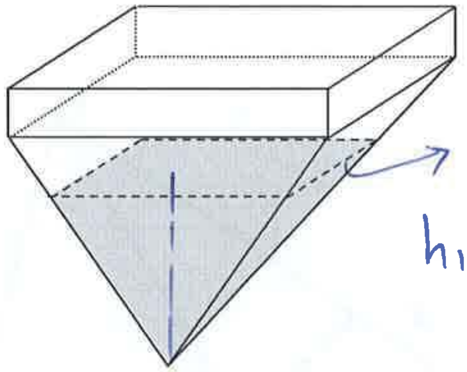
$= 20 \times 20 \times 4$

$= \underline{1600 \text{ cm}^3}$

The container is now inverted as shown in the diagram below.

$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$

let $\frac{h_1}{h_2} = x$



(d) Calculate the depth of the water in the pyramid when inverted. [3]

let x be the ratio of ht of water to the ht of the pyramid

$\frac{1}{3}(20x)(20x)(19.39x) = 1600$ M1

$19.39x^3 = 12$

$x^3 = \frac{12}{19.39}$ M1

$x = 0.8522$

Height of water $= 0.8522 \times 19.39$

$= \underline{16.5 \text{ cm}}$ (3 s.f) A1

(e) Another smaller container, which is geometrically similar, has a square base of 225 cm^2. Both containers are made of the same material. Find the mass of the smaller container in grams, given that the mass of the empty larger container is 1.28 kg. [2]

$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

$\frac{l_1}{l_2} = \sqrt{\frac{225}{400}}$

$= 3/4$

$\frac{V_1}{V_2} = \left(\frac{3}{4}\right)^3$

$= \frac{27}{64}$ M1

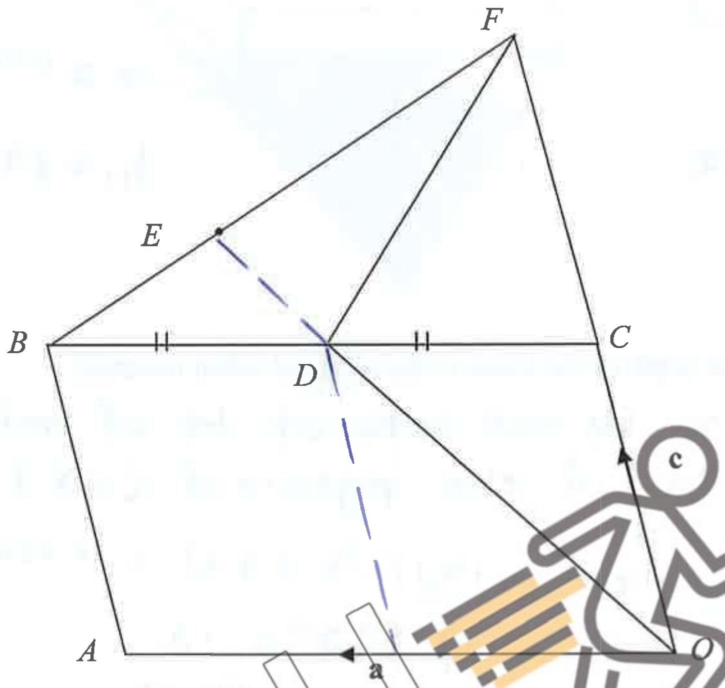
Mass of the smaller container

$= \frac{27}{64} \times 1280 \text{ g}$

$= 27 \times 20 \text{ g}$

$= \underline{540 \text{ g}}$ A1

8 In the diagram, $OABC$ is a parallelogram and D is the midpoint of BC. BE and OC produced intersect at the point F. $BE : BF = 1 : 3$ and $OC : OF = 1 : 2$.
Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.



(a) Express and simply the following vectors in terms of **a** and **c**.

