Name | Reg. No | Class

4E5N

ADDITIONAL MATHEMATICS 4047/01

[80 marks]

SEMESTER ONE EXAMINATION

13 May 2019

2 hours

Additional material: Writing paper

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** questions.

Write your answers on the writing paper provided.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

You are expected to use an electronic calculator to evaluate explicit numerical expressions.

If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to **three** significant figures. Answers in degrees should be given to **one** decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

For Examiner's Use

Brand / Model of Calculator

This question paper consists of **7** printed pages, including the cover page.

Setter: Ms Shen Sirui

Vetter: Mr Nara

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

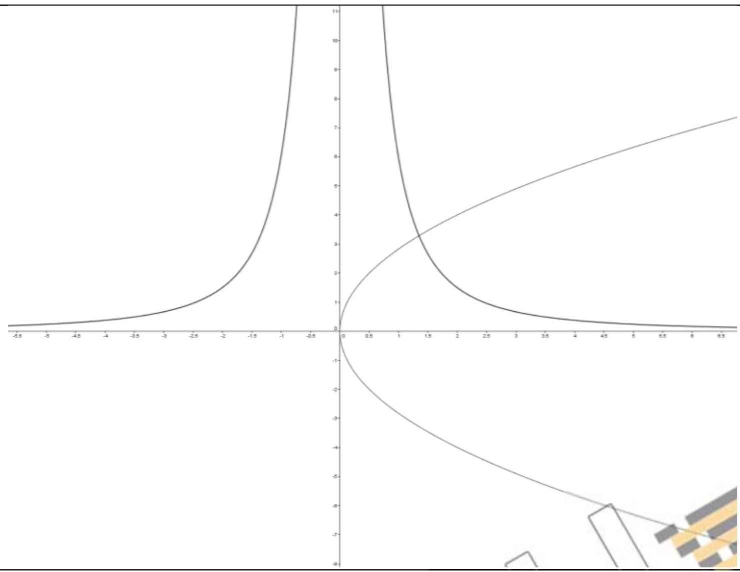
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab \sin C$$

[Turn over

1 **(i)** On the same diagram sketch the curve $y^2 = 8x$ and $y = 6x^{-2}$. [2]

(ii) Find the coordinates of the point of intersection of the two curves. [3]

Qn	Solution	Mark
i		B1 for $y^2 = 8x$ B1 for $y = 6x^{-2}$
ii	$y^2 = 8x - - - (1)$ $y = 6x^{-2} - - - (2)$ Sub (2) into (1): $(6x^{-2})^2 = 8x$ $\frac{36}{x^4} = 8x$ $x^5 = 4.5$ $x = 1.3509$ $y = 3.2877$ Intersection: $(1.35, 3.29)$	M1 for substitution M1 for value of *x* or *y* A1

2 A particle moves along the curve $y = e^{2x}$ in such a way that the y-coordinate of the particle is increasing at a constant rate of 0.3 units per second. Find the y-coordinate of the particle at the instant when the x-coordinate of the particle is increasing at 0.01 units per second.

[4]

Qn	Solution	Mark
	$y = e^{2x}$ $\frac{dy}{dx} = 2e^{2x}$ $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ $2e^{2x} = 0.3 \div 0.01$ $e^{2x} = 15$ $x = \frac{\ln 15}{2}$ Sub $x = \frac{\ln 15}{2}$, $y = e^{2(\frac{\ln 15}{2})} = 15$	M1 for dy/dx M1 for sub into equation connecting dy/dx, dy/dt, dx/dt M1 for $x = \frac{\ln 15}{2}$ or $e^{2x} = 15$ A1

3 The equation of a curve is $y = 3x^2 - kx + 2k - 4$, where k is a constant. Show that the line $y = 2x + 5$ intersects the curve for all real values of k. [5]

Qn	Solution	Mark
	$y = 3x^2 - kx + 2k - 4 - - - (1)$ $y = 2x + 5 - - - (2)$ $(1) = (2)$: $3x^2 - kx + 2k - 4 = 2x + 5$ $3x^2 - kx - 2x + 2k - 9 = 0$ $3x^2 - (k+2)x + 2k - 9 = 0$ $b^2 - 4ac = [-(k+2)]^2 - 4(3)(2k-9)$ $= k^2 + 4k + 4 - 24k + 108$ $= k^2 - 20k + 111$ $= (k-10)^2 - 10^2 + 112$ $= (k-10)^2 + 12$ Since $(k-10)^2 + 12 > 0, b^2 - 4ac > 0$ and line intersects the curve for all real values of k.	M1 for combining equations M1 for $ax^2 + bx + c = 0$ M1 for subs into $b^2 - 4ac$ M1 for $(k-10)^2 + 12$ A1 for conclusion

4 **(a)** Given that $(3^{x+2})(2^{x-2}) = 6^{2x}$, find the value of 6^x . [3]

(b) The side of an equilateral triangle is $6(\sqrt{3}-1)$ cm. **Without using a calculator,** find the exact value of the area of the equilateral triangle in the form $(a + b\sqrt{c})$ cm^2, where a, b and c are integers. [4]

Qn	Solution	Mark
a	$(3^{x+2})(2^{x-2}) = 6^{2x}$ $3^x(3^2)(2^x)(2^{-2}) = 6^{2x}$ $6^x\left(\frac{9}{4}\right) = 6^{2x}$ $6^x = \frac{9}{4}$	M1 for $3^x(3^2)$ or $(2^x)(2^{-2})$ M1 for $6^x\left(\frac{9}{4}\right)$ A1
b	Area $= \frac{1}{2}\left[6\left(\sqrt{3}-1\right)\right]^2 \sin 60$ $= \frac{1}{2}(36)\left(3-2\sqrt{3}+1\right)\left(\frac{\sqrt{3}}{2}\right)$ $= 9\sqrt{3}\left(4-2\sqrt{3}\right)$ $= 36\sqrt{3} - 54$ $= -54 + 36\sqrt{3}$	M1 M1 for $\left(3-2\sqrt{3}+1\right)$ M1 for $\left(\frac{\sqrt{3}}{2}\right)$ A1

