West Spring Secondary School

PRELIMINARY EXAMINATION 2019

Mathematics Paper 1 **4048/01**

Secondary 4 Express/4 Normal Academic (O)/5 Normal Academic

Name _____ ( ) **Date** **2 September 2019**

Class _____ **Duration** **2 hours**

Candidates answer on the question paper
READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

FOR EXAMINER'S USE	
	/80

This document consists of **21** printed pages including this cover page.

Setter **Mr Kok Yeong Haur** **[Turn over**

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Solve $\frac{x}{3}+11=7$

Answer $x=$ ..... [1]

---

2 Given that $\sqrt[3]{2^{18}}=\left(\frac{1}{k}\right)^6$, find k.

Answer $k=$ ..... [2]

---

3 A set of five numbers is shown below.

7 5 18 2 7

(a) Write down the median.

Answer ..... [1]

(b) When one of the number is removed from the set, the median and the range do not change. Which number was removed?

Answer ..... [1]

---

4 n is a positive integer.

Show that $(5n+2)^2-(5n-2)^2$ is a multiple of 8.

Answer

[2]

5 Factorise completely $4ax+15by-20ay-3bx$.

Answer ..... [2]

6 The frequency, f Hz, of a note produced by a string is proportional to the square root of the tension, T newtons, of the string.
For two identical strings, the ratio of the frequencies of the notes produced is 3 : 1.

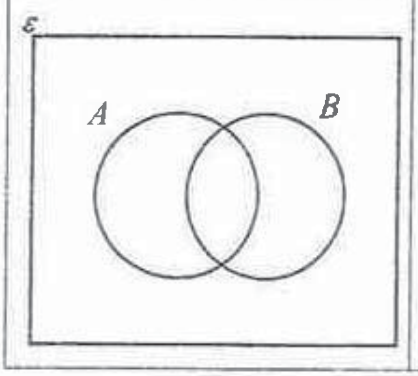
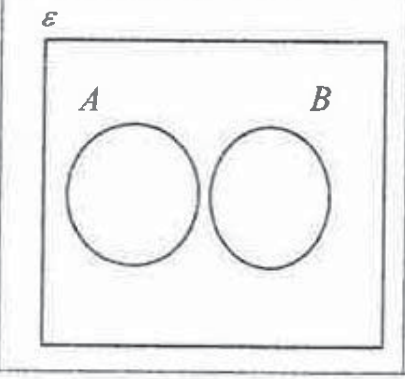
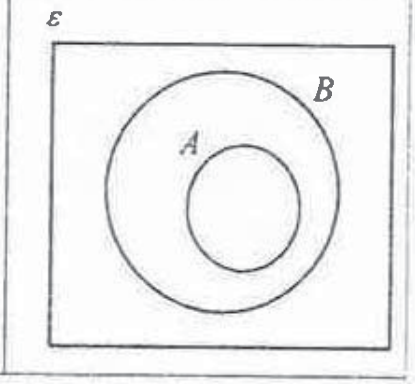
Find the ratio of the tensions in the strings.

Answer ..... : ..... [2]

7 A village of 120 people has two newspapers, the Arirang and the Busan.
35% of the villagers read the Arirang, 60% read the Busan, and 15% read neither.

ε = {all people in the village}
A = {people who read the Arirang}
B = {people who read the Busan}

(a) State which Venn Diagram represents the village.

Diagram 1	Diagram 2	Diagram 3
		

Answer Diagram ..... [1]

(b) In the diagram you have selected in **(a)**, shade the region that represents the people in the village who read Arirang but not Busan. [1]

(c) Find the percentage of the villagers who read both newspapers.

Answer ..... [1]

8 A model of an auditorium is built using a scale of 1 : 250.
The interior volume of the model is 125 000 cm^3.

Find the actual interior volume, in m^3, of the auditorium.
Give your answer to 3 significant figures in standard form.

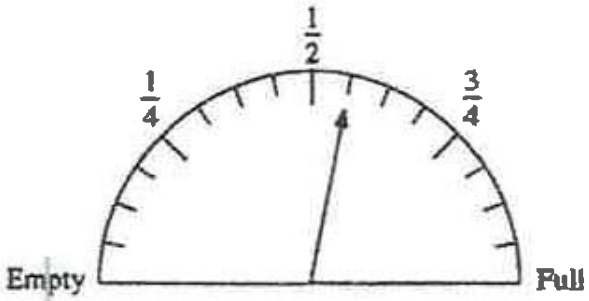
Answer ..... m^3 [3]

9 John can either buy or rent a particular laptop.
The cost of buying is \$1288.
The cost of renting is 25% of the price for the first year, and a monthly rental fee of \$16.50 after the first year.

If x is the number of months after the first year, use the information to form an inequality in x and calculate, in years and months, when it becomes more expensive to rent than to buy the laptop.

