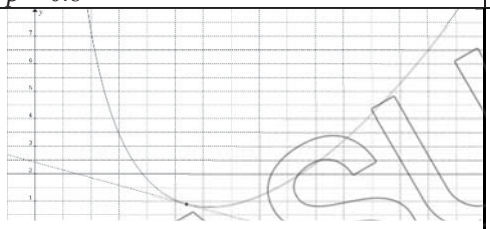
ANSWER KEY

Qn		Qn	
1	$k = 1$	**15a**	999.8 g
2	71	**15b**	9.00 g
3a	2 days	**15c**	For grouped data, only the mid-values of each data are used to calculate the mean and standard deviation. Hence, the mean and standard deviation are estimates.
3b	0.25	**15d**	I will buy machine *A*. Machine *A* is more consistent in producing the packets of 1kg chocolate than machine *B* because the standard deviation of the mass of chocolate by machine *A* is 9.00g which is lower than the standard deviation of the mass of chocolate by machine *B* that is 12.5g.
4	Quadratic Sketch	**16a**	$2\times3\times5^2$
5a	2 m/s	**16b**	$a = 906$
5b	0 m/s^2	**17ai**	$\begin{pmatrix}117\\96\end{pmatrix}$
5c	4 m/s	**17aii**	The elements represent the manufacturing cost of a dress (\$117) and a shirt (\$96) respectively.
5d	Speed-Time graph	**17bi**	(26100)
6a	Venn Diagram	**17bii**	The total manufacturing cost of 100 dresses and 150 shirts.
6b	$\{11,13,17,19\}$	**18a**	$(2a-b+7)^2$
6c	10	**18b**	$(y-3)(xy+3x-y)$
7	212.5 ml	**19a**	4 | 2, 3, 4, 5, 4, 3, 2 | 23 | 25
8a	$n = 6$	**19b**	$k = 9$
8b	20°	**19c**	$S+2=(n+1)^2$ or $S=(n+1)^2-2$
9	Sketch	**19d**	9799
10	Construction	**20**	$m = -38$ $x = 8$
11	$\frac{x-1}{x+1}$	**21a**	$c=\frac{6a+5b}{2(a+b)}$
12	8 : 7	**21b**	$c = 4$
13	$r = 3.00$	**22a**	Since the cost per ml for the 800 ml packet is different from that of the 375 ml packet and the 250 ml packet, the cost of the soya milk is not directly proportional to the quantity of soy milk.
14a	48°	**22b**	16.2 cm
14b	96°		
14c	48°		
14d	6°		

Answer Key

1(a)	$x \geq -3$	**7(a)(i)**	108°
(b)	$\frac{2y^2}{(3-2y)^2}$	**(a)(ii)**	36°
(c)	$\frac{2p^3}{3q^3r^2}$	**(b)**	110.11 cm^2
(d)	$\frac{9t^6}{v^8}$	**(c)(i)**	3850 cm^3
(e)	$x = 2$ or $-1\frac{1}{3}$	**(c)(ii)**	1620 cm^2
2(a)	4.27×10^7	**(d)**	$23.59
(b)	59.5%	**8(a)**	4.619 cm
(c)	1.28×10^8	**(b)(i)**	22.2 cm
(d)	1.28×10^9	**(b)(ii)**	12.7 cm^3
3(a)	$A(9,9)$	**9(a)(i)(a)**	14 minutes
(b)	p = 6 or 12 (rejected)	**(a)(i)(b)**	2 minutes 30 seconds
(c)	$y = -3x+21$	**(a)(ii)**	25%
(d)	$E(0.\ 21)$	**(a)(iii)**	Median for School *X* is 14 minutes while median for School *Y* is 11 minutes 54 seconds. Generally, girls from School *Y* performed better as the median is lower. The interquartile range of School *X* is 2 minutes 30 seconds while interquartile range for School *Y* is 1 minutes 48 seconds. Therefore, the timing for the girls from School *Y* has less variation.
(e)	22.5 $units^2$		
4(a)	$p = 0.8$		
(b)			
(c)	Draw $y = 2$ x = 1.9 or 4.75	**(b)(i)**	$\frac{23}{60}$
(d)	2.75	**(b)(ii)(a)**	$\frac{2093}{14340}$
5(a)(i)	063°	**(b)(ii)(b)**	$\frac{71}{1195}$
(a)(ii)	056°	**10(a)(i)**	Interquartile range is preferred as it is not affected by outliers like 52 hours
(b)(i)	249 m	**(a)(ii)**	Yes. The mean/ median for School *Y* is higher. – 1 mark Generally, students in School *Y* spent more time on HBL – 1 mark
(b)(ii)	215 m	**(b)**	A - $28.50 B - $29.12 C - $35.13 1) I would recommend A. Total cost is lowest. 2) I would recommend B. The value of the voucher is more than the price difference between food delivery service providers A and B. Thus, it is more worthwhile buying from food delivery service provider B.
(c)(i)	210 m		
(c)(ii)	20.9°		
6(a)	100π cm^3		
(b)(i)	$\frac{100\pi}{x}$ seconds		
(b)(ii)	$\frac{100\pi}{2x+1}$ seconds		
(c)	$2x^2+(1-100\pi)x-100\pi = 0$		
(d)	-1.00 or 157.58		
(e)	1.99 seconds		