5 Find the range of values of x for which the gradient of the graph $y = x^4 - 3x^3 - 6x^2 + 6$ is increasing. [5]

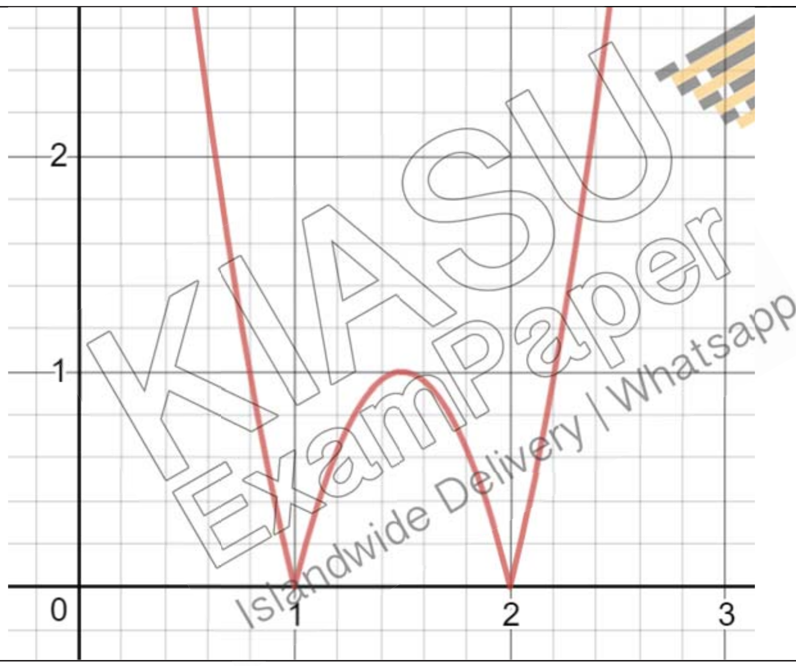
Qn	Solution	Mark
	$y = x^4 - 3x^3 - 6x^2 + 6$ $\frac{dy}{dx} = 4x^3 - 9x^2 - 12x$ $\frac{d^2y}{dx^2} = 12x^2 - 18x - 12$ $12x^2 - 18x - 12 > 0$ $2x^2 - 3x - 2 > 0$ $(2x+1)(x-2) > 0$ $x < -\frac{1}{2}, x > 2$	M1 for $\frac{dy}{dx}$ M1 for $\frac{d^2y}{dx^2}$ M1 for $\frac{d^2y}{dx^2} > 0$ M1 for factorised form A1

6 A curve has the equation $y = (2x - 3)^2 - 1$.

(i) Find the coordinates of the points at which the curve intersects the x-axis. [2]

(ii) Sketch the graph of $y = |(2x - 3)^2 - 1|$. [3]

(iii) Using your graph, state the range of values of k for which $|(2x - 3)^2 - 1| = k$ has 4 solutions. [1]

Qn	Solution	Mark
i	$(2x-3)^2 - 1 = 0$ $2x - 3 = \pm 1$ $x = 1, \quad x = 2$ $(1, 0) \quad (2, 0)$	M1 A1 or B2
ii		T1 for turning point (1.5, 1) P1 for (1, 0) and (2, 0) C1 for shape of graph
iii	$0 < k < 1$	B1 (no mark if students got part ii wrong)

7 It is given that $f'(x) = x + \sin 4x$ and $f(0) = \frac{3}{4}$.

Show that $f''(x) + 16f(x) = 8x^2 + 17$. [5]

Qn	Solution	Mark
	$f(x) = \frac{x^2}{2} - \frac{\cos 4x}{4} + c$	M1 for $\frac{x^2}{2} - \frac{\cos 4x}{4}$
	$\frac{3}{4} = 0 - \frac{1}{4} + c$ $c = 1$	
	$f(x) = \frac{x^2}{2} - \frac{\cos 4x}{4} + 1$	M1 for $f(x) = \frac{x^2}{2} - \frac{\cos 4x}{4} + 1$
	$f''(x) = 1 + 4\cos 4x$	M1 for $1 + 4\cos 4x$
	$f''(x) + 16f(x) = 1 + 4\cos 4x + 16(\frac{x^2}{2} - \frac{\cos 4x}{4} + 1)$	M1 for sub into $f''(x) + 16f(x)$
	$= 1 + 4\cos 4x + 8x^2 - 4\cos 4x + 16$ $= 8x^2 + 17$	A1

8 Solve the equation $6\sin^2 x + 5\cos x = 5$ for $0° < x < 360°$. [5]

Qn	Solution	Mark
	$6(1 - \cos^2 x) + 5\cos x = 5$	M1 for $1 - \cos^2 x$
	$6 - 6\cos^2 x + 5\cos x - 5 = 0$ $6\cos^2 x - 5\cos x - 1 = 0$	M1 for equation
	$(6\cos x + 1)(\cos x - 1) = 0$	
	$\cos x = -\frac{1}{6}$, $\cos x = 1$	M1 for $\cos x = -\frac{1}{6}$
	$\alpha = 80.405$ (Rej)	M1 for basic angle
	$x = 180 - \alpha, 180 + \alpha$ $x = 99.6°,\ 260.4°$	A1 for both answers Ignore if students do not reject $\cos x = 1$

9 **(a)** Given that the first two non-zero terms in the expansion, in ascending powers of x, of $(1+bx)(1+ax)^6$ are 1 and $-\frac{21}{4}x^2$ and that $a>0$, find the value of a and of b. [5]

(b) Find the term independent of x in the expansion of $\left(2x+\frac{1}{x^2}\right)^9$. [3]

Qn	Solution	Mark
a	$(1+ax)^6 = 1+\binom{6}{1}(1)^5(ax)^1+\binom{6}{2}(1)^4(ax)^2+\cdots$ $=1+6ax+15a^2x^2+\cdots$ $(1+bx)(1+ax)^6=(1+bx)(1+6ax+15a^2x^2+\cdots)$ $=1+6ax+bx+15a^2x^2+6abx^2+\cdots$ $6a+b=0$ $b=-6a---(1)$ $15a^2+6ab=-\frac{21}{4}---(2)$ sub (1) into (2): $15a^2+6a(-6a)=-\frac{21}{4}$ $21a^2=\frac{21}{4}$ $a^2=\frac{1}{4}$ $a=\frac{1}{2}$ $b=-3$	M1 for $1+6ax+15a^2x^2$ M1 for $6a+b=0$ M1 for $15a^2+6ab=-\frac{21}{4}$ A1 A1
b	$T_{r+1}=\binom{9}{r}(2x)^{9-r}\left(\frac{1}{x^2}\right)^r$ For x^0, $x^{9-r}(x)^{-2r}=x^0$ $r=3$ $T_{3+1}=\binom{9}{3}(2x)^{9-3}\left(\frac{1}{x^2}\right)^3$ $=84(2x)^6(x)^{-6}$ $=5376$	M1 for $\binom{9}{r}(2x)^{9-r}\left(\frac{1}{x^2}\right)^r$ M1 for $r=3$ A1