Answer ..... years ..... months [3]

10 The diagram shows the fuel gauge of Kumar's car.
The fuel gauge indicates the amount of petrol in the car.



(a) Find the fraction of the car tank that is not filled with petrol.

Answer ..... [1]

The fuel tank can store a maximum of 50 litres of petrol.
For cars travelling into Johor Bahru, their fuel tanks must be at least $\frac{3}{4}$ full.

(b) Calculate how much fuel must Kumar top up in Singapore before he can enter Johor Bahru.

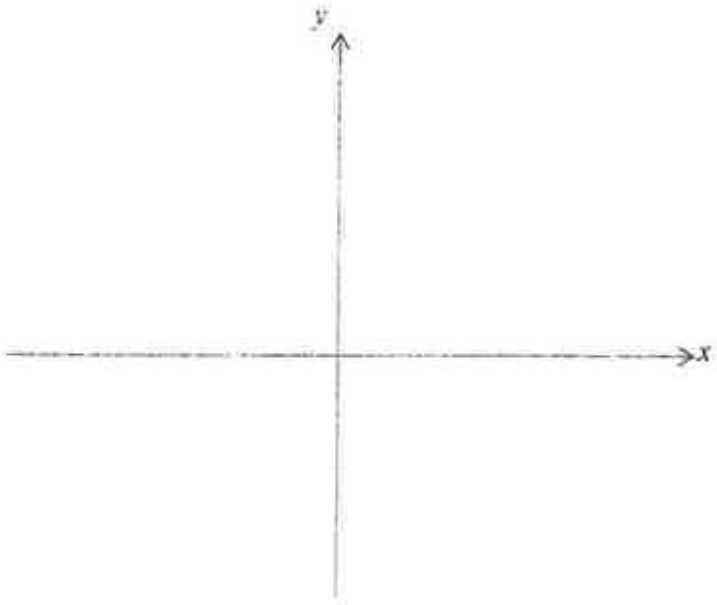
Answer ..... litres [2]

11 **(a)** Express $x^2 - 8x + 19$ in the form $(x-a)^2 + b$.

Answer ..... [1]

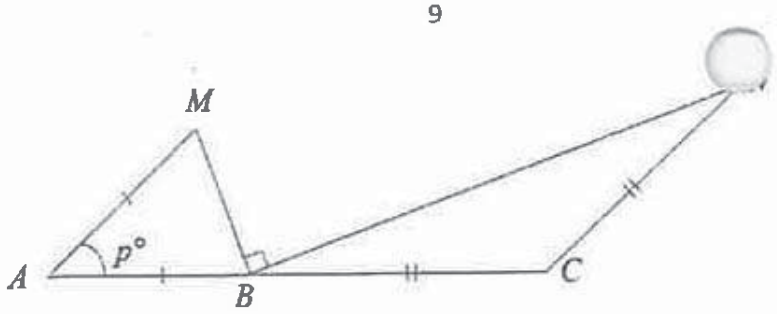
(b) Hence, sketch the graph of $y = x^2 - 8x + 19$ on the axes below, labelling clearly the turning point and intercept(s).

Answer



[2]

12



ABC is a straight line.
$AB = AM$ and $CB = CN$.
Angle $MAB = p^\circ$ and angle $MBN = 90^\circ$.

(a) Find angle CBN in terms of p.

Answer ..... [1]

(b) Explain with workings if AM is parallel to CN.

.....

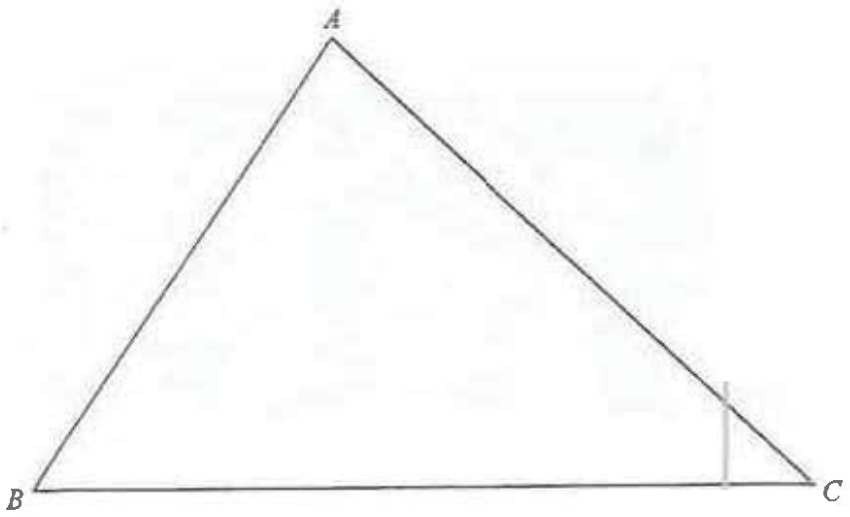
.....

