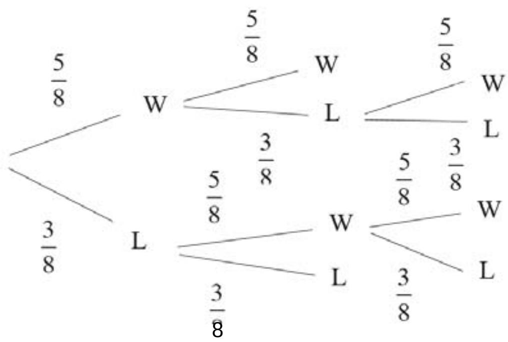
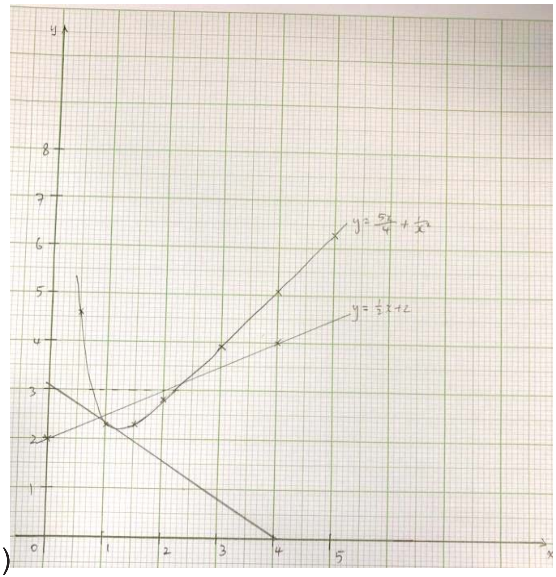
Answer key for paper 2

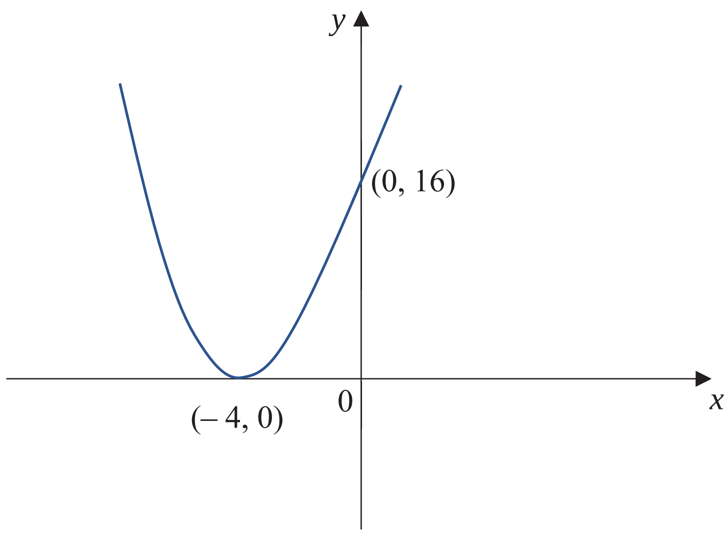
1	**(a)(i)** $(2p+3q)(x-1)$ (ii) $x=1$ (b) $\frac{9q^4}{p^2}$	2	**(a)** 1.29 radians **(b)** $4.09cm^2$
3	(b) A(0, 9). D(0, 3) (c) *E* (5, 15) (d) 45 unit^2	4	**(a) (i)** $\$\frac{65}{x}$ (ii) $\$\frac{65}{x-6}$ (c) $x=17.28$ or $x=-11.28$ (d) $5.76 (to nearest cent)
5	(b)(i) 061.6° (ii) 4.41 km (c) 0.231 km.		
7	(a) (i) 74^o (ii) $74°$ (iii) $41°$ (b) (i) 13 (Tangent from ext. point) (ii) 6 cm	6	**(a)** 0.0015, 0.0018, 0.225(egg) (b) **E** = $\begin{pmatrix} 0.0015 \\ 0.0018 \\ 0.225 \end{pmatrix}$ (c) $\begin{pmatrix} 0.5625 \\ 2.49 \\ 0.684 \end{pmatrix}$ (c) The elements in **DE** represents the cost price (in dollars) needed for the making of one cookie, one cake and one pancake respectively (d) $196.16
8	(a) (ii) $130\frac{2}{3}mm^2 \quad or \quad 131mm^2(3sf)$ (iii) $64.4mm^2$ (b) $42.9cm(3sf)$		
9	(a)(i) $26,399.70 (ii) $1847.36 (b) $924 (Accept answers from **$939 to $1064.40** as some students may assume up to 23 working days and 4 weekends or 21 working days and 5 weekends) (c) Acceptable responses such as 1. Renting is better as there is no need to pay down payment and monthly instalment. 2. Although it is more expensive to buy a car, it is very convenient. 3. Any other reasonable justification.		
10 (a)	(a)(i)  (ii) $\frac{175}{256}$ (b) (i) 53.875 kg or $\frac{431}{8}kg$ or $53\frac{7}{8}kg$ (ii) 9.87 kg (iii) (1) The mean weight is lower. Jogging has helped to reduce weight. (2) Standard deviation is higher implies that weight loss after exercising is more spread out. (accept "not consistent") (c) (i) 20 students (ii) 15 students (iii) 56 marks (iv) 90 marks (v) $\frac{3}{199}$	11	(a) $p \approx 6.3$ (b)  (c) From the graph, when $y=3$, $x=0.7$ or $x=2.2$ (Accept ± 0.2 0) (d) From the tangent at (1, 2.3), the gradient is – 0.75 (± 0.2) (e) (i) Draw graph of $y=\frac{1}{2}x+2$ (ii) The *x* coordinates are 0.9 and 2.3 (iii) $3x^3-8x^2+4=0$

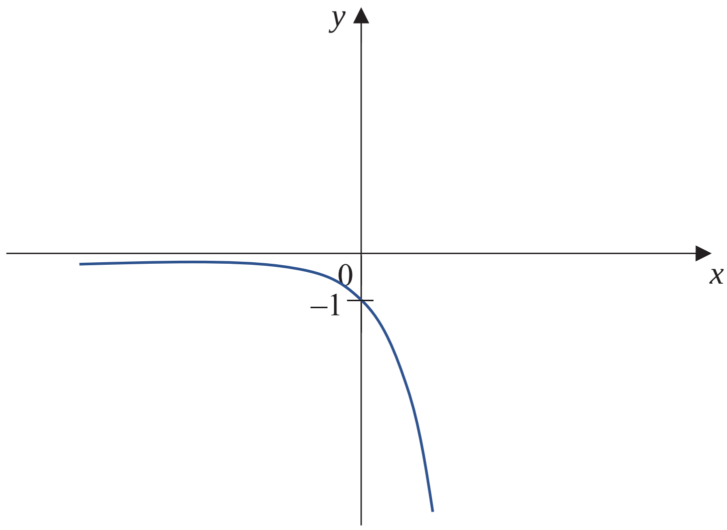
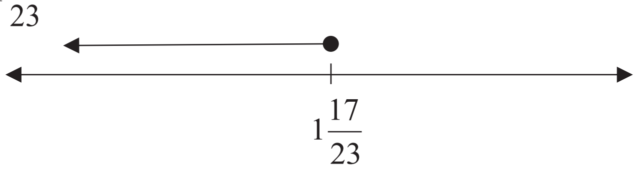
Anglican High School
Preliminary Examination 2020
Secondary Four Mathematics Paper 1 (4048/01)