10 The equation of a curve is $y = \frac{x^2}{2x-1}$.

(i) Find the coordinates of the stationary points of the curve. [4]

(ii) Determine the nature of each of the stationary points of the curve. [4]

<table>
<tr><th>Qn</th><th>Solution</th><th>Mark</th></tr>
<tr><td>i</td><td>$y = \frac{x^2}{2x-1}$
$\frac{dy}{dx} = \frac{2x(2x-1) - 2x^2}{(2x-1)^2}$
$= \frac{2x^2 - 2x}{(2x-1)^2}$

when $\frac{dy}{dx} = 0, \frac{2x^2 - 2x}{(2x-1)^2} = 0$
$2x(x-1) = 0$
$x = 0, x = 1$
$y = 0, y = 1$

Stationary points: $(0, 0)$ and $(1, 1)$</td><td>M1 for quotient or product rule

M1 for $\frac{2x^2-2x}{(2x-1)^2} = 0$

M1 for both x

A1 for both coordinates</td></tr>
<tr><td>ii</td><td>$\frac{d^2y}{dx^2} = \frac{(4x-2)(2x-1)^2 - 4(2x-1)(2x^2-2x)}{(2x-1)^4}$
when $x = 0, \frac{d^2y}{dx^2} = -2 < 0$
$(0, 0)$ is maximum point.

when $x = 1, \frac{d^2y}{dx^2} = 2 > 0$
$(1, 1)$ is minimum point.

OR

<table><tr><td>x</td><td>-0.1</td><td>0</td><td>0.1</td></tr><tr><td>$\frac{dy}{dx}$</td><td>> 0</td><td>0</td><td>< 0</td></tr></table>
$(0, 0)$ is maximum point.

<table><tr><td>x</td><td>0.9</td><td>1</td><td>1.1</td></tr><tr><td>$\frac{dy}{dx}$</td><td>< 0</td><td>0</td><td>> 0</td></tr></table>
(1, 1) is minimum point.</td><td>M1 for $\frac{d^2y}{dx^2}$

M1 for sub either $x = 0$ or $x = 1$ into $\frac{d^2y}{dx^2}$

A1 for (0, 0) max pt

A1 for (1, 1) min pt

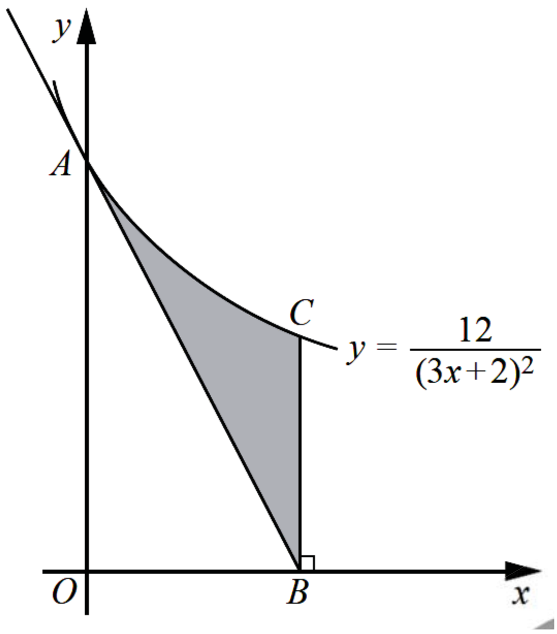
M1 for 1st derivative test

A1 for (0, 0) max pt

M1 for 1st derivative test

A1 for (1, 1) min pt</td></tr>
</table>

11



The diagram shows part of the curve $y = \frac{12}{(3x+2)^2}$ meeting the y-axis at point A. The tangent to the curve at A intersects the x-axis at point B. Point C lies on the curve such that BC is parallel to the y-axis. Find

(i) the equation of AB, [4]

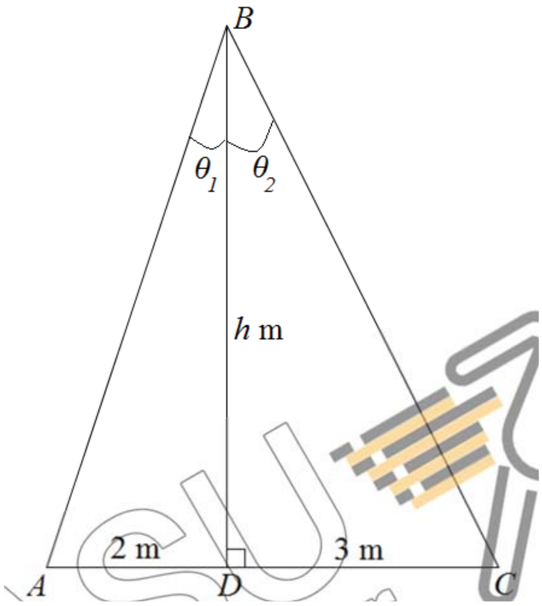
(ii) the area of the shaded region. [5]

Qn	Solution	Mark
i	$y = \frac{12}{(3x+2)^2}$ $\frac{dy}{dx} = -24(3x+2)^{-3}(3)$ $= -\frac{72}{(3x+2)^3}$ when $x = 0, \frac{dy}{dx} = -9$ when $x = 0, y = 3$ Line AB: $y = -9x + 3$	M1 for dy/dx M1 for dy/dx at A M1 for $y = 3$ A1
ii	sub $y = 0, 0 = -9x + 3$ $x = \frac{1}{3}$ $B = \left(\frac{1}{3}, 0\right)$	M1 for x-coordinate of B

	Area of $OACB = \int_0^{\frac{1}{3}} 12(3x+2)^{-2}\,dx$ $= \left[\frac{12(3x+2)^{-1}}{-1(3)}\right]_0^{\frac{1}{3}}$ $= \left[-\frac{4}{3x+2}\right]_0^{\frac{1}{3}}$ $= -\frac{4}{3\left(\frac{1}{3}\right)+2} - \left(-\frac{4}{3(0)+2}\right)$ $= \frac{2}{3}$ Area of $\Delta OAB = \frac{1}{2}\left(\frac{1}{3}\right)(3) = \frac{1}{2}$ Area of shaded region $= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$ unit^2	M1 for $-\frac{4}{3x+2}$ (independent of limits) M1 for area of *OACB* M1 for area of tri *OAB* A1

12 **(a)** State the values between which the principal value of $\tan^{-1} x$ must lie. Give your answer in terms of π. [1]

(b) The diagram below shows triangle *ABC* where *AD* = 2 m, *DC* = 3 m and *BD* = *h* m. *BD* is perpendicular to *AC* and $\theta_1 + \theta_2 = 45°$.