.....

.....

[2]

13 The diagram represents a park ABC.



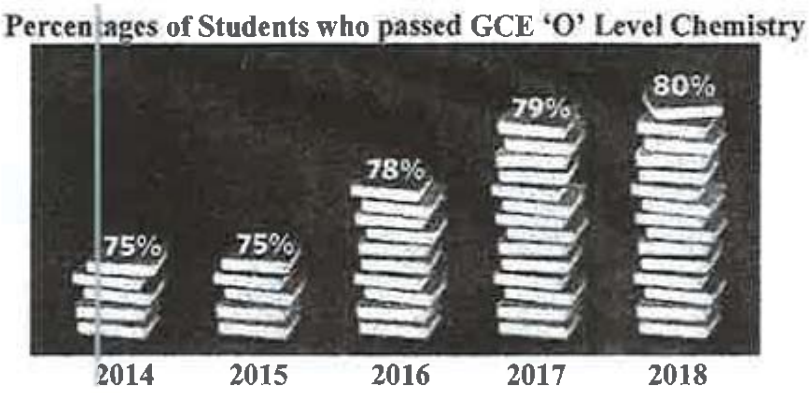
(a) Construct the perpendicular bisector of BC. [1]

(b) Construct the bisector of angle ABC. [1]

(c) A café is to be built in the park, nearer to B than to C and nearer to AB than to BC.

Shade the region where the café is to be built. [1]

14 Sam draws this graph to show the percentages of his students that passed Chemistry exam for the last four years.



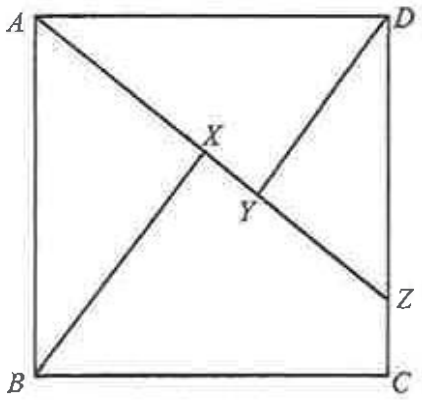
(a) State and explain one aspect of the graph that may be misleading.

.....
.....
.....
.....
..... [2]

(b) Based on the statistic, explain if Sam can also claim that the number of students passing Chemistry has increased?

.....
.....
.....
.....
..... [1]

15



ABCD is a square.
Point *Z* lies on *CD* such that *A*, *X*, *Y* and *Z* form a straight line.
Angle *AXB* = angle *DYA* = 90°.

By considering angle *DAY* = θ, prove that triangles *ABX* and *DAY* are congruent.

Answer

[3]

16 The table shows the times taken by 140 girls to complete the West Spring Cross Country 2019.

Time (in minutes)	$10 \leq x < 20$	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$
Number of girls	25	39	62	14

(a) Calculate an estimate of

(i) the mean time,

Answer ..... minutes [1]

(ii) the standard deviation.

Answer ..... minutes [1]

(b) The mean time for the boys to complete the run was 23.8 minutes and the standard deviation was 10.4 minutes.
Make two comments comparing the times of the girls and the boys.

1. .....

.....

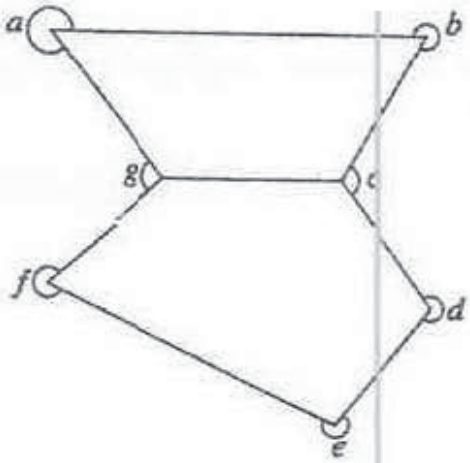
2. .....

..... [2]

17 (a) Calculate the sum of interior angles of a pentagon.

Answer ..... ° [2]

(b) Calculate the sum of the angles a, b, c, d, e, f and g in the diagram.



Answer ..... ° [2]

18 The table shows the travel times in minutes between some stations on an MRT route.

Admiralty	Sembawang	Yishun	Khatib	Yio Chu Kang	Ang Mo Kio
3					
9	6				
13	10	4			
a	15	9	5		
20	17	b	7	2	

(a) Find the values of a and b.

Answer $a =$ ..... , $b =$ ..... [2]

(b) A train leaves Sembawang MRT station and reaches Ang Mo Kio MRT station at 20 08.

(i) Calculate the time when the train leaves Sembawang MRT station.