Name: **Marking Scheme**	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIM EXAMINATION

MATHEMATICS
Paper 1

4048/01
31 August 2020
2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST
Write your name and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **80**.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10	11
Marks											
Question	12	13	14	15	16	17	18	19	20	21	22
Marks											

Table of Penalties		**Qn. No.**		
Presentation	–1			
Units	–1			
Significant Figures	–1		**Parent's/ Guardian's Signature**	**80**

This question paper consists of 22 printed pages.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Given that $\frac{8^{-1}}{8^k} \times 64 = 1$, find k.

$$\frac{8^{-1}}{8^k} \times 64 = 1$$
$$8^{-1-k} \times 8^2 = 8^0 \quad \text{M1}$$
$$8^{1-k} = 8^0$$
$$1-k = 0$$
$$k = 1 \quad \text{A1}$$

Answer $k =$ 1 [2]

---

2 Find the difference between the greatest and the least integer values of x that satisfies $-4 \le \frac{x-3}{4} - \frac{x+2}{3} < 2$.

$$-4 \le \frac{x-3}{4} - \frac{x+2}{3} < 2$$
$$-48 \le 3x-9-4x-8 < 24 \quad \text{M1}$$
$$-48 \le -x-17 < 24$$
$$-31 \le -x < 41$$
$$-41 < x \le 31 \quad \text{M1}$$

Alternative method:

$$-4 \le \frac{x-3}{4} - \frac{x+2}{3} \quad \text{and} \quad \frac{x-3}{4} - \frac{x+2}{3} < 2 \quad \text{M1}$$
$$-48 \le 3x-9-4x-8 \qquad 3x-9-4x-8 < 24$$
$$-48 \le -x-17 \qquad x > -41$$
$$x \le 31$$
$$\therefore -41 < x \le 31 \quad \text{M1}$$

$$\text{Difference} = 31-(-40)$$
$$= 31+40$$
$$= 71 \quad \text{A1}$$

Answer 71 [3]

---

3 **(a)** 12 engineers worked together and completed 30 identical model cars in 6 days. Assuming all the engineers worked at the same rate, how many days would 18 engineers take to complete 15 identical model cars?

No. of engineers	No. of model cars	No. of days	
12	30	6	
18	30	4	M1
18	15	2	A1

Answer 2 days [2]

(b) The kinetic energy of a moving object, E joules, is directly proportional to the square of its speed, v m/s.
When the speed of the moving object is halved, the corresponding kinetic energy of the object is p times its original kinetic energy.

Find the value of p.