By using a suitable formula for $\tan(\theta_1 + \theta_2)$, find the value of *h*. [5]

Qn	Solution	Mark
a	$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$	B1
b	$\tan(\theta_1 + \theta_2) = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}$	M1 for tan addition formula
	$\tan 45 = \frac{\frac{2}{h} + \frac{3}{h}}{1 - \left(\frac{2}{h}\right)\left(\frac{3}{h}\right)}$	M1 for either $\tan\theta_1 = \frac{2}{h}$ or $\tan\theta_2 = \frac{3}{h}$
	$1 = \frac{\frac{2}{h} + \frac{3}{h}}{1 - \left(\frac{2}{h}\right)\left(\frac{3}{h}\right)}$	M1 for tan 45 = 1
	$1 - \frac{6}{h^2} = \frac{5}{h}$	
	$h^2 - 5h - 6 = 0$	M1 for $h^2 - 5h - 6 = 0$
	$(h-6)(h+1) = 0$	
	$h = 6, \quad h = -1$ (rej)	A1

13 The Ultraviolet Index describes the level of solar radiation on the earth's surface. The Ultraviolet Index, U, measured from the top of a building is given by $U = 6 - 5\cos qt$, where t is the time in hours, $0 \leq t \leq 20$, from the lowest value of Ultraviolet Index and q is a constant. It takes 10 hours for the Ultraviolet Index to reach its lowest value again.

(i) Explain why it is impossible to measure a Ultraviolet Index of 12. [1]

(ii) Show that $q = \frac{\pi}{5}$. [1]

(iii) The top of the building is equipped with solar panels that supply power to the building when the Ultraviolet Index is at least 3.5. Find the duration, in hours and minutes, that the building is supplied with power by the solar panels. [5]

Qn	Solution	Mark
i	Max $U = 6 + 5 = 11$ Since max value of $U = 11$, we cannot measure a Ultraviolet Index of 12.	B1 for stating max value of U
ii	$10 = \frac{2\pi}{q}$ $q = \frac{2\pi}{10} = \frac{\pi}{5}$	B1 for $q = \frac{2\pi}{10}$
iii	$6 - 5\cos\frac{\pi}{5}t = 3.5$ $\cos\frac{\pi}{5}t = \frac{1}{2}$ $\alpha = \frac{\pi}{3}$ $\frac{\pi}{5}t = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, \frac{\pi}{3} + 2\pi, 2\pi - \frac{\pi}{3} + 2\pi$ $t = 1.6666, 8.3333, 11.66, 18.33$ Duration $= (8.3333 - 1.6666) + (18.33 - 11.66)$ $= 13.3367$ $=$ 13 hours 20 mins	M1 for forming equation M1 for basic angle M1 for $\frac{\pi}{3}, 2\pi - \frac{\pi}{3}$ M1 for all 4 values A1

END OF PAPER

Name | Reg. No | Class

MARK SCHEME

4E/5N

ADDITIONAL MATHEMATICS — 4047/02

Paper 2 [100 marks]

SEMESTER ONE EXAMINATION

May 2019

2 hours 30 minutes

Candidates answer on the question paper.

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.
Write your name, register number and class on all the work you hand in.
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You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **ALL** questions.
Write your answers in the spaces provided on the question paper.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

You are expected to use a scientific calculator to evaluate explicit numerical expressions.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three** significant figures. Give answers in degrees to **one** decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

Brand / Model of Calculator

For Examiner's Use	
Total	100

This question paper consists of **15** printed pages.

Setter: Mr. Gabriel Cheow
Vetter: Mr. Narayanan

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \quad \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \quad \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

[Turn over

For Examiner's Use

1 The roots of the quadratic equation $2x^2 - 8x + 9 = 0$ are α and β.

(i) Show that the value of $\alpha^3 + \beta^3$ is 10. [3]

$\alpha + \beta = 4, \alpha\beta = \frac{9}{2}$ **M1 – sum & pdt**

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= 4^2 - 9$
$= 7$ **M1**

$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$
$= (4)\left(7 - \frac{9}{2}\right)$
$= 10\ (shown)$ **A1**

(ii) Find a quadratic equation whose roots are $\frac{1}{\alpha^2 + \beta}$ and $\frac{1}{\alpha + \beta^2}$. [4]

$New\ sum: \frac{1}{\alpha^2 + \beta} + \frac{1}{\alpha + \beta^2} = \frac{\alpha + \beta^2 + \alpha^2 + \beta}{(\alpha^2 + \beta)(\alpha + \beta^2)}$ **Allow for ECF**

$= \frac{\alpha + \beta^2 + \alpha^2 + \beta}{\alpha^3 + \beta\alpha + \alpha^2\beta^2 + \beta^3}$

$= \frac{4 + 7}{10 + \frac{9}{2} + \left(\frac{9}{2}\right)^2}$ **M1**

$= \frac{44}{139}$ **M1**

$New\ pdt: \frac{1}{\alpha^2 + \beta} \times \frac{1}{\alpha + \beta^2} = \frac{1}{(\alpha^2 + \beta)(\alpha + \beta^2)}$

$= \frac{1}{10 + \frac{9}{2} + \left(\frac{9}{2}\right)^2}$

$= \frac{4}{139}$ **M1**

$New\ eqn: x^2 - \frac{44}{139}x + \frac{4}{139} = 0$ **A1**

$139x^2 - 44x + 4 = 0$

For Examiner's Use

2 The function $f(x) = 6x^3 + ax^2 + bx - 12$, where a and b are constants, is exactly divisible by $x + 2$ and leaves a remainder of 5 when divided by $x + 1$.