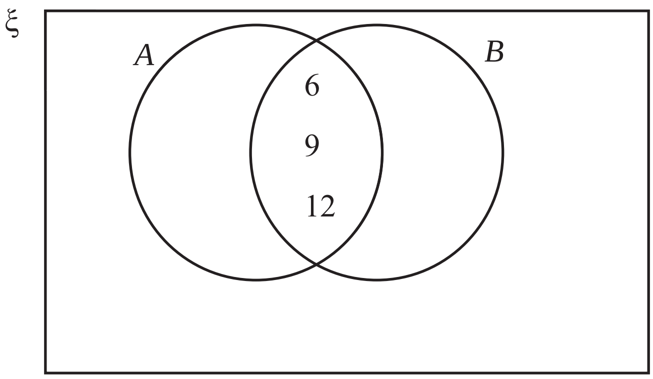
Marking Scheme

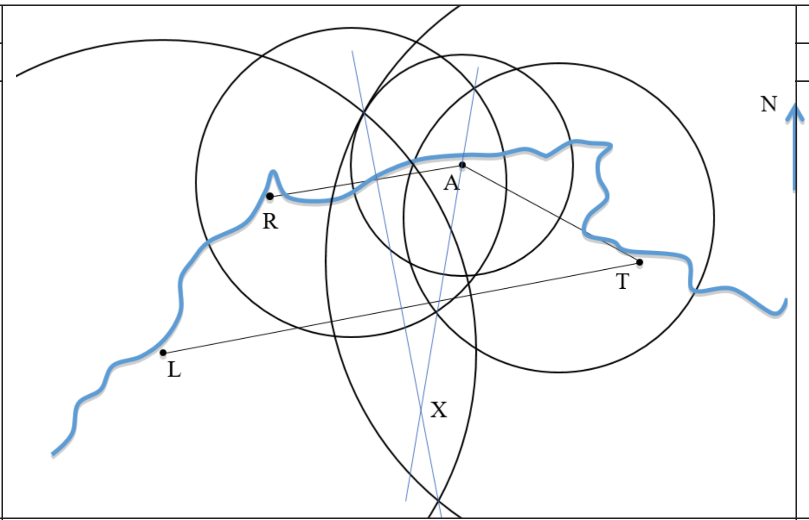
Qn		Solution	Marks	Remarks
1	**(a)**	$\dfrac{1458}{23.4-3.699^2}=150.0401$ (correct to 4 d.p.)	B1	
	(b)	$\dfrac{1458}{23.4-3.699^2}=150.0$ (correct to 4 s.f.)	B1	
2		Greatest possible value of L = $\dfrac{7\times10^9}{2.47\times10^7}$ $=2.83\times10^2$ (3 s.f.)	M1 A1	
3	**(a)**	Percentage of countries visited by Ryan = $\dfrac{20}{195}\times100\%$ $=10.3\%$ (to 3s.f)	B1	
	(b)	Original price = $\dfrac{100}{77.5}\times\$139.50=\$180$	B1	
4		$3^{3x+2}-9^{\frac{3}{2}x}+(27)^{x+1}=3^{3x+2}-3^{3x}+3^{3x+3}$ $=3^{3x}(3^2-1+3^3)$ $=3^{3x}(35)$ Since 35 is a multiple of 5, hence $3^{3x+2}-9^{\frac{3}{2}x}+\left(\dfrac{1}{27}\right)^{-x-1}$ is divisible by 5 for all positive integer values.	M1 A1	M1 – express all in index notation with base 3 Any multiple of 5
5	**(a)**	Prime factorization of the numbers HCF and LCM are $12=2^2\times3$ $2376=2^3\times3^3\times11$ Prime factorization of the numbers are $72=2^3\times3^2$ $108=2^2\times3^3$ Let x be the third number. Smallest possible value of $x=2^2\times3\times11=132$	M1 A1	M1 – prime factorisation
	(b)(i)	$6468=2^2\times3\times7^2\times11$	B1	
	(b)(ii)	$k=3\times11=33$	B1	

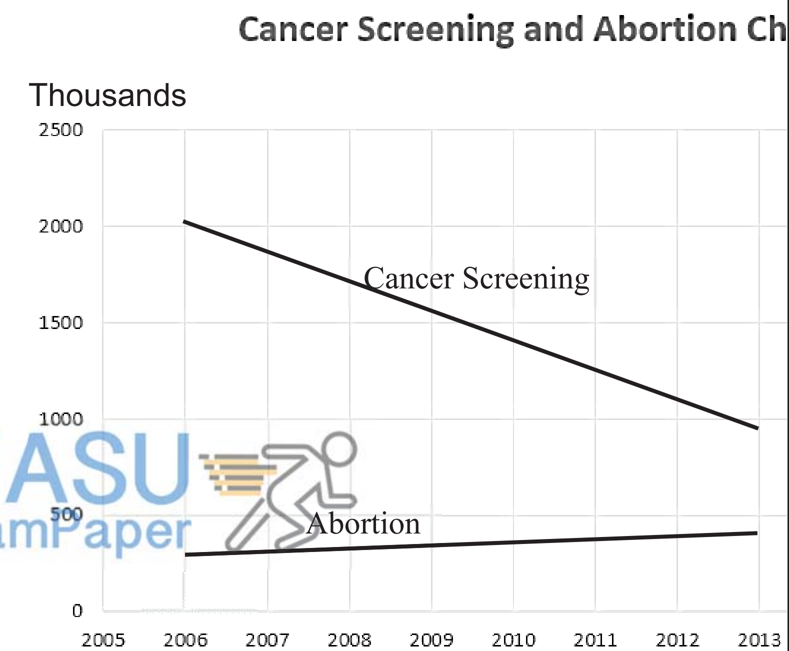
6	**(a)**	1 man will take 24 hours to dig 6m trench. 10 men will take 2.4hours to dig 6m trench. Time taken to dig a 5m trench = $2.4\times\frac{5}{6}$ hours $= 2$ hours	M1 A1	Accept procedures
	(b)	$y = k\sqrt{x}$ When $x = a$, $y = 24$ $24 = k\sqrt{a}$ $k = \frac{24}{\sqrt{a}}$ When $x = 4a$, $y = \left(\frac{24}{\sqrt{a}}\right)\sqrt{4a}$ $y = 48$	M1 A1	No mark if k expressed in terms of x
7	**(a)**	1 cm on map represents 0.3 km on actual ground. Actual length of park = 8×0.3 km $= 2.4$ km	B1	
	(b)	Area scale of map $= 1\text{ cm}^2 : 0.09\text{ km}^2$ Area of park on map = $\frac{4.5}{0.09} = 50\text{ cm}^2$ Breadth of park on the map = $\frac{50}{8} = 6.25$ cm	M1 A1	
8	**(a)**	$\frac{9\text{ km}}{1\text{ h}} = \frac{9000\text{ m}}{3600\text{ s}} = 2.5$ m/s	B1	
	(b)	Distance travelled with 60 litres of fuel $= \frac{250}{17.25}\times 60 = 869.6 \approx 870$ km (3 s.f.)	B1	Accept $869\frac{13}{23}$
9		$\frac{1}{18x^2-2}+\frac{2}{3x-1}$ $=\frac{1}{2\left(9x^2-1\right)}+\frac{2}{3x-1}$ $=\frac{1}{2(3x-1)(3x+1)}+\frac{2}{3x-1}$ $=\frac{1+2(2)(3x+1)}{2(3x-1)(3x+1)}$ $=\frac{1+12x+4}{2(3x-1)(3x+1)}$ $=\frac{12x+5}{2(3x-1)(3x+1)}$	M1 M1 A1	