$E = kv^2$, k is a constant

$E_2 = k\left(\frac{V_1}{2}\right)^2$ M1

$E_2 = \frac{1}{4}k(V_1)^2$

$p = \frac{1}{4}$ A1

Alternative method:

$\frac{E_1}{E_2} = \left(\frac{V_1}{V_2}\right)^2$

$\frac{E_1}{E_2} = \left(\frac{V_1}{\frac{1}{2}V_1}\right)^2$ M1

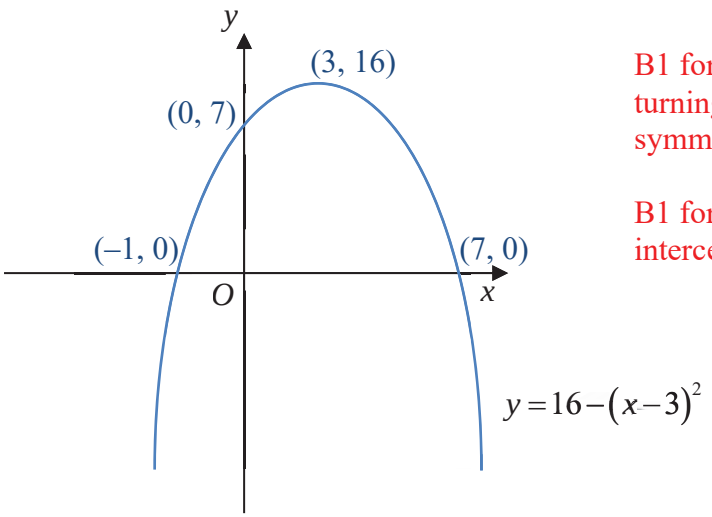
$\frac{E_1}{E_2} = 4$

$\frac{E_2}{E_1} = \frac{1}{4}$ A1

Answer $p =$ $\frac{1}{4}$ [2]

4 Sketch the graph of $y = 16 - (x-3)^2$ on the axes below.

Indicate clearly the coordinates of the points where the graph crosses the axes and the maximum point on the curve.

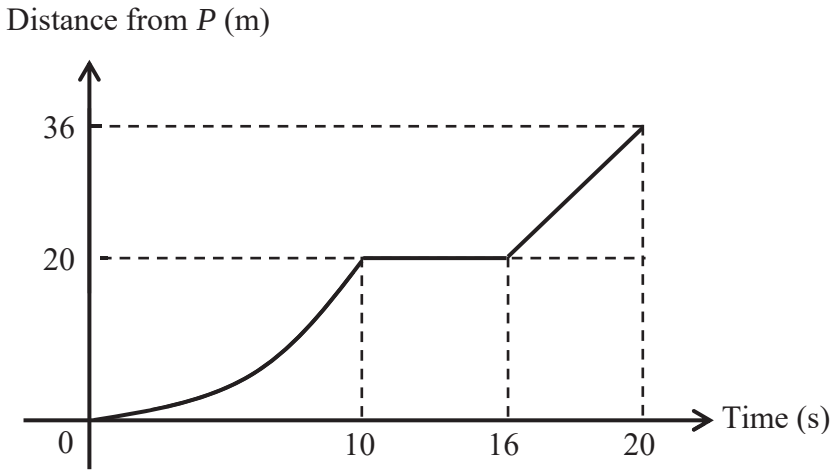


B1 for shape and turning point and symmetry of curve

B1 for x- and y-intercepts

[2]

5 The diagram below shows the distance-time graph of a particle from point P for the first 20 seconds of its journey.



(a) Find the average speed during the first 10 seconds.

$$\text{Average speed} = \frac{20}{10}$$
$$= 2 \text{ m/s} \quad \text{B1}$$

Answer 2 m/s [1]

(b) Find the acceleration of the particle when $t = 17$.