(i) Find the value of a and of b. [4]

$$f(-2) = 0$$
$$-48 + 4a - 2b - 12 = 0 \quad \textbf{M1}$$
$$2a - b = 30 \cdots\cdots Eqn\ 1$$

$$f(-1) = 5$$
$$-6 + a - b - 12 = 5 \quad \textbf{M1}$$
$$a - b = 23 \cdots\cdots Eqn\ 2$$

$$Eqn\ 1 - Eqn\ 2: a = 7 \quad \textbf{A1}$$
$$Sub\ into\ Eqn\ 1: b = -16 \quad \textbf{A1}$$

(ii) By showing your working clearly, factorise $f(x)$. [3]

$$6x^3 + 7x^2 - 16x - 12 = (x + 2)(Ax^2 + Bx + C)$$

$$By\ observation: A = 6, C = -6$$

$$\Rightarrow 6x^3 + 7x^2 - 16x - 12 = (x + 2)(6x^2 + Bx - 6) \quad \textbf{M1}$$

$$Let\ x = 1:$$
$$6 + 7 - 16 - 12 = (3)(6 + B - 6)$$
$$-15 = 3B$$
$$B = -5$$

$$6x^3 + 7x^2 - 16x - 12 = (x + 2)(6x^2 - 5x - 6) \quad \textbf{M1}$$
$$= (x + 2)(3x + 2)(2x - 3) \quad \textbf{A1}$$

(iii) Hence, solve the equation $6(2^{3y}) + 2^{2y+3} - 2^{y+4} - 12 = 2^{2y}$ [4]

$$6(2^{3y}) + 2^{2y+3} - 2^{y+4} - 12 = 2^{2y}$$
$$6(2^{3y}) + 8(2^{2y}) - 16(2^y) - 12 = 2^{2y}$$
$$6(2^y)^3 + 7(2^y)^2 - 16(2^y) - 12 = 0 \quad \textbf{M1}$$

$$Let\ x = 2^y$$

$$\Rightarrow (x + 2)(3x + 2)(2x - 3) = 0 \quad \textbf{M1}$$
$$x = -2, -\frac{2}{3}, \frac{3}{2}$$
$$2^y = -2\ (rej.), -\frac{2}{3}(rej.), \frac{3}{2} \quad \textbf{M1}$$
$$y \ln 2 = \ln\frac{3}{2}$$
$$y = \frac{\ln 1.5}{\ln 2} = 0.585\ (3sf) \quad \textbf{A1}$$

For Examiner's Use

3 **(i)** Express $\dfrac{2x+16}{(x^2+4)(2x-1)}$ in partial fractions. [5]

$$\frac{2x+16}{(x^2+4)(2x-1)} = \frac{Ax+B}{x^2+4} + \frac{C}{2x-1}$$

$$2x+16 = (Ax+B)(2x-1) + C(x^2+4)$$ **M1**

Let $x = 0.5$:

$17 = C\left(\frac{17}{4}\right)$

$C = 4$ **M1**

Let $x = 0$:

$16 = B(-1) + 4(4)$

$B = 0$ **M1**

Let $x = -1$:

$14 = -A(-3) + 20$

$3A = -6$

$A = -2$ **M1**

$$\therefore \frac{2x+16}{(x^2+4)(2x-1)} = \frac{-2x}{x^2+4} + \frac{4}{2x-1}$$ **A1**

(ii) Differentiate $\ln(x^2+4)$ with respect to x. [2]

$$\frac{d}{dx}[\ln(x^2+4)] = \frac{2x}{x^2+4}$$ **B2**

(iii) Hence, using your results in **(i)** and **(ii)**, find $\displaystyle\int \frac{x+8}{(x^2+4)(2x-1)}\,dx$. [4]

$$\int \frac{x+8}{(x^2+4)(2x-1)}\,dx = \frac{1}{2}\int \frac{2x+16}{(x^2+4)(2x-1)}\,dx$$

$$= \frac{1}{2}\int \left(\frac{-2x}{x^2+4} + \frac{4}{2x-1}\right) dx$$ **M1 partial frac**

$$= -\frac{1}{2}\int \frac{2x}{x^2+4}\,dx + \frac{1}{2}(2\ln(2x-1) + c_1)$$ **M1**

$$= -\frac{1}{2}\ln(x^2+4) + c_2 + \ln(2x-1) + \frac{1}{2}c_1$$ **M1**

$$= \ln(2x-1) - \frac{1}{2}\ln(x^2+4) + c$$ **A1**

For Examiner's Use

4 Prove the following identities.

(a) $(\sec x - \tan x)(\operatorname{cosec} x + 1) = \cot x$ [3]

$$LHS = (\sec x - \tan x)(cosec\ x + 1)$$

$$= \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x} + \frac{\sin x}{\sin x}\right)$$ **M1** sin and cos only

$$= \frac{(1 - \sin x)(1 + \sin x)}{\cos x \sin x}$$ **M1** single fraction

$$= \frac{1 - \sin^2 x}{\cos x \sin x}$$

$$= \frac{\cos^2 x}{\cos x \sin x}$$ **M1** cos^2 + sin^2 = 1

$$= \frac{\cos x}{\sin x}$$

$$= \cot x$$

$$= RHS\ (proven)$$

(b) $\dfrac{1 - \cos 2x + \sin x}{\sin 2x + \cos x} = \tan x$ [3]

$$LHS = \frac{1 - \cos 2x + \sin x}{\sin 2x + \cos x}$$

$$= \frac{1 - (1 - 2\sin^2 x) + \sin x}{2\sin x \cos x + \cos x}$$ **M1** cosine double angle; **M1** sine double angle

$$= \frac{2\sin^2 x + \sin x}{\cos x\,(2\sin x + 1)}$$

$$= \frac{\sin x\,(2\sin x + 1)}{\cos x\,(2\sin x + 1)}$$ **M1** factorise and cancel

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

$$= RHS\ (proven)$$

For Examiner's Use

For Examiner's Use

5 The lines $y=8$ and $4x+3y=30$ are tangent to a circle C at the points $(-1,8)$ and $(3,6)$ respectively.

(i) Show that the equation of C is $x^2+y^2+2x-6y-15=0$. [5]

Let centre of circle be O.
Horizontal tangent at $(-1,8)$ means that O is on the line $x = -1$. **M1**

To find normal of circle at $(3,6)$:

$$4x+3y=30$$
$$y=-\frac{4}{3}x+10$$
$$\therefore m_{normal}=\frac{3}{4}$$

$eqn\ of\ normal: y-6=\frac{3}{4}(x-3)$ **M1**

$When\ x=-1, y=3. \Rightarrow O(-1,3)$ **M1**

Horizontal tangent is $y = 8$. Hence radius is 5! **M1**

$$(x+1)^2+(y-3)^2=5^2$$ **M1**
$$x^2+2x+1+y^2-6x+9=25$$
$$x^2+y^2+2x-6y-15=0\ (shown)$$

(ii) Explain whether or not the x-axis is tangent to C. [3]

C has centre $(-1,3)$ and radius 5.
Hence its horizontal tangents are $y=3\pm5 \Rightarrow y=8\ or\ y=-2$ **M1**

x-axis is $y=0$, which is between the two horizontal tangents. **M1**
Hence the x-axis will cut through C at two points.
Hence the x-axis is **not** tangent to C. **A1**

Alternative solution: Sub $y=0$ into eqn of C, show that $b^2-4ac\neq0$.