10	**(a)**	$4a^2b+4a^2-b-1$ $=4a^2(b+1)-(b+1)$ $=(b+1)(4a^2-1)$ $=(b+1)(2a-1)(2a+1)$	M1 A1	Grouping Special product
	(b)	$f=3g-2x^3$ $2x^3=3g-f$ $x^3=\frac{3g-f}{2}$ $x=\sqrt[3]{\frac{3g-f}{2}}$	M1 A1	
11	**(a)**	$6x^2-7x-20=(3x+4)(2x-5)$	B1	
	(b)	$6(y-1)^2-7y+7=20$ $6(y-1)^2-7(y-1)-20=0$ $[3(y-1)+4][2(y-1)-5]=0$ $(3y+1)(2y-7)=0$ $y=-\frac{1}{3}$ or $y=\frac{7}{2}$	M1 M1 A1	Deduct 1 mark for skipping step
12	**(a)(i)**	$y=x^2+8x+16$ $y=(x+4)^2-4^2+16$ $y=(x+4)^2$	B1	
	(a)(ii)		G2	G1 for correct shape G1 for both correct coordinates or label on axes
	(b)	$x=0$	B1	
	(c)	$(-2,0)$	B1	

13	(a)(i)	$n=-1$ or $n=-3$	B1	Any negative odd number
	(a)(ii)	$k<0$	B1	
	(b)		G1	Curve must pass through $(0,-1)$
14	(a)	$2x-y=11$....... Eq (1) $5y-2px+7=0$....... Eq (2) From Eq (1), $y=2x-11$....... Eq (3) Substitute Eq (3) into Eq (2), $5(2x-11)-2px+7=0$ $10x-55-2px+7=0$ $2x(5-p)=48$ $x=\dfrac{24}{5-p}$	M1 A1	M1 for correct substitution
	(b)	$p=5$	B1	
	(c)	When $p=5$, $x=\dfrac{24}{4}=6$ $y=2(6)-11=1$	B1	
15	(a)	$\dfrac{1}{3}(4x-5)\leq 1-\dfrac{1}{5}x$ $5(4x-5)\leq 15-3x$ $20x+3x\leq 15+25$ $23x\leq 40$ $x\leq 1\dfrac{17}{23}$ 	M1 A1 B1	
	(b)	0, 1	B1	

16	Let $x°$ be the exterior angle. Interior angle of the polygon = $1.5x°$ $x° + 1.5x° = 180°$ $2.5x° = 180°$ $x° = 72°$ Hence, $n = \frac{360°}{72°}$ $n = 5$		M1 A1	
17		$\xi = \{1,2,3,4,5,6,7,8,9,10,11,12,13\}$ $A = \{4,6,8,9,10,12\}$ $B = \{3,6,9,12\}$ $C = \{6,12\}$		
	(a)(i)	$A' = \{1,2,3,5,7,11,13\}$ or $1,2,3,5,7,11,13$	B1	
	(a)(ii)	$A' \cap C = \{\ \}$ or $\varnothing$ or null/empty set or no element	B1	
	(b)	 	B1 B1	B1 for correct Venn Diagram B1 for correct elements in $A \cap B$
	(c)	$\{\ \}, \{6\}, \{12\}$	B1	
18	**(a)**	Distance travelled by Anne in 15 min = $0.3 \times 15 = 4.5$ km	B1	
	(b)	Time taken from rest to the end of journey $= \frac{10 - 4.5}{0.5} = 11$ min Total time taken = 15 + 5+ 11= 31 min	B1	
	(c)	Jane needs to complete the 10 km in $(31 - 3) = 28$ min Average speed of Jane is $\frac{10}{28} \approx 0.36$ km/min (2 d.p.)	M1 A1	

19	**(a)**	$\angle QRP = \dfrac{180° - 4x°}{2} = 90° - 2x°$ (supplementary angles)	B1	
	(b)	$\angle APR = 4x°$ (alternate angles, AB // CD) Since QP bisects $\angle APR$, $\angle QPR = 2x°$ $\angle PQR + \angle QRP + \angle QPR = 180°$ (sum of angles in triangle) $\angle PQR + 90° - 2x° + 2x° = 180°$ $\angle PQR = 180° - 90°$ $\therefore \angle PQR = 90°$ Hence, PQ is perpendicular to QR.(shown)	M1 M1 A1	Deduct 1 for presentation without reason
20	**(a)(i)**		G1	
	(a)(ii)		G1	
	(b)(i)		G1	
	(b)(ii)	$AX = 4.5 \times 200 = 900$ km	B1	Accept $\pm$ 20 km
21	**(a)**	Given $CD = CB$. Given $\angle AFE = 45°$ and $\angle FAE = 90°$, $\angle AEF = 45°$. $\therefore \Delta FAE$ is an isosceles right-angled triangle. Hence, $AF = AE$ and $FB = ED$. By Pythagoras' Theorem, $FC = EC$. $\Delta CDE \equiv \Delta CBF$ (SSS)	 B1 B1	B1 for valid proof and reasons B1 for correct pair and congruency condition, accept other valid solution
	(b)	Let sides of square be $3a$. Area of square = $9a^2$ Area of ΔCEF = $= 9a^2 - \left(\frac{1}{2} \times 3a \times a\right) - \left(\frac{1}{2} \times 3a \times a\right) - \left(\frac{1}{2} \times 2a \times 2a\right)$ $= 9a^2 - \frac{3}{2}a^2 - \frac{3}{2}a^2 - 2a^2$ $= 4a^2$ Area of ΔCEF : Area of square = $4a^2 : 9a^2 = 4 : 9$	 M1 A1	Accept other solutions.

	(c)	Area of square $= \dfrac{16 \text{ cm}^2}{4} \times 9 = 36 \text{ cm}^2$ Length of side of square = $\sqrt{36} = 6$ cm	M1 A1	
22	**(a)**	$AC = \sqrt{25^2 - 7^2} = 24$ cm Given that $\tan \angle CAD = \dfrac{3}{4}$, $\dfrac{CD}{24} = \dfrac{3}{4}$ $CD = 18$ cm	M1 A1	
	(b)	$AD = \sqrt{24^2 + 18^2} = 30$ cm $\cos \angle ADE = -\cos \angle ADC$ $= -\dfrac{18}{30}$ $= -\dfrac{3}{5}$	M1 A1	
23	**(a)**	Rate of decrease of cancer screening = $\dfrac{2007371 - 935573}{8}$ $= 133974.75$ persons per year	B1	
	(b)	The scale on the vertical axis is not defined.	B1	
	(c)		G1	

Anglican High School
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Secondary Four Mathematics Paper 2 (4048/2)

1 **(a)** **(i)** Factorise $2px-2p+3qx-3q$ completely. [2]

	$2px-2p+3qx-3q$ $=2p(x-1)+3q(x-1)$ $=(2p+3q)(x-1)$	M1 A1 Grouping $(2p+3q)$

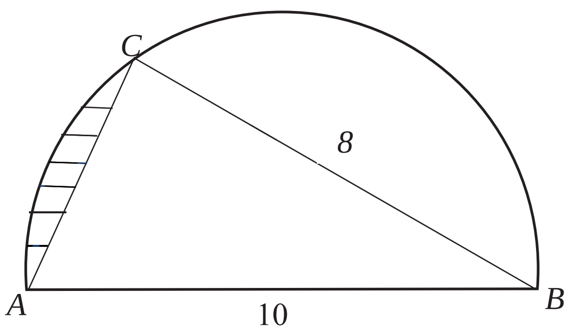
(ii) Given that p and q are positive constants, find the value of x for which $2px-2p+3qx-3q=0$. [1]