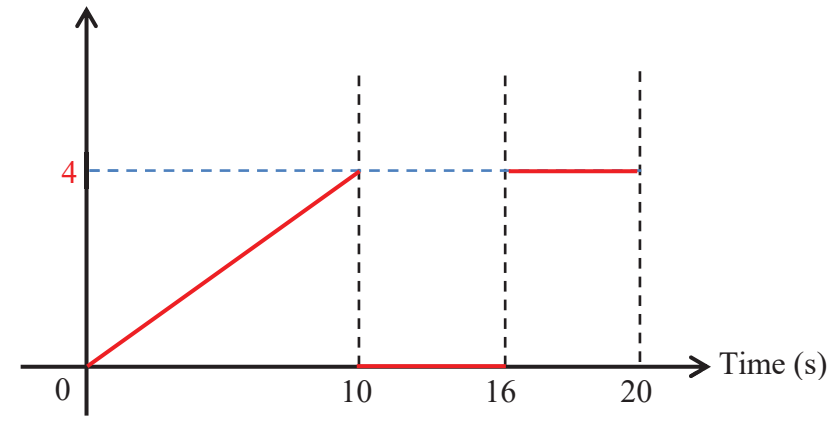
Answer 0 B1 m/s^2 [1]

(c) Find the speed of the particle when $t = 18$.

$$\text{Speed} = \frac{16}{4}$$
$$= 4 \text{ m/s} \quad \text{A1}$$

Answer 4 m/s [1]

(d) The speed increases uniformly for the first 10 seconds.
Sketch the speed-time graph for the first 20 seconds on the answer space, indicating the corresponding speeds clearly. [3]



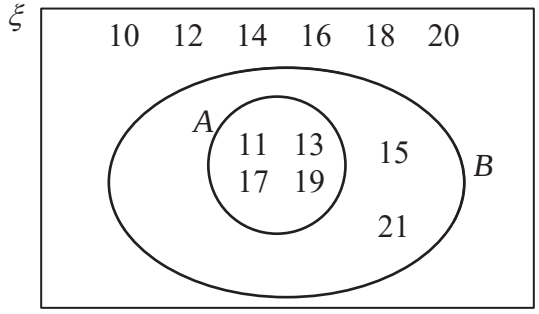
B1 for each interval

6 Given $\xi = \{x \text{ is an integer: } 10 \le x < 22\}$, and

$A = \{x : x \text{ is a prime number}\}$

$B = \{x : x \text{ is an odd number}\}$

(a) Draw the Venn diagram to show the elements of ξ and two sets A and B. [1]



B1

(b) List the elements in the set $A \cap B$.

$A \cap B = \{11,13,17,19\}$ B1

Answer $\{11,13,17,19\}$ [1]

(c) Find $n(A \cup B')$.

$(A \cup B') = \{10,11,12,13,14,16,17,18,19,20\}$

$n(A \cup B') = 10$ B1

Answer 10 [1]

7 500 ml of liquid fertiliser was made by mixing 200 ml of water with a 300 ml mixture of chemical *A* and chemical *B*.
The ratio of chemical *A* to chemical *B* is 1 : 9.
Abigail wants to decrease the concentration of chemical *B* to 20% of the liquid fertiliser.
Calculate the volume of chemical *B* to be removed from the original 500 ml of liquid fertiliser.

$$\text{Amount of chemical } B = \frac{9}{10} \times 300$$
$$= 270 \text{ ml}$$

Let the volume of chemical *B* to be reduced be x ml.

$$\frac{270 - x}{500 - x} = \frac{1}{5} \quad \text{M1}$$
$$1350 - 5x = 500 - x$$
$$4x = 850$$
$$x = 212.5 \quad \text{A1}$$

Answer 212.5 ml [2]

8 The number of sides of a regular polygon B is triple the number of sides of regular polygon A.
The ratio of each interior angle of the polygon A to that of the polygon B is 3 : 4.

(a) Find the number of sides of polygon A.

$$\frac{(n-2)\times 180}{n} = \frac{3}{4}\times\frac{(3n-2)\times 180}{3n} \quad \text{M1}$$

$$n-2 = \frac{3n-2}{4}$$

$$4n-8 = 3n-2$$

$$n = 6 \quad \text{A1}$$

Answer 6 [2]