(iii) The points Q and R also lie on the circle, and the length of the chord QR is 2 units. Calculate the shortest distance from the center of C to the chord QR. [2]

Let M be midpoint of QR. Hence OM perpendicular to QR.
Hence, OM is shortest distance from C to chord QR.
Consider right-angled triangle OMR.
By Pythagoras Theorem,

$$OM=\sqrt{5^2-\left(\frac{2}{2}\right)^2}$$ **M1**
$$=\sqrt{24}=2\sqrt{6}$$
$$=4.90\ (3sf)$$ **A1**

For Examiner's Use

For Examiner's Use

6 The table shows experimental values of two variables *x* and *y*, which are known to be connected by the equation $yx^n = A$, where *n* and *A* are constants.

x	1.0	1.5	2.0	2.5	3.0
y	22.0	13.0	8.9	6.9	5.3

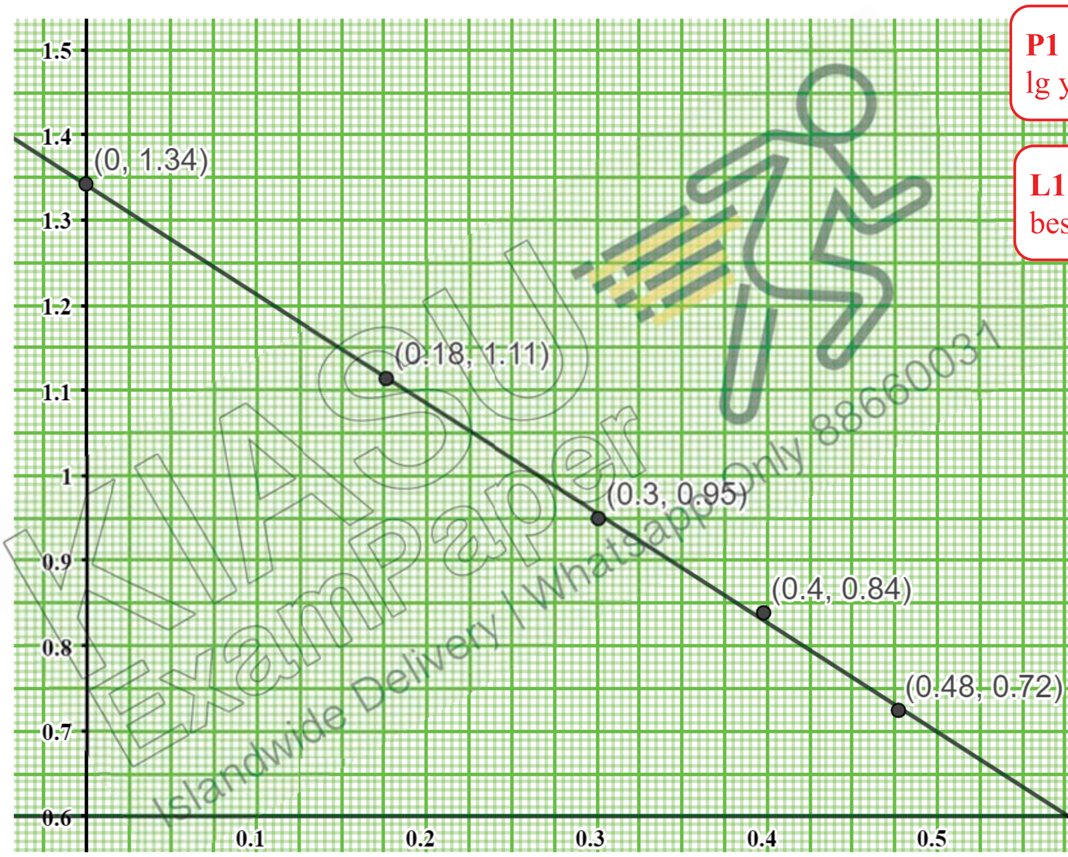
(i) Plot lg *y* against lg *x* and draw a straight line graph. [3]

lg x	0	0.176	0.301	0.398	0.477
lg y	1.34	1.11	0.949	0.839	0.724

T1 table of values

P1 plot of lg y / lg x

L1 scale & best fit line



Scale: 4 cm to 0.1 units on X-axis, 2 cm to 0.1 units on Y-axis.
Scale used must be appropriate in order to award L1.

For Examiner's Use

(ii) Use your graph to estimate the value of A and of n. [4]

$$yx^n = A$$

$$\lg y + n\lg x = \lg A$$ **M1**

$$\lg y = -n\lg x + \lg A$$

$$Y = mX + c$$

$$\Rightarrow m = -n, c = \lg A$$

$$m = \frac{0.7 - 1.34}{0.5 - 0} = -1.28$$ **M1**

$$n = 1.28$$ **A1**

$$c = 1.34$$

$$\lg A = 1.34$$

$$A = 10^{1.34} = 21.9$$ **A1**

(iii) On the same diagram, draw the line representing the equation $y = x^2$ and hence find the value of x which satisfies the equation $x^{n+2} = A$. [2]

$$y = x^2$$

$$\lg y = 2\lg x$$

$Draw: Y = 2X$ **M1 for drawing line**

$$x^{n+2} = A$$

$$(n+2)\lg x = \lg A$$

$$2\lg x = -n\lg x + \lg A$$

Let graph 1 be $\lg y = 2\lg x$, and
Let graph 2 be $\lg y = -n\lg x + \lg A$

From graph, let intersection be (X, Y).