	$(2p+3q)(x-1)=0$ $(x-1)=0$ or (2p+3q)=0 [optional in this case.] $x=1$	B1

(b) Simplify $\left(-3p^2q^{-1}\right)^2\left(p^{-2}q^2\right)^3$, expressing your final answer in positive indices. [2]

	$\left(-3p^2q^{-1}\right)^2\left(p^{-2}q^2\right)^3$ $=9p^4q^{-2}p^{-6}q^6$ $=9p^{-2}q^4$ $=\dfrac{9q^4}{p^2}$	M[1] (correct use of $(ab)^n=a^nb^n$) A[1]

2 In the diagram, BC = 8 cm and AB = 10 cm is the diameter of the semi-circle.



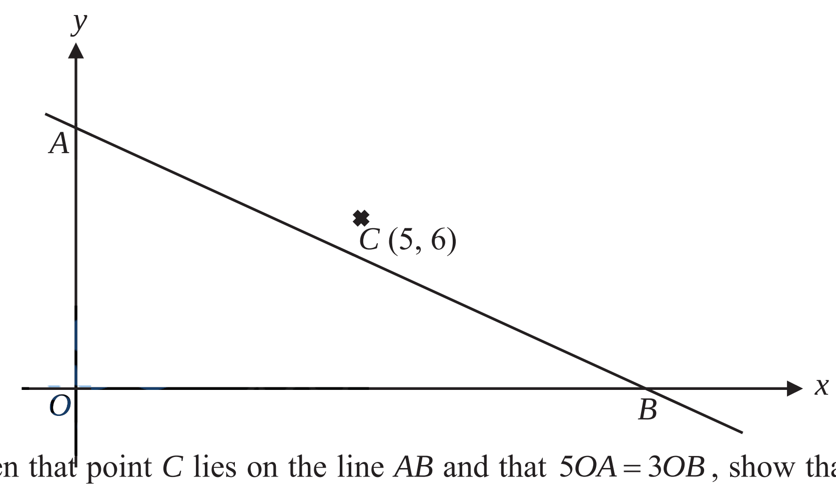
Find,

(a) $\angle COA$ in radians. [2]

(b) Area of the shaded region. [3]

(a)	$\cos\angle ABC = \frac{8}{10}$ $\angle ABC = 0.64350$ Using ext $\angle$= sum of int $\angle$ of Δ $\angle COA = 2\times\angle ABC$ $= 1.2870$ $= 1.29$ radians *Alternatively*, $\cos\angle CAB = \frac{6}{10}$ $\angle CAB = 0.92730$ $\angle COA = \pi - 0.92730 - 0.92730$ $= 1.2870$ $= 1.29$ radians	M1 A1 M1 A1
(b)	Area of shaded region = Area of sector *COA* - Area of triangle *COA* $= \frac{1}{2}\times 5^2 \times 1.2870 - \frac{1}{2}\times 5^2 \times \sin 1.2870$ $= 16.088 - 12.000$ $= 4.088$ $= 4.09 cm^2$	M1,M1for each correct term. A1

3 In the diagram, not drawn to scale, point *A* lies on the *y*-axis and point *B* lies on the *x*-axis. The coordinates of *C* is (5, 6).



(a) Given that point *C* lies on the line *AB* and that $5OA = 3OB$, show that the *y*-intercept of the line *AB* is 9. [3]

(b) Given that point D lies on the *y*-axis, state the coordinate of *D* such that triangle*ACD* forms an isosceles triangle, [1]

(c) Given further that *OCEA* is a parallelogram, find the coordinates of the point *E*. [1]

(d) Find the area of the parallelogram, *OCEA*. [2]

(a)	$\frac{OA}{OC}=\frac{3}{5}$ Hence gradient of $AB=-\frac{3}{5}$ Equation of *AB*: $y-6=-\frac{3}{5}(x-5)$ $y=-\frac{3}{5}x+9$ Since the *y*-intercept of $AB=9$, therefore the line AB cuts the *y*-axis at 9. (shown)	M[1] (deducing gradient) M[1] (correct equation) A[1]
(b)	Coordinates of $A=(0, 9)$. Coordinates of $D=(0, 3)$	B[1]
(c)	Distance of $OA=9$ units $E=(5, 15)$	B[1]
(d)	Area of parallelogram $=9\times 5$ $=45$ unit^2	M[1] A[1]

4 Mrs Tan bought *x* kg of rice for $65 in December 2019. In February 2020, the price of rice increased and she received 6 kg less for the same amount of money spent.

(a) Write down an expression for the price of rice per kilogram, in terms of *x*,

(i) in December 2019, [1]

(ii) in February 2020. [1]

(b) If the increase in price is $2 per kilogram of rice, form an equation in *x* and show that it reduces to $x^2-6x-195=0$. [3]

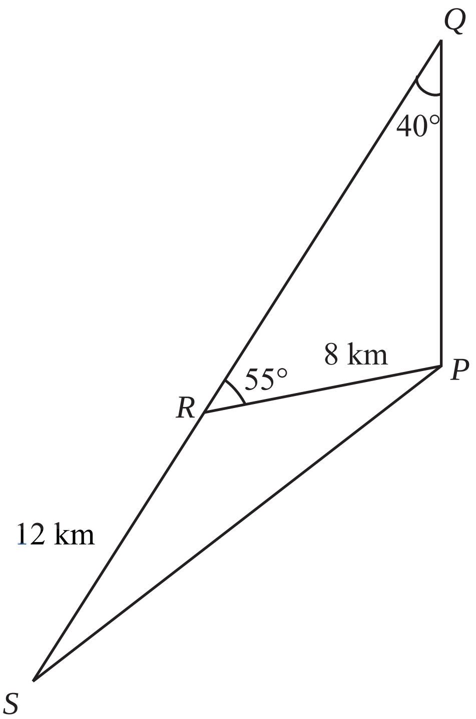
(c) Solve the equation, giving your answers correct to 2 decimal places. [2]

(d) Hence, find the price of rice per kilogram in February 2020, leaving your answer to the nearest cent. [1]

4(a)	(i)	Price per kg in Dec 2019 = $ $\frac{65}{x}$	B1	
	(ii)	Price per kg in Jan 2020 = $ $\frac{65}{x-6}$	B1	

4(b)	$\frac{65}{x-6}-\frac{65}{x}=2$ $65x-65(x-6)=2x(x-6)$ $65x-65x+390=2x^2-12x$ $2x^2-12x-390=0$ $x^2-6x-195=0$	M1 M1 A1	
4(c)	$x^2-6x-195=0$ $x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(-195)}}{2(1)}$ $x=\frac{6\pm\sqrt{816}}{2}$ $x=17.28$ or $x=-11.28$	M1 A1	M1- show substitution into quadratic formula A1 for both correct answers
4(d)	Price per kg in Jan 2020 = \$ $\frac{65}{17.28-6}=\$5.76$ (to nearest cent)	B1	

5 In the diagram, $PQRS$ are four points on level ground, and Q is due north of P.



Given that angle $PQR = 40°$, angle $PRQ = 55°$, $PR = 8$ km, $RS = 12$ km and that QRS is a straight line,

(a) show that the distance PS, corrected to 5 significant figures, is 17.836 km, [2]

(b) using the result in **(a)**, find,

(i) the bearing of P from S. [3]