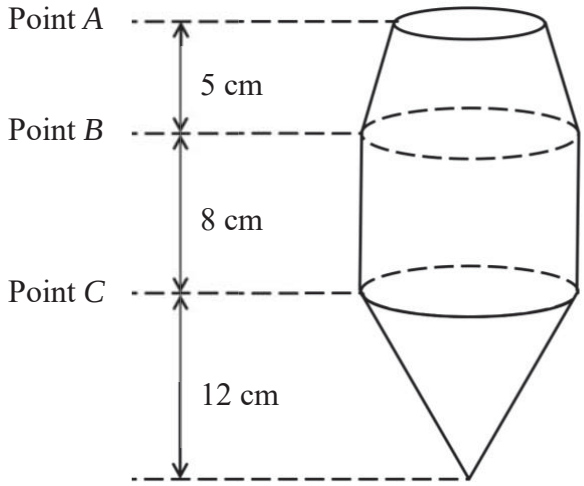
(b) Hence, find the size of each exterior angle of polygon B.

$$\text{Size of each exterior angle of polygon B} = \frac{360°}{3\times 6} \quad \text{M1}$$

$$= 20° \quad \text{A1}$$

Answer 20 ° [2]

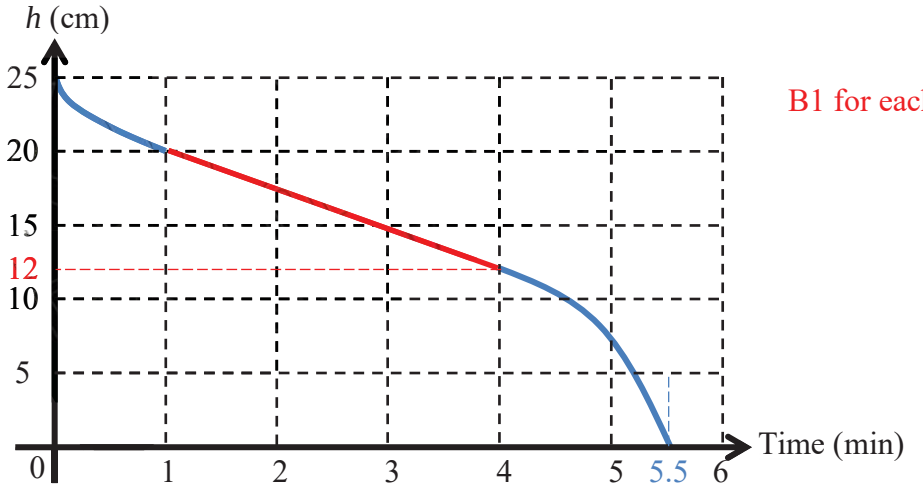
9 The container, shown in the diagram, is initially full of water.
There is a small hole at the bottom and water is leaking through the hole at a constant rate.



It takes one minute for the water level to drop from Point *A* to Point *B* and another three minutes to drop from Point *B* to Point *C* and a further 90 seconds for the water to be fully drained from the container.

On the axes below, sketch the graph showing the depth of the water, h cm, in the container.