Answer ..... [1]

(ii) Given that the distance between Sembawang MRT and Ang Mo Kio MRT stations is 12.5 km, find the average speed of the train in km/h, between these two stations.

Answer ..... km/h [1]

19 **(a)** Express 600 as the product of its prime factors.

Answer ..... [1]

(b) The number $600k$ is a perfect cube.

Find the smallest positive integer value of k.

Answer $k =$ ..... [1]

(c) x is a number between 950 and 1000.
The highest common factor of x and 600 is 20.

Find the smallest possible value of x.

Answer $x =$ ..... [2]

20 A factory makes wooden tables and chairs.

A table requires 8 hours of labour (L), 9 planks of wood (W) and 3 tins of paint (P).
A chair requires x hours of labour (L), 2 planks of wood (W) and 1 tin of paint (P).

(a) Represent this information in a 2 x 3 matrix, **P**.

$$\mathbf{P} = \begin{array}{c} \begin{matrix} \text{L} & \text{W} & \text{P} \end{matrix} \\ \begin{pmatrix} & & \\ & & \end{pmatrix} \begin{matrix} \text{Table} \\ \text{Chair} \end{matrix} \end{array}$$

[1]

(b) The cost of labour is \$10 per hour, the cost of wood is \$20 per plank and the cost of paint is \$4 per tin.

Find, in terms of x, the matrix $\mathbf{R} = \mathbf{P}\begin{pmatrix} 10 \\ 20 \\ 4 \end{pmatrix}$.

Answer $\mathbf{R} = \begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(c) Explain what each element in matrix **R** represents.

.....

..... [1]

(d) The cost of a table is four times the cost of a chair.
Calculate x.

Answer $x =$ ..... [1]

21 The diagram shows part of the speed-time graph of an object over a period of 50 seconds.
The object accelerates uniformly from 10 m/s to v m/s in 20 seconds.
It then decelerates uniformly for the next 15 seconds.
Thereafter it maintains a constant speed of 10 m/s.
The object travelled 450 m in the first 20 seconds.

(a) Calculate the value of v.

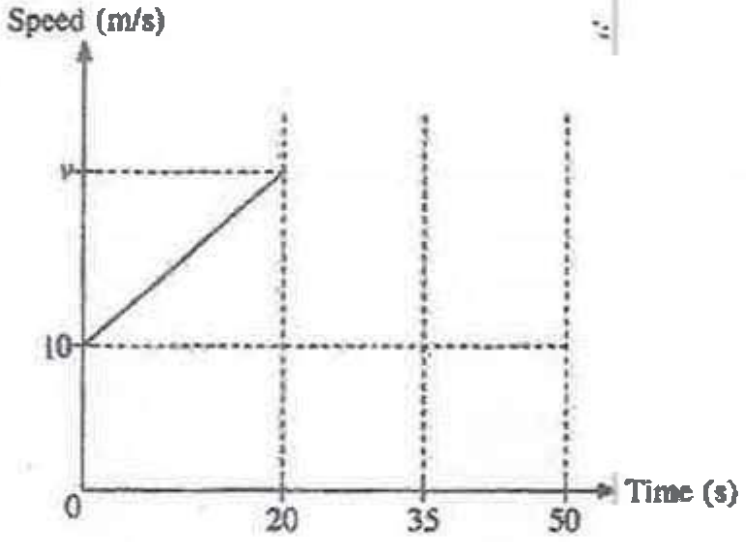
Answer $v =$ ..... m/s [2]

(b) Find the acceleration of the object after 7 seconds.

Answer ..... m/s^2 [1]

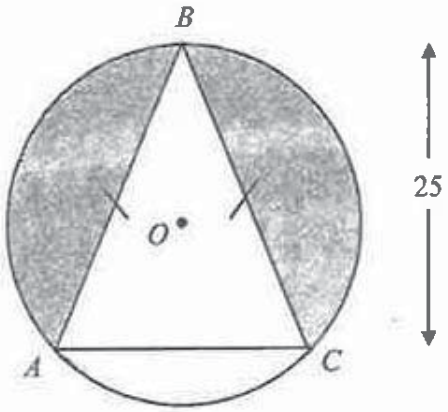
(c) Complete the speed-time graph.

Answer



[2]

22



ABC is an isosceles triangle with vertices on a circle with centre *O* and radius 15 cm. The height of the triangle *ABC* is 25 cm.

Calculate the area of the shaded region.

Answer ..... [5]

23 A company sells two sizes of the same brand of drink.

1.5 litre
$3.40

6.5 litres
$16.80

(a) Show that the cost of the drink is **not** directly proportional to the volume of the drink.

Answer

[2]

(b) The bottles are all geometrically similar.
The height of the 1.5 litre is 18 cm.

Calculate the height of the 6.5 litres bottle.

Answer ..... cm [3]

24 A is the point $(1, 1)$.

$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.