(ii) the shortest distance from R to PS. [2]

(c) A vertical tower stands at point R.
Evan, walking along PS and stops at a point where the greatest angle of elevation of the top of the tower, T, is 3°. Find the height of the tower. [2]

(a)	angle $SRP = 180° - 55°$ (angles on a straight line) $= 125°$ $PS^2 = 12^2 + 8^2 - 2(12)(8)\cos 125°$ $= 318.127$ $PS = 17.836$ km (to 5 sf) (shown)	M1 if 125° is shown M1 (correct angle, formula) M1 or with implicit square root
(b)(i) (Mtd 1)	angle $QPR = 180° - 40° - 55°$ (angles in a triangle) $= 85°$ $\frac{\sin RPS}{12} = \frac{\sin 125}{17.836}$ angle $RPS = 33.444$ (to 3 dp) angle $NSP = 180° - 85° - 33.444°$ (interior angles) $= 61.6°$ (to 1 dp) Bearing of P from S = 061.6°	M[1] M[1] A[1]
(b)(i) (Mtd 2)	$\frac{\sin RSP}{8} = \frac{\sin 125}{17.836}$ angle $RSP = 21.556$ (to 3 dp) angle $QSN = 40°$ (alternate angles) $= 40°$ angle $NSP = 40° + 21.556°$ $= 61.6°$ (to 1 dp) Bearing of P from S = 061.6°	M[1] M[1] A[1]
(b)(ii) (Mtd 1)	Let the shortest distance be d km. Area of $RPS = \frac{1}{2}(8)(17.836)\sin 33.444°$ $\frac{1}{2}(17.836)(d) = \frac{1}{2}(8)(17.836)\sin 33.444°$ $d = 4.4090$ (to 5 sf) $= 4.41$ (to 3 sf) Hence the shortest distance is 4.41 km.	M[1] A[1]

(b)(ii) (Mtd 2)	Let the shortest distance be d km. Area of $RPS = \frac{1}{2}(8)(12)\sin 125°$ $\frac{1}{2}(17.836)(d) = \frac{1}{2}(8)(12)\sin 125°$ $d = 4.4090$ (to 5 sf) $= 4.41$ (to 3 sf) Hence the shortest distance is 4.41 km.	M[1] A[1]
(c)	Let the height of the tower be h km. $\tan 3° = \frac{h}{4.4179}$ $h = 0.23153$ (to 5 sf) $h = 0.231$ (to 3 sf) The height of the tower is 0.231 km.	M[1] for correct equation A[1]

6 **(a)** The table below shows the amount of flour, sugar and number of eggs needed for each type of pastry sold in a cafe.

	Flour (g)	Sugar (g)	Number of eggs
Cookie	90	50	1.5
Cake	220	200	8
Pancake	60	80	2

The above information is represented by a matrix $\mathbf{D} = \begin{pmatrix} 90 & 50 & 1.5 \\ 220 & 200 & 8 \\ 60 & 80 & 2 \end{pmatrix}$.

Each kg of flour and sugar costs \$1.50 and \$1.80 respectively. A dozen eggs costs \$2.70. Complete the cost in \$ for every gram of flour and sugar as well as the cost in \$ for each egg in the table below:

Flour(\$/g)	Sugar(\$/g)	1 Egg(\$)

[2]

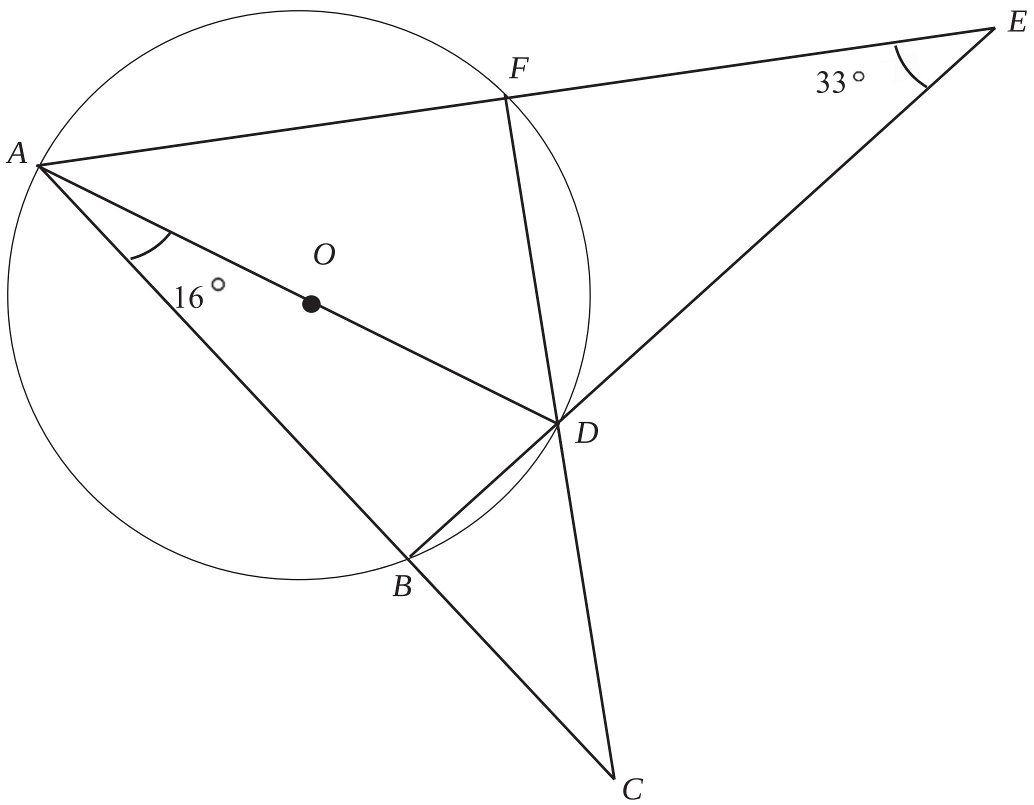
(b) Write down a 3×1 matrix **E** such that its elements represent the unit cost of each ingredients needed for the various pastries. [1]

(c) Calculate **DE** and state what the elements of **DE** represents. [2]

(d) The cafe prepared 70 cookies, 30 cakes and 120 pancakes on a particular day. Using matrix multiplication, calculate the total cost of the basic ingredients for all the pastries prepared on this day. [2]

Q6a)	0.0015, 0.0018, 0.225(egg)	B1when both ans for flour and sugar are correct. B1 for egg.
b)	$\mathbf{E} = \begin{pmatrix} 0.0015 \\ 0.0018 \\ 0.225 \end{pmatrix}$	B1
c)	$\mathbf{DE} = \begin{pmatrix} 90 & 50 & 1.5 \\ 220 & 200 & 8 \\ 60 & 80 & 2 \end{pmatrix}\begin{pmatrix} 0.0015 \\ 0.0018 \\ 0.225 \end{pmatrix}$ $= \begin{pmatrix} 0.5625 \\ 2.49 \\ 0.684 \end{pmatrix}$ The elements in **DE** represents the cost price (in dollars) needed for the making of one cookie, one cake and one pancake respectively.	B1 B1
d)	$\text{Total cost} = \begin{pmatrix} 70 & 30 & 120 \end{pmatrix}\begin{pmatrix} 0.5625 \\ 2.49 \\ 0.684 \end{pmatrix}$ $= \begin{pmatrix} 196.155 \end{pmatrix}$ The total cost is $196.16	M1 A1

7 **(a)** In the diagram, AD is the diameter of the circle $ABDF$ with centre O. Given BD and AF produced meet at E, AB and FD produced meet at C, $\angle FED = 33^o$ and $\angle DAB = 16^o$.