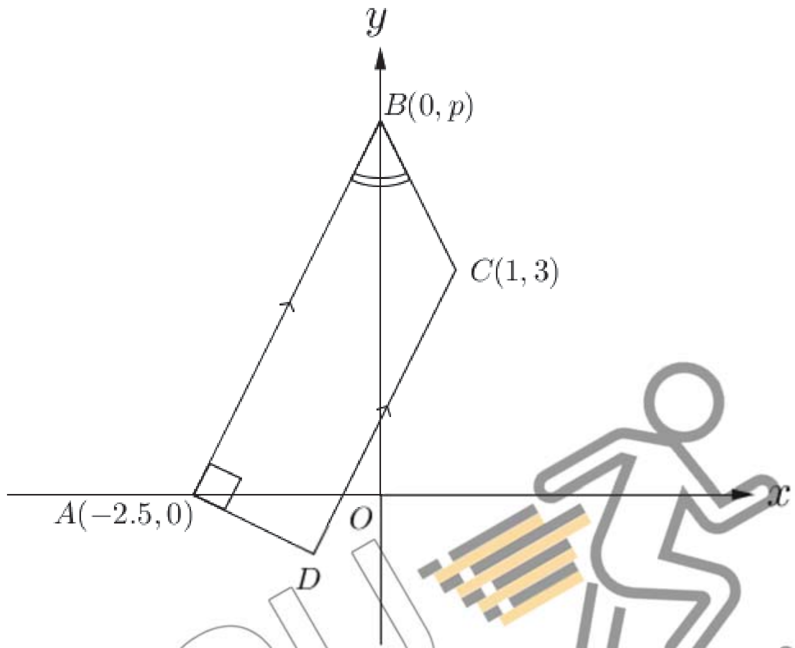
$$(X, Y) = (0.41, 0.82)$$

$$\lg x = 0.41$$

$$x = 10^{0.41} = 2.57$$ **A1**

For Examiner's Use

7 The diagram shows a trapezium with vertices $A(-2.5, 0)$, $B(0, p)$, $C(1, 3)$ and D.
The sides AB and DC are parallel and the angle DAB is 90°.
Angle ABO is equal to angle CBO.



(i) Express the gradients of the lines AB and CB in terms of p and hence, or otherwise, show that $p = 5$. [3]

$m_{AB} = \frac{p}{2.5}$

$= \frac{2p}{5}$ **M1**

$m_{BC} = \frac{3-p}{1}$ **M1**

$m_{AB} = -m_{BC}$

$\frac{2p}{5} = \frac{p-3}{1}$ **M1**

$2p = 5p - 15$

$3p = 15$

$p = 5\ (shown)$

For Examiner's Use

(ii) Find the coordinates of *D*. [4]

$m_{CD} = m_{AB} = 2$

$m_{AD} = -\frac{1}{2} \because AD \perp CD$ **M1** $m_1m_2 = -1$

$Let\ D(k, h)$

$m_{CD} = \frac{3-h}{1-k}$

$2 = \frac{3-h}{1-k}$

$3 - h = 2 - 2k$

$h = 2k + 1 \cdots\cdots Eqn\ 1$ **M1** form eqn of k, h

$m_{AD} = \frac{h-0}{k+2.5}$

$-\frac{1}{2} = \frac{h-0}{k+2.5}$

$2h = -k - 2.5 \cdots\cdots Eqn\ 2$

$Eqn\ 1\ in\ Eqn\ 2: \quad 2(2k+1) = -k - 2.5$

$5k = -2 - 2.5$

$k = -0.9$ **M1** solving either unknown

$Eqn\ 1 + 2 \times Eqn\ 2: \ 5h = -4$

$h = -0.8$

$\therefore\ D(-0.9, -0.8)$ **A1**

Alternative method: finding eqn of line *AD* and eqn of line *CD*.

(iii) Find the area of the trapezium *ABCD*. [2]

$Area = \frac{1}{2}\begin{vmatrix} 1 & 0 & -2.5 & -0.9 & 1 \\ 3 & 5 & 0 & -0.8 & 3 \end{vmatrix}$ **M1** shoelace method Or attempt to cut up trapezium

$= \frac{1}{2}\left[\left(5 + 2 - \frac{2}{7}\right) - (-12.5 - 0.8)\right]$

$= 21\ units^2$ **A1**

For Examiner's Use

8 **(a)** Solve the equation $3\log_x 3 = 8 - 4\log_3 x$. [5]

$$3\log_x 3 = 8 - 4\log_3 x$$

$$\frac{3}{\log_3 x} = 8 - 4\log_3 x$$ **M1** common log base 3 eqn

$$Let\ y = \log_3 x$$

$$\frac{3}{y} = 8 - 4y$$

$$3 = 8y - 4y^2$$

$$4y^2 - 8y + 3 = 0$$ **M1** simplify to quad eqn

$$(2y-3)(2y-1) = 0$$

$$y = 1.5\ or\ 0.5$$ **M1**

$$x = 3^{1.5}\ or\ 3^{0.5}$$

$$= \sqrt{27}\ or\ \sqrt{3}$$ **A1, A1**

(b) It is given that $\log_a x = p$ and $\log_a y = q$.

Express $\log_y ax^2y^3$ in terms of p and q. [3]

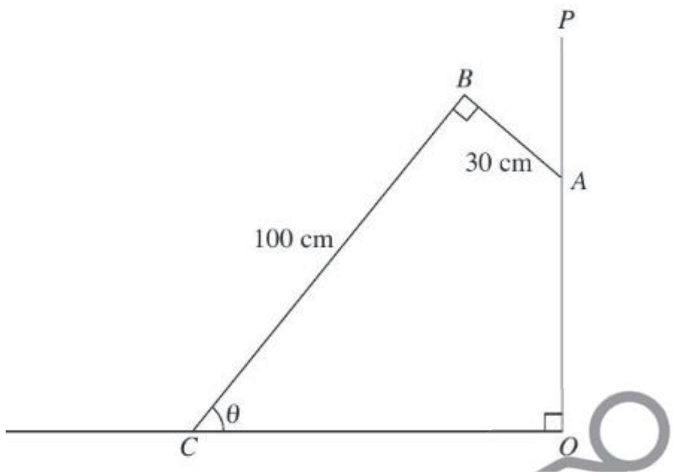
$$\log_y ax^2y^3 = \log_y a + 2\log_y x + 3\log_y y$$ **M1** splitting of logs

$$= \frac{1}{\log_a y} + 2\cdot\frac{\log_a x}{\log_a y} + 3$$ **M1**

$$= \frac{1}{q} + \frac{2p}{q} + 3$$ **A1**

For Examiner's Use

9 The figure shows a stage prop ABC used by a member of the theatre, leaning against a vertical wall OP. It is given that $AB = 30$ cm, $BC = 100$ cm, $\angle ABC = \angle AOC = 90°$ and $\angle BCO = \theta$.