D divides BC such that $BD : DC = 1 : 1$.

(a) Find $\overrightarrow{BC}$.

Answer $\overrightarrow{BC} = \begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [2]

(b) Find $\left|\overrightarrow{AD}\right|$

Answer $\left|\overrightarrow{AD}\right| =$ ..... [2]

(c) P is the point $(3, 9)$.
Use vectors to show whether or not $ABPC$ is a parallelogram.

Answer

[2]

End of Paper 1

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Simplify $\frac{9a^2}{4b^5} \div \frac{15a^4}{12ab^3}$ [1]

(b) Express as a single fraction in its simplest form $\frac{5}{p+2} - \frac{2}{2p-3}$ [2]

(c) Solve the inequality $7-2x \geq 3x-8$ [2]

(d) It is given that $a = \frac{1}{2}\sqrt{\frac{\pi(b^2-6)}{c}}$.

Express b in terms of a and c. [2]

2 The diagram show patterns of grey and white squares.

Diagram 1 Diagram 2 Diagram 3 Diagram 4

Diagram	Number of grey squar (*G*)	Number of white squares (*W*)	Total number of squar (*T*)
1	1	0	1
2	1	8	9
3	17	8	25
4	17	32	49
5	⋮	⋮	⋮
6	49	x	y

(a) Find the values of x and y. [2]

(b) **(i)** Write down an expression, in terms of n, for the total number of squares (T) for Diagram n. [1]

(ii) Explain why it is not possible to have a diagram with a total of 226 squares. [1]

(c) Write an equation connecting G, W and T. [1]

(d) If there are 161 grey squares in Diagram 10, calculate the number of white squares. [2]

3 **(a)** One litre of petrol costs $2.25 in a particular petrol station.
During National Day, there was a promotion of 7% fuel discount.
Members are entitled to an additional 10% discount on their fuel purchase.
Karen, who is a member, paid $61.58 for her petrol on National Day.

Calculate to 1 decimal place, the number of litres she pumped. [2]

(b) The Singapore Savings Bonds (SSB) for the month of Aug pays the following interest rate for the first three years. The interest is paid out twice a year for every year.

Rate	1.95%	2.00%	2.10%
Year	1st	2nd	3rd

Sam bought $20 000 of the SSB for Aug.

(i) Calculate the interest Sam will receive in the first half of the first year. [1]

(ii) Calculate the total interest earned after three years. [1]

Sam also invested $20 000 in a savings account of a bank with a rate of compound interest of 1.98% per year. He leaves the money in the account for 3 years.

(iii) Calculate the total amount of interest he will earn after 3 years.
Give your answer to the nearest cent. [2]

(iv) Based on the given information, give one possible reason why Sam would prefer to invest more in SSB than in the bank. [1]

(c) Din booked a hotel in China using his credit card.
The hotel costs RMB 390 per night. Din booked the hotel for 4 nights and has two payment options:

Option A: The hotel charge in Singapore dollars using the hotel's exchange rate.
Option B: The hotel charge in RMB, after which Din's credit card company will convert to Singapore dollars using the company's exchange rate. There is a fee of 0.5% charged by the credit card company for the currency conversion.

Din found the currency exchange rates for the hotel and the credit card company

Hotel:	S$ 1 = RMB 4.97
Credit Card:	S$1 = RMB 5.06

Explain with workings, which payment option Din should choose. [2]

4 **(a)** The equation of line p is $2x+3y=12$.
The line cuts the x-axis at point A and the y-axis at point B.

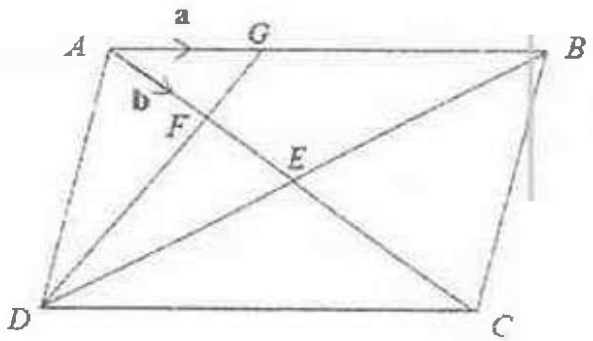
(i) Find the length of AB, [3]

A point C lies on the line p such that it is equidistant from the coordinate axes.

(ii) Show that the coordinates of point C is (2.4, 2.4). [2]

(iii) Write down the equation of the line which passes through C and is parallel to the y-axis. [1]

(b) The diagram shows a parallelogram $ABCD$.
AC and BD intersects at E.
G is a point on AB such that $2AG = GB$.
$AF : AE = 1 : 2$.
$\overrightarrow{AG} = \boldsymbol{a}$ and $\overrightarrow{AF} = \boldsymbol{b}$.