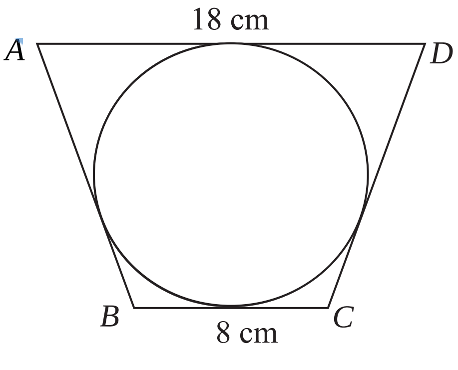
Calculate, stating your reasons clearly,

(i) $\angle ADB$ [2]

(ii) $\angle AFB$, [1]

(iii) $\angle DAF$. [2]

(b) Given $ABCD$ is a trapezium with AD = 18 cm, BC = 8 cm and $AB = DC$. A circle is inscribed in the trapezium as shown.

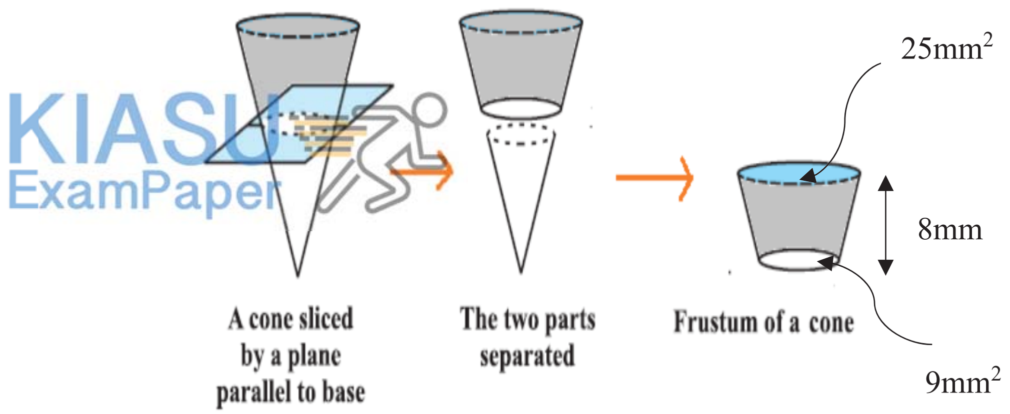


(i) Show that the length of AB is 13 cm, stating your reason(s) clearly. [2]

(ii) Calculate the radius of the circle. [2]

7ai)	$\angle ABD = 90^o$ (angle in semi-circle) $\angle ADB = 180^o - 90^o - 16^o$ $= 74^o$ (sum of angles in a triangle)	M1 A1
ii)	$\angle AFB = \angle ADB$(Angles in the same segment) $= 74°$	B1
iii)	$\angle DAF + 33° = 74°$(ext angle =sum of opp int angle of triangle) $\angle DAF = 74° - 33°$ $= 41°$ Alternatively, $\angle ADE = 180° - 74°$(angle on a straight line) $\angle DAF + \angle ADE + 33° = 180°$(sum of angle in triangle) $\angle DAF = 180° - 33° - 106°$ $= 41°$	M1 A1 M1 A1 Minus 1 mark for incorrect reasons, or self created reasons.
bi)	AB =4 + 9 (Tangent from ext point) =13	B1 for stating 4+9 B1 for quoting tangent from external point.
ii)	Let this distance be x cm. By Pythagoras Theorem, $x = \sqrt{169 - 25}$ $= 12$ Hence, the radius of the circle = 6 cm	M1 A1

8 **(a)** A piece solid cone is cut into a smaller piece of cone and a frustum as shown in the diagram.



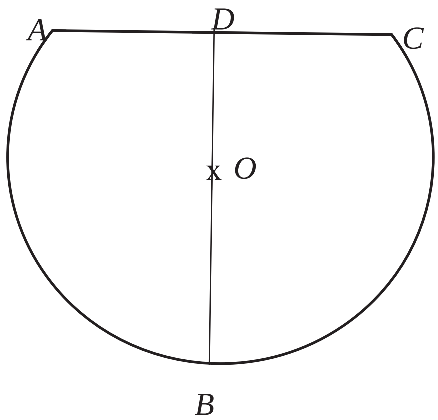
Given that the height of the frustum is 8mm and the two circular base areas of 9 mm^2 and 25 mm^2 respectively.

(i) Show that the height of the smaller cone is 12 mm. [2]

(ii) Find the volume of the frustum. [3]

(iii) Find the curved surface area of the smaller cone. [3]

(b) In the figure, *ADCB* is a major segment of a circle, centre *O* and radius 7 cm. *BD* = 12 cm. *AC* is perpendicular to the line *BOD*. Find the perimeter of the major segment. [4]



8ai) Let x be the height of the smaller cone Using area of similar objects, $\left(\frac{x}{x+8}\right)^2=\frac{9}{25}$ $\frac{x}{x+8}=\frac{3}{5}$ $5x=3x+24$ $2x=24$ $x=12mm(shown)$	M1 A1
8aii) Volume of frustum = Volume of big cone - volume of small cone $=\frac{1}{3}\pi R^2H-\frac{1}{3}\pi r^2h$ $=\frac{1}{3}\left[25\times(8+12)\right]-\frac{1}{3}(9\times12)$ $=\frac{500}{3}-36$ $=130\frac{2}{3}mm^2 \quad or \quad 131mm^2(3sf)$	Accept calc of volume using R and r M1 for big cone answer M1 for small cone answer A1
8aiii) Curved surface area of small cone $=\pi rl$ Since area of small circle, $\pi r^2=9$, $r=\sqrt{\frac{9}{\pi}}\approx1.6926mm$ Slant height of small cone $=\sqrt{1.6926^2+12^2}\approx12.1187mm$ Therefore curved surface area of small cone $=\pi(1.692)(12.1187)$ $=64.441mm^2$ $=64.4mm^2$	M1 M1 A1
8b) $AD=\sqrt{7^2-5^2}$ $=\sqrt{24}cm$ $AC=2\sqrt{24}$ $\approx9.79796cm$ $\angle AOD=\cos^{-1}\frac{5}{7}$ $\approx0.77519rad\ or\ 44.415°$ $Arc\ ABC=7\times2(\pi-0.77519)cm$ OR $x=2\pi\times7\times\frac{271.17°}{360°}$ $\approx33.1296cm$ $\approx33.1296cm$ $Perimeter=9.79796+33.1296$ $\approx42.927cm$ $=42.9cm(3sf)$	M1 M1 M1 A1

9 Mr Tan is looking to purchase a new car to drive to and from work on weekdays and for leisure on weekends. He estimates the total distance travelled is about 1500 km per month. The average cost of petrol is $2.20 per litre. To buy a new car, he must pay a down payment of 30% of the selling price of the car before he can take a car loan for the remaining amount from a bank. He has to pay back the loan by monthly instalment. A 5-year car loan simple interest rate offered by most banks is 2.28% per annum. Mr Tan shortlisted 3 cars with all the relevant cost as shown in the table.