(i) Show that $OC = (100\cos\theta + 30\sin\theta)$ cm. [2]

Let D be foot of B on OC, let E be foot of A on BD.

$\cos\theta = \frac{CD}{100} \Rightarrow CD = 100\cos\theta$ **M1**

$\sin\theta = \frac{AE}{30} \Rightarrow AE = 30\sin\theta$ **M1**

$OC = CD + AE = 100\cos\theta + 30\sin\theta$

(ii) Express OC in terms of $R\cos(\theta - \alpha)$, where R is a positive constant and α is an acute angle. [3]

$R = \sqrt{100^2 + 30^2}$

$= 100\sqrt{109}$ **M1 for R**

$\alpha = \tan^{-1}\left(\frac{30}{100}\right)$ **M1 for alpha**

$= 16.7°(1dp)$

$\therefore OC = 10\sqrt{109}\cos(\theta - 16.7°)$ **A1**

(iii) State the maximum value of OC and the corresponding value of θ. [2]

$OC_{max} = 10\sqrt{109}$ **B1**

$\theta = 16.7°$ **B1**

(iv) Find the value of θ for which $OC = 80$ cm. [3]

$80 = 10\sqrt{109}\cos(\theta - 16.7°)$

$\cos(\theta - 16.7°) = \frac{8}{\sqrt{109}}$ **M1**

$\theta - 16.7° = 39.98°\ (\theta \text{ is acute})$ **M1**

$\theta = 56.7°$ **A1**

For Examiner's Use

10 Given that $y = a + b\cos 4x$, where a and b are integers, and x is in radians,

(i) state the period of y. [1]

$\frac{\pi}{2}$ **B1**

Given that the maximum and minimum values of y are 3 and – 5 respectively, find

(ii) the amplitude of y, [1]

$$amplitude = \frac{3-(-5)}{2}$$
$$= 4$$ **B1**

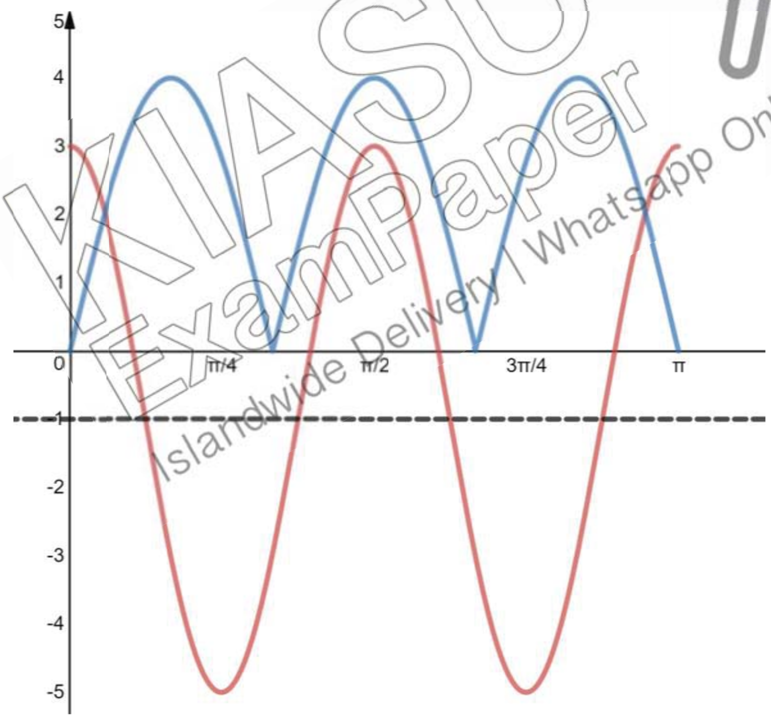
(iii) the value of a and of b. [2]

$b = 4$ **B1**

$a = -1$ **B1**

Using the values of a and b found in part **(iii)**,

(iv) sketch the graph of $y = a + b\cos 4x$ for $0 \le x \le \pi$. [3]



C1 shape (cos)

P1 two periods

I1 impt coords

ECF max 2 marks

S1 shape (sin)

M1 modulus

(v) On the same set of axes, sketch the graph of $y = |4\sin 3x|$, and hence state the number of solutions of $a + b\cos 4x = |4\sin 3x|$. [3]

Number of solutions = 2 **A1**

For Examiner's Use

11 The dimensions of a cuboid are $3x$ cm by $2x$ cm by h cm and its total surface area is 312 cm^2. The volume of the cuboid is V cm^3.

(i) Express h in terms of x. [2]

$$2[3x(2x) + 3xh + 2xh] = 312$$
M1 form eqn of total SA

$$6x^2 + 5xh = 156$$

$$h = \frac{156 - 6x^2}{5x}$$
A1

(ii) Show that $V = \frac{36}{5}x(26 - x^2)$. [2]

$$V = (3x)(2x)\left(\frac{156 - 6x^2}{5x}\right)$$
M1

$$= 6x\left(\frac{156 - 6x^2}{5}\right)$$
M1

$$= \frac{36}{5}x(26 - x^2)$$

(iii) Find the maximum volume of the cuboid as x varies, giving your answer to the nearest cm^3. [5]

$$\frac{dV}{dx} = \frac{36}{5}[(26 - x^2) + x(-2x)]$$

$$= \frac{36}{5}[-3x^2 + 26]$$
M1 differentiate

$$\frac{dV}{dx} = 0$$

$$3x^2 - 26 = 0$$

$$x^2 = \frac{26}{3}$$

$$x = \pm\sqrt{\frac{26}{3}} \; (rej. -ve \because x > 0)$$
M1 solve for x

$$\frac{d^2V}{dx^2} = \frac{36}{5}(-6x)$$

$$\left.\frac{d^2V}{dx^2}\right|_{x=\sqrt{\frac{26}{3}}} = \frac{36}{5}(-6)\left(\sqrt{\frac{26}{3}}\right) < 0 \Rightarrow max$$
M1 2^{nd} deriv. test

$$V = \frac{36}{5}\left(\sqrt{\frac{26}{3}}\right)\left(26 - \frac{26}{3}\right)$$
M1

$$= 367.4\ldots$$

$$= 367 \; cm^3$$
A1

For Examiner's Use

– End of Paper –