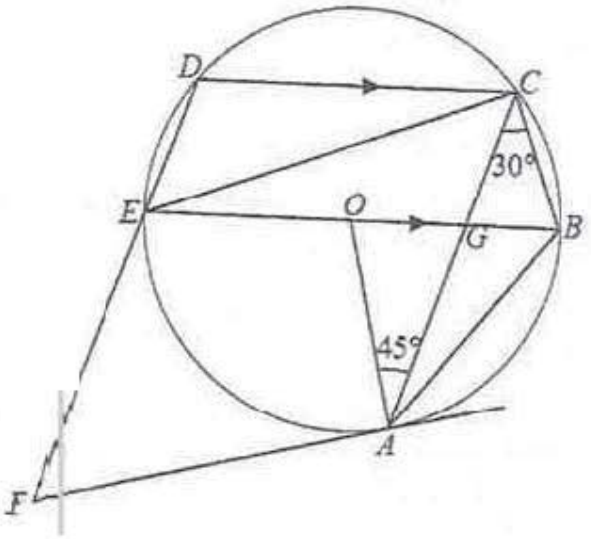
(i) Use vectors to determine if D, F and G lie on a straight line. [3]

(ii) Find the ratio of

(a) $\dfrac{\text{the area of } \Delta AFG}{\text{the area of } \Delta DFC}$, [1]

(b) $\dfrac{\text{the area of } \Delta AFG}{\text{the area of } \Delta DEC}$. [1]

5 (a)



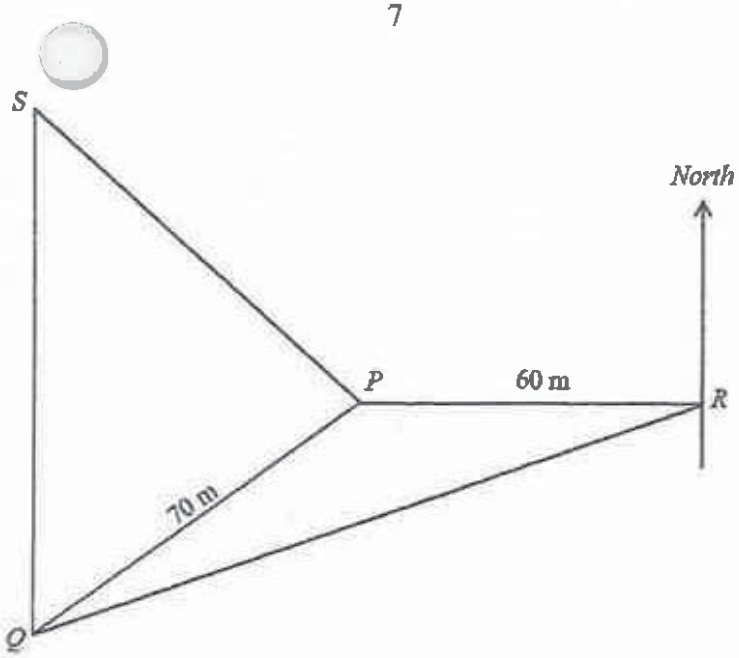
The diagram shows a circle, centre O.
AF is a tangent to the circle.
The line CD is parallel to the diameter of the circle BE.
DEF is a straight line.
The lines AC and BE intersect at G.
Angle $OAC = 45^\circ$ and angle $ACB = 30^\circ$

(i) Find, giving reasons for each answer,
(a) angle ECG, [1]
(b) angle CAB, [2]
(c) angle EFA. [3]

(ii) Determine with workings if triangles GAB and DCE are similar [2]

(b) A sector has radius 8 cm and angle 0.873 radians.
It is then formed into a cone by joining the two radii together
Calculate the perpendicular height of the cone. [3]

6



Points P, S, Q and R are at ground level.
Q is on a bearing of 240° from R.
R is 60 m due East of P.
S is due North of Q.
$PQ = 70$ m.

(a) Find angle PQR. [2]

(b) Calculate the bearing of P from Q. [1]

(c) Calculate QR. [3]

(d) An engineer, X, walks along a straight line from S to Q.

Calculate the shortest distance of X from P during this journey. [2]

(e) S is the base of a vertical tower.
T is the point on top of the tower vertically above S.
The angle of depression of R from T is 27°.