(i) $\overrightarrow{AC}$ **[1]**

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$
$= -\underset{\sim}{a} + \underset{\sim}{c}$ A1

(ii) $\overrightarrow{BF}$ **[1]**

$\overrightarrow{BF} = \overrightarrow{BC} + \overrightarrow{CF}$
$= \overrightarrow{AO} + \overrightarrow{OC}$
$= -\underset{\sim}{a} + \underset{\sim}{c}$ A1

(iii) $\overrightarrow{OD}$ **[1]**

$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD}$
$= \underset{\sim}{c} + \frac{1}{2}\underset{\sim}{a}$ A1

(iv) $\overrightarrow{OE}$ [2]

$\overrightarrow{OE} = \overrightarrow{OF} + \overrightarrow{FE}$

$= 2\underset{\sim}{c} + \frac{2}{3}\overrightarrow{FB}$ (M1)

$= 2\underset{\sim}{c} - \frac{2}{3}(-\underset{\sim}{a} + \underset{\sim}{c})$

$= \frac{4}{3}\underset{\sim}{c} + \frac{2}{3}\underset{\sim}{a}$

$= \frac{4}{3}(\underset{\sim}{c} + \frac{1}{2}\underset{\sim}{a})$ (A1)

(b) State two facts about the vectors $\overrightarrow{OD}$ and $\overrightarrow{OE}$ from the results in **(a)**. [2]

$\overrightarrow{OE} = \frac{4}{3}\overrightarrow{OD}$

$\frac{\overrightarrow{OE}}{\overrightarrow{OB}} = 4:3$

1) $\overrightarrow{OE}$ is parallel to $\overrightarrow{OD}$ $\Rightarrow$ O, D and E are collinear. (A1)

2) OE : OD = 4 : 3 (A1)

(c) Find the ratio of the areas of

(i) ΔODF and ΔOEF, [1]

Area of ΔODF : Area of ΔOEF

$= 3 : 4$ (A1)

(ii) ΔOCD and $OABC$, [1]

Area of ΔOCD : Area of OABC

$= 1 : 4$

(iii) ΔOCD and $OABF$. Areas of ΔOBC : $\Delta BCF = 1 : 1$ [2]

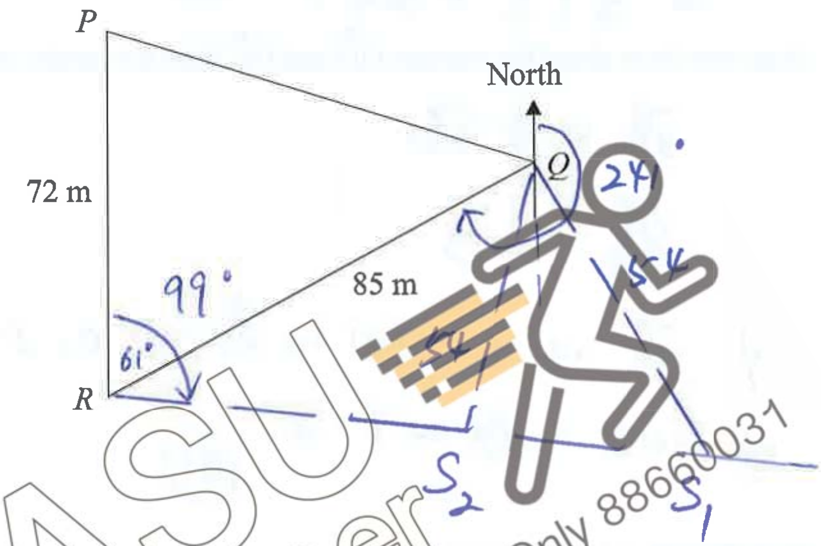
ΔOCD	OABC	ΔBCF
1	4	
	2	1
1	4	2

(M1)

Area of ΔOCD : Area of OABF $= 1 : 6$ (A1)

9 Two school teams, *Novotel* and *Temasek*, are participating in an Amazing Race in Bishan Park. The diagram shows the paths in the park.
The teams assemble at P before heading to Q to start the race.
P is due north of R.
The bearing of R from Q is 241°.
The distance PR is 72 metres and the distance RQ is 85 metres.