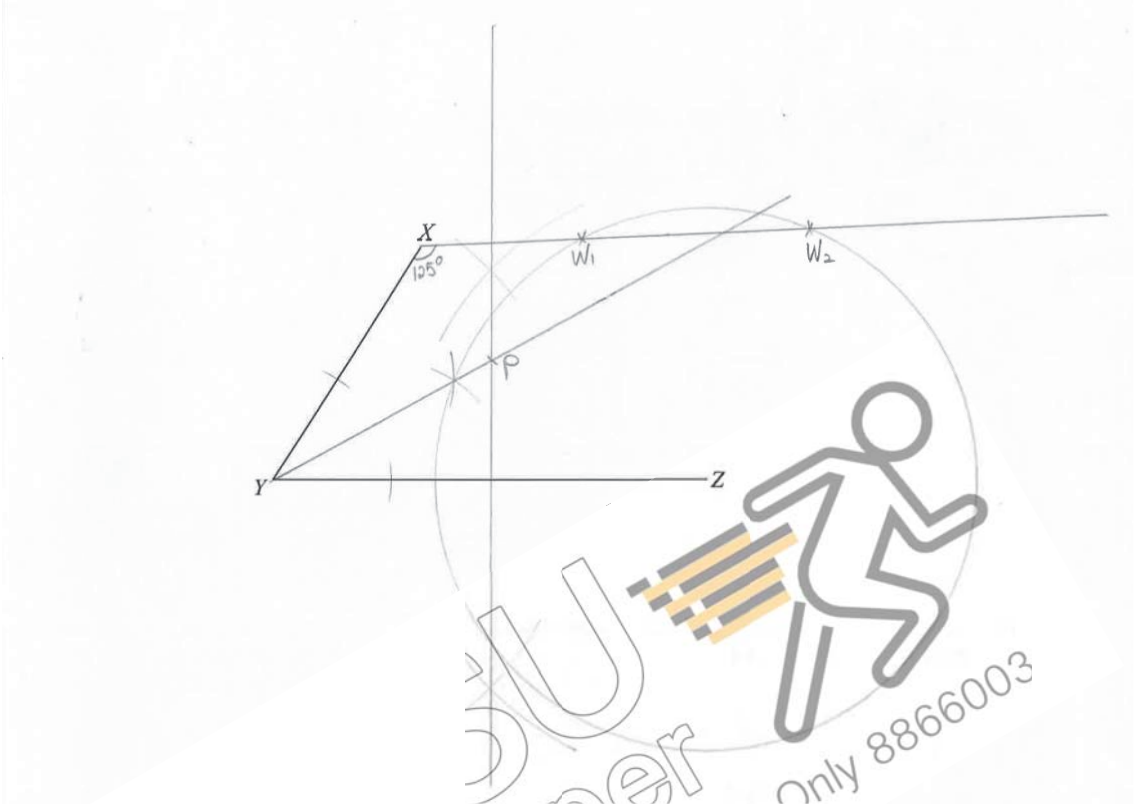
Answer



B1 for each part

[3]

10 Three points X, Y and Z are shown below.



(a) P is equidistant from the points Y and Z.
P is equidistant from the lines YX and YZ.
Construct and label the point P.

B1 for perpendicular bisector
B1 for angle bisector and point P [2]

(b) The point W is such that $\angle YXW = 125°$ and WZ is 5 cm.
Find the two possible positions of W and label them W_1 and W_2. [2]

B1 each

11 Express $\dfrac{(3x+1)(x-1)}{x^2-1}-2$ as a single fraction in the simplest form.

$$\frac{(3x+1)(x-1)}{x^2-1}-2$$

$$=\frac{(3x+1)(x-1)}{(x+1)(x-1)}-\frac{2(x+1)}{x+1}$$ M1

$$=\frac{3x+1-2(x+1)}{x+1}$$ M1 - single fraction

$$=\frac{x-1}{x+1}$$ A1

Answer $\dfrac{x-1}{x+1}$ [3]

---

12 Q is 25% greater than P and R is 30% smaller than Q.
What is the ratio of P : R?

P	:	Q	:	R	
1	:	1.25			
		1	:	0.7	
4	:	5	:	3.5	M1 (o.e.)
8	:	10	:	7	

$P:R=8:7$ A1

Answer 8 : 7 [2]

---

13 A bank guarantees an interest amount of \$750.46 at the end of 3 years with a compound interest of r% per annum, compounded every three months, for a principal amount of \$8000. Find the value of r.

$$8000+(750.46)=8000\left(1+\frac{\frac{r}{4}}{100}\right)^{3\times4} \quad \text{M1}$$

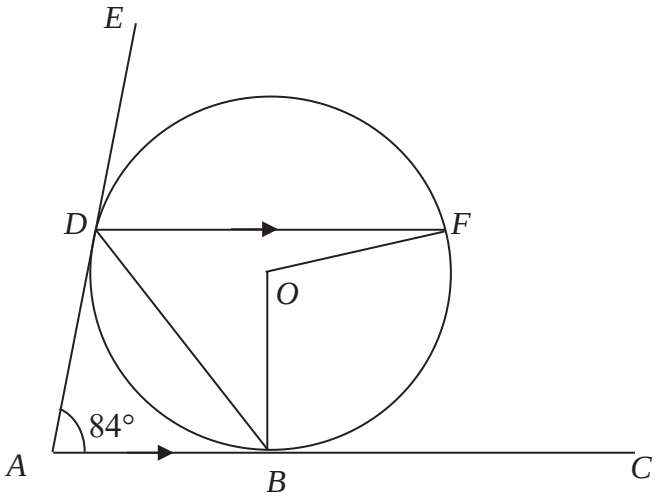
$$1.0938075=\left(1+\frac{r}{400}\right)^{12}$$