	Car A	Car B	Car C
Selling Price	$87 999	$108 999	$107 888
Fuel Consumption(km per litre)	17.2	14.9	17.8
Car Insurance per year	$ 1 200	$ 1500	$ 1 500
Engine Capacity(in cubic cm)	1 598	1 499	1 197
Monthly Car Maintenance	$200	$200	$200
Monthly Car Park charges	$150	$150	$150

He also finds out that the annual road tax of the car is determined by the engine capacity of the car is as follows:

$$\text{Annual Road Tax} = [\$500 + 0.75(\text{Engine Capacity minus } 1000)] \times 0.782$$

From the information provided,

(a) Calculate

(i) the down payment to buy **car A.** [1]

(ii) the total **monthly** expenditure including monthly bank instalment and all the other monthly costs to own a car. [7]

(b) An alternative to buying a car is to rent an electric car that is easily accessible from his house and workplace. Subscription per month is $15 and it costs 33 cents per minute of use. Mr Tan estimates that the average daily travel time to and from work is about 1 hour and 20 minutes. On weekends, he needs to use the car for about 5 hours for leisure activities. Calculate his monthly expenditure to rent a car. [2]

(c) Do you think Mr Tan should buy or rent a car? Justify your answer. [1]

9*ai*) Car A down payment(30% of \$87,999) =\$26,399.70	B1
9*aii*) *For Car A* Loan Amount=\$87,999-\$26,399.70 =\$61,599.30 Loan Interest over 5 years $=61{,}599.30\times\frac{2.28}{100}\times 5$ $=\$7{,}022.32$ Monthly Car Instalment $=\frac{61{,}599.3+7{,}022.32}{60}$ $=\$1{,}143.69$ Monthly Distance covered by car = 1500km Amount of petrol needed= $\frac{1500}{17.2}$ $=87.209 litre$ (1)Monthly petrol consumption=$\$2.20\times 87.209$ $=\$191.86$ (2) Monthly car insurance= $\frac{1200}{12}=\$100$ (3) Monthly road tax= $\frac{[500+0.75(1598-1000)]\times 0.782}{12}$ $=\$61.81$ (4) Monthly Maintenance cost=\$200 (5) Monthly Car Park charges=\$150 (6) Total monthly cost =\$1143.69+703.67 =\$1847.36	M1 A1 M1 A1 B1 B1 B1
9b) Monthly cost of renting a car based on 20 workdays and 4 weekends $=\$15+80\min\times 20\, working\ days\times 0.33+5\times 60\min\times 4 weekends\times 0.33$ $=\$924$ Accept answers from **\$939 to \$1064.40** as some students may assume up to 23 working days and 4 weekends or 21 working days and 5 weekends.	M1 A1
9c) Acceptable responses such as a) Renting is better as there is no need to pay down payment and monthly instalment. b) Although it is more expensive to buy a car, it is very convenient. c) Any other reasonable justification.	B1for any of the 4.

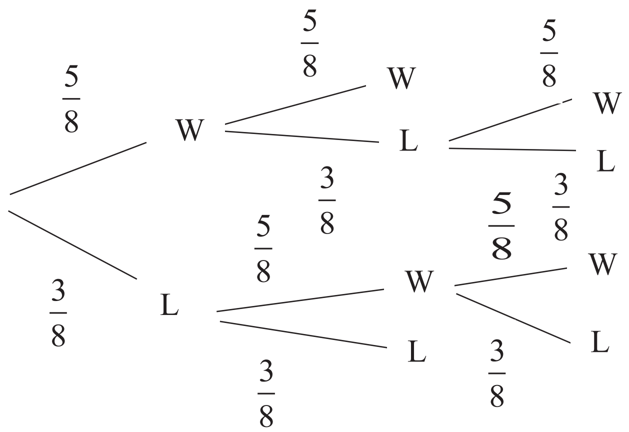
	Breakdown of cost of owning a car	Car A			
9ai	Down payment	$26,399.70			B1
9aii	Loan Interest	$7,022.32			M1
	Monthly Instalment	$1,143.69			A1
	Monthly Petrol needed in litres	87.209			M1
	Monthly Petrol Cost	$191.86			A1
	Monthly Car Insurance	$100			B1
	Monthly Road Tax	$61.81			B1
	Monthly Maintenance/Servicing	$200			
	Monthly Car Park Charges	$150			
9aiii)	Total Monthly Cost	$1847.36			B1
9b)	Monthly Cost Renting Electric car	Fr $939 to $1064.40			M1, A1
9c)	Acceptable responses. a) Renting is better as there is no need to pay down payment and monthly instalment. b) Although it is more expensive to buy a car, it is very convenient. c) Any other reasonable justification.				B1fo r any of the 4

10(a) Team Alpha and Delta are competing in a best of 3 games badminton finals. Each game will only result in a win or a loss. The competition ends when either one wins 2 games out of 3.

The probability of Alpha team winning in any one game is $\frac{5}{8}$.

(i) Draw a tree diagram to show all the possible outcomes for Alpha team. [2]

(ii) Calculate the probability, expressing your answers in fraction, that Alpha team wins the completion. [2]

10ai)  	B1 for 2nd branch with proper label B1 for 3rd branch with proper label
10*ii*) P(Alpha team wins the match) =P(Alpha wins, Alpha wins)+P(Alpha wins, Delta wins, Alpha wins) +P(Delta wins, Alpha wins, Alpha wins) $=\frac{5}{8}\times\frac{5}{8}+\frac{5}{8}\times\frac{3}{8}\times\frac{5}{8}+\frac{3}{8}\times\frac{5}{8}\times\frac{5}{8}$ $=\frac{175}{256}$	M1 A1

10 **(b)** The frequency table shows the weight in kg of 80 persons who join an exercise club.

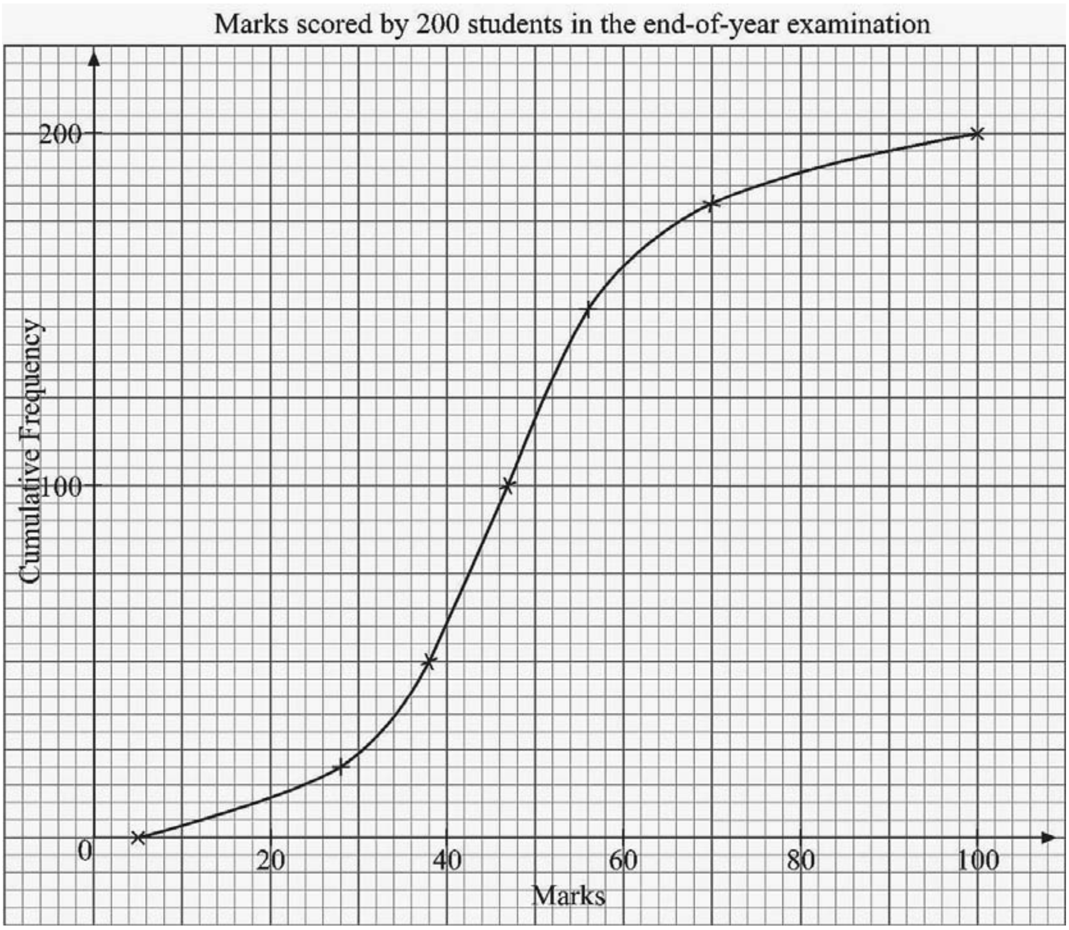
Weight	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$	$60 < x \leq 70$	$70 < x \leq 80$
Frequency	8	17	34	18	3