(a) Find the distance PQ. [3]



Bearing of Q from R is 241° − 180° = 61° (alt ∠s) (M1)

$\angle PRQ = 61°$

$PQ^2 = 72^2 + 85^2 - 2 \times 72 \times 85 \cos 61°$ (M1)

$PQ = 80.467$

$= \underline{80.5 \text{ m}}$ (3 s.f) (A1)

(b) The final station of the race is at R, each team is required to find a ***clue*** that is hidden at point S before completing the race at R. [3]
The bearing of S from R is 099° and QS is 54 metres.
Given that there are **two** possible locations for S, find the two possible values of angle RSQ.

$\angle QRS = 99° - 61° = 38°$

In $\triangle RQS$

$$\frac{\sin \angle RSQ}{85} = \frac{\sin 38°}{54} \quad \text{(M1)}$$

$$\sin \angle RSQ = \frac{85 \times \sin 38°}{54}$$

$\angle RS_1Q = 75.7188°$

$= \underline{75.7°}$ (1 d.p) (A1)

OR $\angle RS_2Q = 180° - 75.7188°$

$= 104.2812°$

$= \underline{104.3°}$ (1 d.p) (A1)

(c) Both teams manage to find the ***clue*** at the same time and team *Novotel* is closer to *R* than team *Temasak*. [5]
Team *Novotel* claims that they are the winner.
Given that the speed of team *Novotel* is 30% less than the speed of team *Temask* when they travel from *S* to *R*.
Do you agree with team *Novotel* that they will win the race?
Justify your answer with clear working in your calculations.

Novotel — S_2 Temasak — S_1

Team Novotel $\angle RQS_2 = 180° - 38° - 104.2812°$
$= 37.7188°$
$= 37.7°$

Team Temasak $\angle RQS_1 = 180° - 38° - 75.7188°$
$= 66.2812°$
$= 66.3°$

M1

Novotel $\dfrac{RS_2}{\sin 37.7188°} = \dfrac{54}{\sin 38°}$

$RS_2 = 53.66 m$
$= 53.7 m$ M1

Temasak $\dfrac{RS_1}{\sin 66.2812°} = \dfrac{54}{\sin 38°}$

$RS_1 = 80.3017$
$= 80.3 m$ M1

Let the speed of team Temasak be x

$Time_{Novotel} = \dfrac{53.66}{0.7x} = \dfrac{76.657}{x}$

$Time_{Temasak} = \dfrac{80.3017}{x}$ A1

Time taken by Team Novotel is less than time taken by Team Temasak.

Yes, I agree that Team Novotel will win the race.

A1

10 ***Answer the whole of this question on a sheet of graph paper.***

The table below gives some values of x and the corresponding values of y for $y = x(1 + x)(5 - x)$.

x	-2	-1	-0.5	1	2	3	4	5
y	14	0	-1.375	8	18	p	20	0

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-2 \leq x \leq 5$.
Using a scale of 2 cm to represent 5 units, draw a vertical y-axis for $-5 \leq y \leq 25$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve where $x = 4$. [2]

(d) **(i)** On the same axes, draw the line $2x + y = 12$ for $-2 \leq x \leq 5$. [1]

(ii) Write down the x-coordinates of the points where this line intersects the curve. ~~[1]~~ [2]

(iii) The x-coordinates of the points where the two graphs intersect are solutions of the equation $x^3 + Ax^2 + Bx + 12 = 0$. Find the value of A and the value of B. [2]