Calculate the height of the tower. [2]

7 The variables x and y are connected by the equation

$$y = x + \frac{4}{x} - 5.2.$$

Some corresponding values of x and y, correct to 2 decimal places, are given in the table below.

x	0.7	1.0	1.5	2.0	3.0	4.0	5.0	7.0	7.5
y	1.21	–0.20	–1.03	–1.2	p	–0.20	0.60	2.37	2.83

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 < x \leq 8$.
Using a scale of 4 cm to represent 1 unit, draw a vertical y-axis for $-2 \leq y \leq 3$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to solve the equation $x^2 - 4x + 4 = 0$. [2]

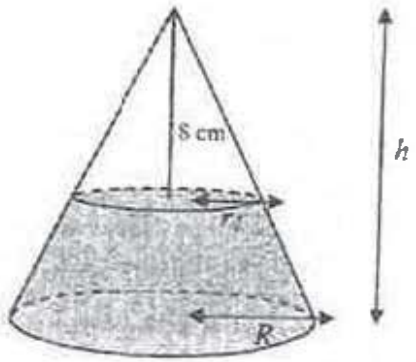
(d) By drawing a tangent, find the value of x where the gradient of the curve is 0.75. [2]

(e) **(i)** On the same axes, draw the line $y = \frac{3}{2}x - 2$, for $0 < x \leq 8$. [1]

(ii) Write down the x-coordinate of the point where this line intersects the curve. [1]

(iii) This value of x is a solution of the equation $x^2 + Ax + B = 0$.
Find the value of A and the value of B. [2]

8 The diagram shows a conical bottle of height h and radius R that is filled with water. When rests on its base, the water in the bottle is 8 cm from its vertex.

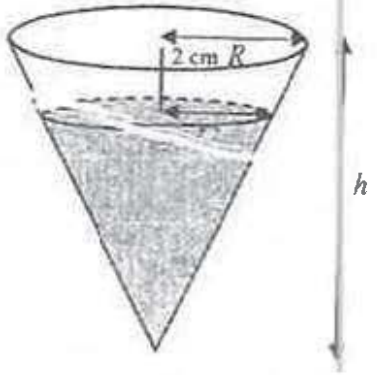


(a) Express r in terms of R and h.

Hence show that the volume of the water can be expressed as $=\frac{1}{3}\pi R^2\left(h-\frac{512}{h^2}\right)$

[3]

When the same conical bottle is turned upside down, the water level is 2 cm from its base.



(b) Show that the volume of water $=\frac{1}{3}\pi R^2\frac{(h-2)^3}{h^2}$. [2]

(c) Using your answers from part **(a)** and **(b)**, or otherwise, write down an equation in h and show that it reduces to