(i) Find the mean weight. [1]

(ii) Find the standard deviation. [1]

(iii) After 6 months of exercising, the mean and standard deviation of these 80 persons are 50kg and 12.2 kg respectively. Give 2 comments on the effect of exercise. [2]

10 **(c)** The cumulative frequency graph below shows the end-of-year Mathematics examination marks of a group of 200 students.



From the graph, find the

(i) number of students who scored 28 or less marks. [1]
(ii) number of students who scored more than 76 marks. [1]
(iii) upper quartile [1]

(iv) minimum mark attained by the top 2.5% of the cohort. [1]

(v) probability that 1 student scored 28 marks or less and the other student score more than 76 marks when 2 students are chosen at random. [2]

10bi) Mean = 53.875 kg or $\frac{431}{8}$ *kg or* $53\frac{7}{8}$ *kg*	B1
Standard deviation $=\sqrt{\frac{8\times35^2+17\times45^2+34\times55^2+18\times65^2+3\times75^2}{80}-\left(\frac{431}{8}\right)^2}$	
= 9.8734	
10bii) = 9.87 kg	B1
10biii) (1) The mean weight is lower. Jogging has helped to reduce weight.	B1
(2) Standard deviation is higher implies that weight loss after exercising is more spread out.(accept "not consistent")	B1
10ci) 20 students	
10cii) 200-185= 15 students	B1
10ciii) Upper quartile = 56 marks	B1
10civ) 90 marks	B1
	B1
10*cv*) Probability(1 stud $\le$ 28 and 1 stud $\ge$ 76)	
$=2\times\frac{20}{200}\times\frac{15}{199}$ or $\frac{20}{200}\times\frac{15}{199}+\frac{15}{200}\times\frac{20}{199}$	
$=\frac{3}{199}$	M1
	A1

11 The variables x and y are connected by the equation

$$y=\frac{5x}{4}+\frac{1}{x^2}.$$

The table below shows some values of x and the corresponding values of y, correct to 1 decimal place.

x	0.5	1	1.5	2	3	4	5
y	4.6	2.3	2.3	2.8	3.9	5.1	p

(a) Calculate the value of p.

Answer p = ... [1]

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw a horizontal x-axis for $0 \le x \le 5$ and a vertical y-axis for $0 \le y \le 8$. On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) From the graph, find the value of x in the range $0 \le x \le 5$ for which $\frac{5x}{4} + \frac{1}{x^2} - 3 = 0$ [2]

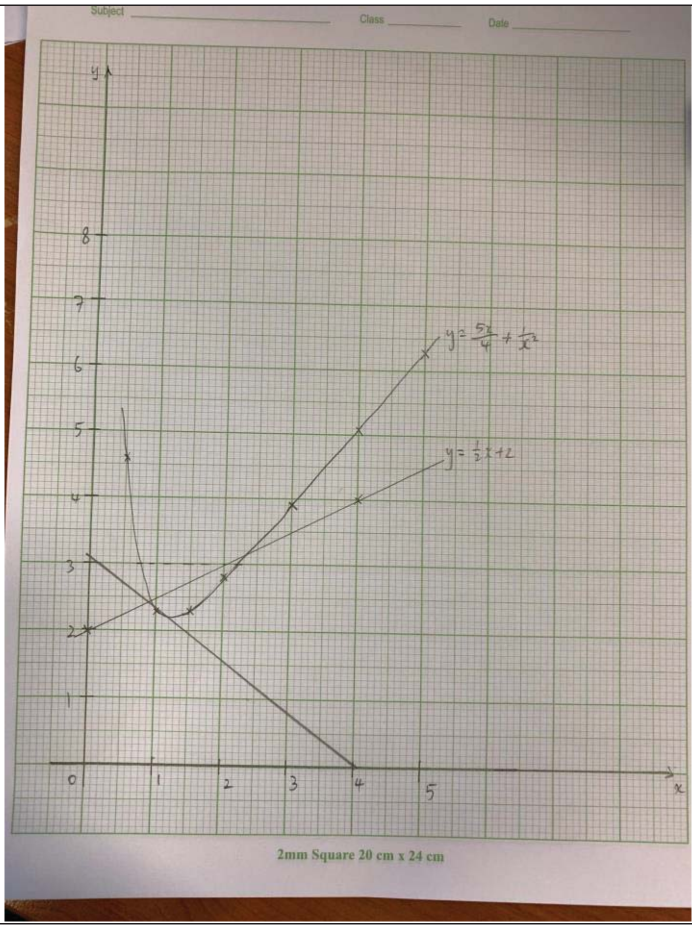
(d) By drawing a tangent, find the gradient of the curve at the point $(1,\ 2.3)$. [2]

(e) **(i)** On the same axes, draw the graph of $y = \frac{1}{2}x + 2$. [2]

(ii) Write down the x-coordinates of the points where the two graphs intersect. [1]

(iii) Find the equation in the form $3x^3 + ax^2 + bx + c = 0$, which is satisfied by the values of x found in part **(e)(ii).** [1]

Qn	Solution	Marks	Remarks
11(a)	$p = \frac{5(5)}{4} + \frac{1}{(5)^2} = 6.29 \approx 6.3$	B1	

11(b)		G3	G1 – correct scale for axes and labelling. G1 – all points correctly marked G1 – for smooth curve
11(c)	$\frac{5x}{4} + \frac{1}{x^2} - 3 = 0$ $\frac{5x}{4} + \frac{1}{x^2} = 3$ $y = 3$ From the graph, when $y = 3$, $x = 0.7$ or $x = 2.2$	G1 B1	G1 for drawing the line $y = 3$ Accept ± 0.2
11(d)	From the tangent at (1, 2.3), the gradient is – 0.75 (± 0.2)	G1+B1	G1 for drawing tangent
11(e)	(i) Draw graph of $y = \frac{1}{2}x + 2$	G1+G1	G1 for correct graph G1 for labelling graph
	(ii) The x coordinates are 0.9 and 2.3	B1	Accept ± 0.2
	(iii) $\frac{5x}{4} + \frac{1}{x^2} = \frac{1}{2}x + 2$ $5x^3 + 4 = 2x^3 + 8x^2$ $3x^3 - 8x^2 + 4 = 0$	B1	