d(i) $y = -2x + 12$

x	-2	0	4
y	16	12	4

d(iii) $y = -2x + 12$

$y = x(5 + 4x - x^2)$

$= -x^3 + 4x^2 + 5x$

$-x^3 + 4x^2 + 5x = -2x + 12$

$-x^3 + 4x^2 + 7x - 12 = 0$

$x^3 - 4x^2 - 7x + 12 = 0$

$A = -4$ A1

$B = -7$ A1

~~~ End of Paper ~~~

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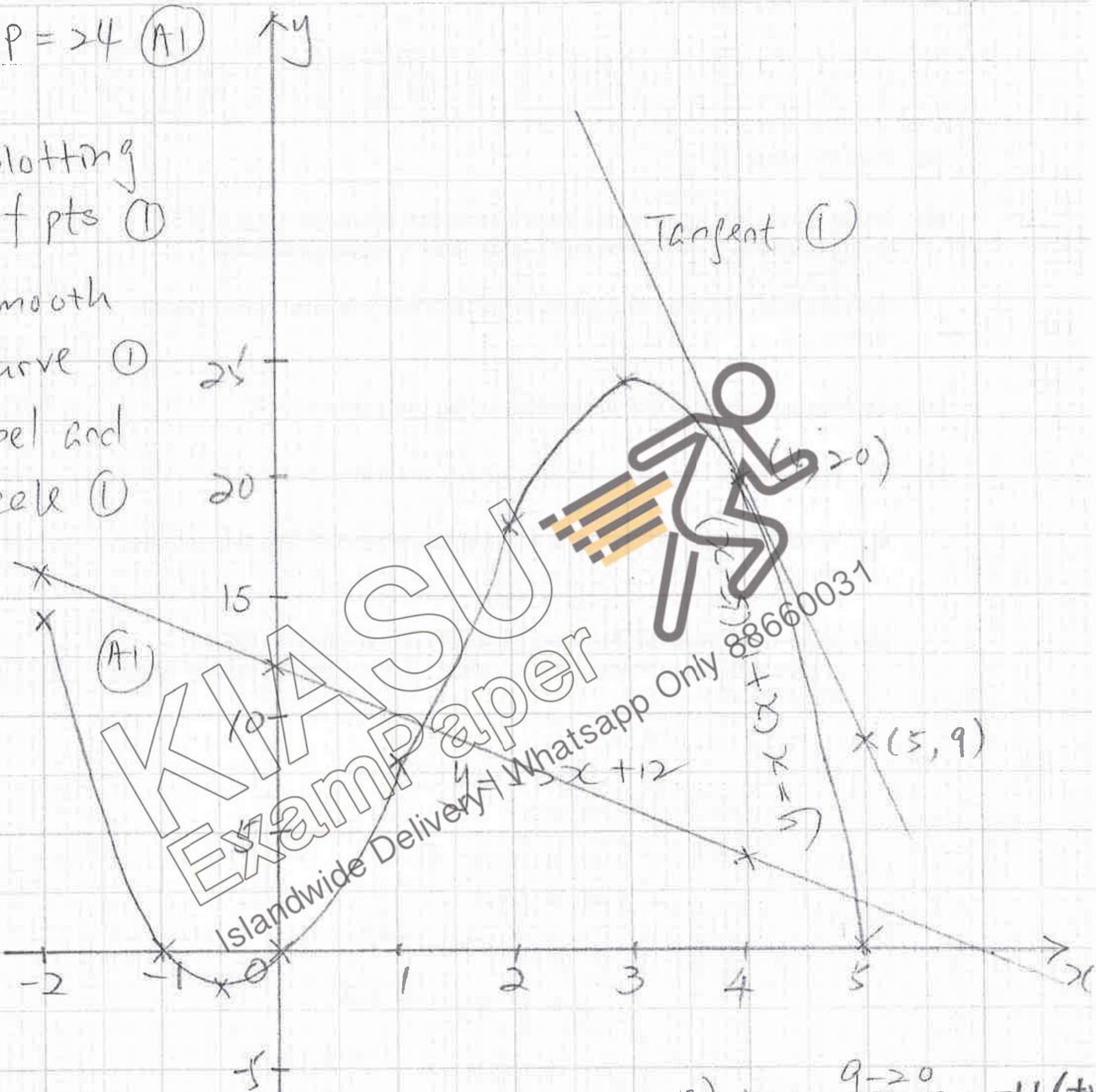
Q10

a) $P = 24$ (A1)

b) plotting of pts (1)

Smooth curve (1)

label and scale (1)



c) $m = \frac{9-20}{5-4} = -11$ (± 1) (A1)

d i) $y = -2x + 12$

d (ii) $x = 1.2 \pm 0.1$
or $x = 4.9 \pm 0.1$ (A2)

d (iii) $A = -4$ (A1) $B = -7$ (A1)