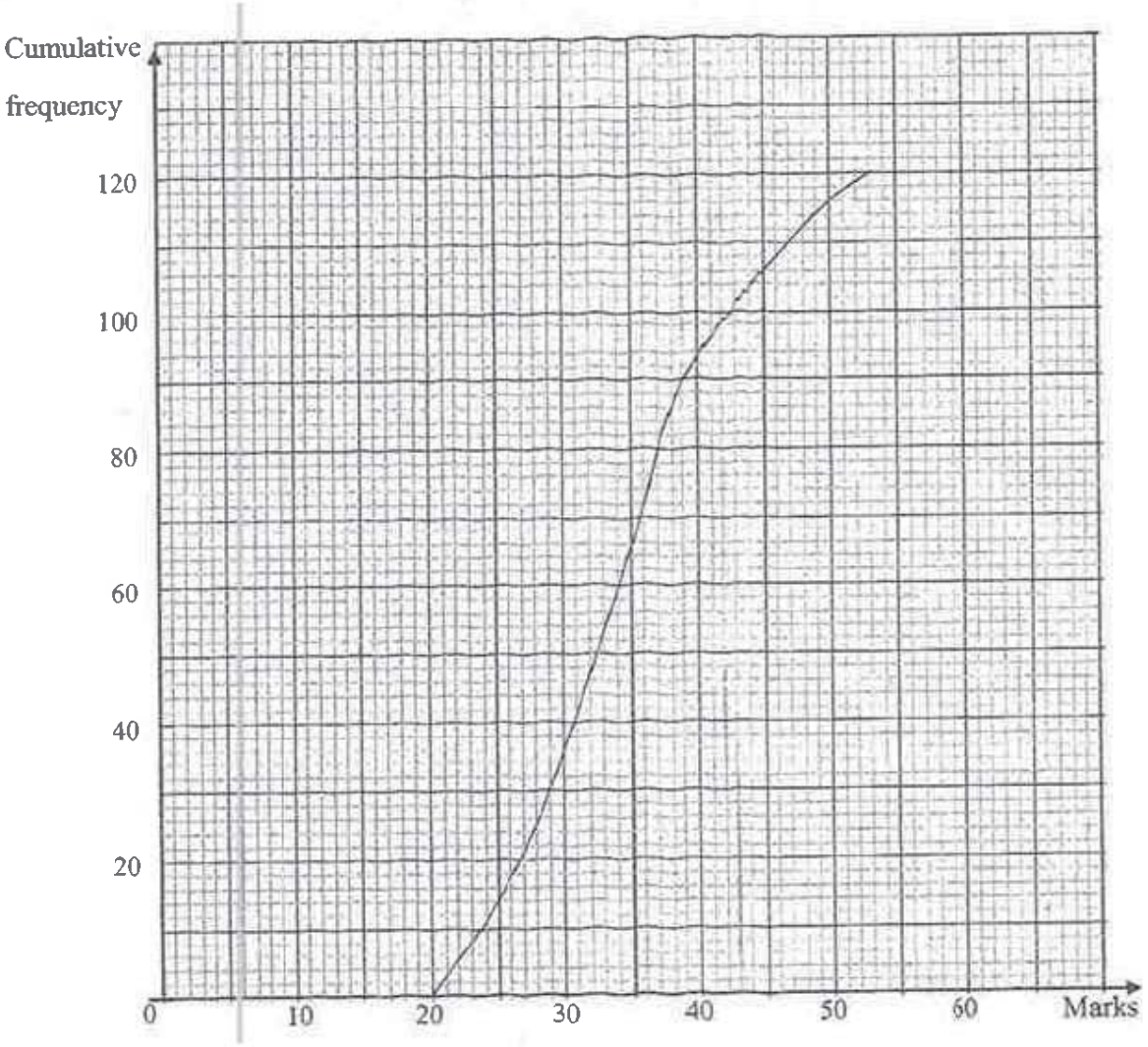
$$h^2-2h-84=0.$$

[2]

(d) Solve the equation $h^2-2h-84=0$, giving your solutions correct to one decimal place. [3]

(e) Calculate the volume of water in the conical bottle if $R=7$ cm. [2]

9 (a) The marks of 120 students in a Physics test were recorded.
The cumulative frequency curve below shows the distribution of the marks.



(i) Use the curve to estimate
(a) the median mark, [1]
(b) the interquartile range of the marks. [2]

(ii) The criteria for distinction is 45 marks.
Estimate the percentage of students who scored distinction. [2]

(iii) marks of the same 120 students in a Chemistry test were also recorded. The box-and-whisker plot shows the distribution of the marks.

Make two comments comparing the marks of the students for Physics and for Chemistry. [2]

(b) The table summarises the number of practice papers each student did before taking the Physics test.

Number of practice papers	0	1	2	3	4
Number of students	23	40	19	26	12

(i) One student is selected at random.
Find the probability that the student did not do any practice papers. [1]

(ii) Two students are selected at random.
Find, as a fraction in its simplest form, the probability that
(a) they both did three practice papers, [2]
(b) one had done more than two practice papers and the other had done fewer than two practice papers. [2]

10 Zander runs a restaurant in a shopping mall for the month of June.

His full-service, non-corner, 24-hour restaurant is made up of the following sections:

Percentage of Restauran Floor Area	Purpose
60%	Dining Area
30%	Kitchen
10%	Others (e.g. cashier, dish washing, receiving, storage etc)

The restaurant has 70 seats in the dining area.
Full-service restaurants typically have about one seat per 12 square feet (sqf).

(a) Estimate the floor area of the restaurant in sqf. [1]

(b) The monthly rental of the restaurant is calculated in dollars based on the table.

Location in mall	Monthly Rent in Dollars per square feet (S/psf		
	< 1000	1000 to 2000	> 2000
Corner	50	40	30
Non-Corner	40	30	20

The mall management also charge a "Maintenance & Advertising" cost every month. This cost is based on the floor area of the tenant, and is $200 per 100 sqf.

Use the table and information provided to calculate the monthly rental cost, inclusive of the "Maintenance & Advertising" cost. [2]

(c) In addition to the monthly rental cost calculated in part **(b)**, Zander estimates that he will these costs each month

- Food Raw Materials & Ingredients $14,400*
- Utilities $21,600**
- Other administrative cost (fixed) $2,000

* Dependent on hours of operation; based on 24-hour and $20 per hour
** Dependent on hours of operation; based on 24-hour and $30 per hour

The shopping mall requires all restaurant tenants to open at least 12 hours each day, and at least till 1 am.

Zander requires 20 workers who are paid according to their working hours (Table 1).

To help understand and improve his business, Zander also collected information on his restaurant revenue at different times of the day (Table 2).

Table 1: Workers' Wage

Regular Hours (8 am to 12am)	Irregular Hours (12 am to 8am)
$12/h	$18/h

Table 2: Estimated Revenue at different hours in a day

Hours	Sales
12 pm to 2 pm (Lunch)	$800 / h
7 pm to 11 pm (Dinner)	$900 / h
Other hours	$250 / h

Zander needs a monthly profit of at least $7000 for his repayment for the loan he took for his restaurant business.

(i) Determine with workings, if Zander is able to repay his loan in June. [4]

(ii) Suggest a sensible opening hours for Zander's restaurant in July that will allow him to pay his loans. Justify your decision and show all calculations clearly. [2]

End of Paper ☺