$$1+\frac{r}{400}=\sqrt[12]{1.0938075} \quad \text{M1}$$

$$r=3.00\ (3\text{sf}) \quad \text{A1}$$

Answer $r =$ 3.00 [3]

14 In the figure below, *ABC* and *ADE* are tangents to the circle with centre *O*.



Given that *DF* is parallel to *ABC* and angle *DAB* = 84°, **stating your reasons** clearly, find

(a) angle *ADB*,

$AD = AB$ (tangents from ext pt)

$\angle ADB = \dfrac{180° - 84°}{2}$ (Base angles of isosceles triangle)

$= 48°$ A1

Answer 48 ° [1]

(b) angle *BOF*,

$\angle BDF = 48°$ (alt angles, *DF* // *AB*) M1

$\angle BOF = 96°$ (angle at the centre = 2 × angles at circumference) A1

Answer 96 ° [2]

(c) angle *FBC*,

$\angle OBC = 90°$ (tan ⊥ rad)

$\angle OBF = \dfrac{180° - 96°}{2}$ M1

$= 42°$ (base angles of isosceles triangle)

$\angle FBC = 90° - 42°$

$= 48°$ A1

Alternate segment theorem award 2m

Answer 48 ° [2]

(d) angle *DFO*.

$\angle DFB = \angle FBC = 48°$ (alt angles, *DF* //*AB*) M1

$\angle DFO = 48° - 42°$

$= 6°$ A1

Answer 6 ° [2]

15 The table below shows the mass of 500 packets of 1 kg chocolate packed by machine *A*.

Mass (x grams)	$980 < x \leq 990$	$990 < x \leq 1000$	$1000 < x \leq 1010$	$1010 < x \leq 1020$
Frequency	74	178	182	66

(a) Calculate an estimate of the mean mass.

Mean $= 999.8g$ A1

Answer 999.8 g [1]

(b) Calculate an estimate of the standard deviation.

Standard Deviation $= 9.00g$ A1

Answer 9.00 g [1]

(c) Explain why the mean and standard deviation are estimates.

Answer [1]

For grouped data, only the mid-value of each data is used to calculate the mean and standard deviation. Hence, the mean and standard deviation are estimates. A1

(d) The mean and standard deviation of mass of chocolate packed by machine *B* is given below.

Mean (grams)	1000.2
Standard Deviation (grams)	12.5

If you are the owner of the chocolate distribution business, which machine will you buy? Explain your reason.

Answer [1]

I will buy machine *A*.
Machine *A* is more consistent in producing the packets of 1kg chocolate than machine *B* because the standard deviation of the mass of chocolate by machine *A* is 9.00g which is lower than the standard deviation of the mass of chocolate by machine *B* that is 12.5g. A1

16 **(a)** Express 150 as a product of its prime factors.

$150 = 2 \times 3 \times 5^2$ A1

Answer $2 \times 3 \times 5^2$ [1]

(b) Find the smallest integer a, such that $\left(150 + 150^2\right)a$ is a perfect square.

$\left(150 + 150^2\right)a$
$= 150\left(151\right)a$ M1
$= \left(2 \times 3 \times 5^2\right)\left(151\right)a$

$\therefore a = 2 \times 3 \times 151$
$= 906$ A1

Answer $a =$ 906 [2]

17 A tailor shop makes dresses and shirts.
The following table shows the different components of manufacturing each clothing item.

	Labour (Hours)	Cloth (Metres)	Buttons (Rows)
Dress	7	5	2
Shirt	4	5	3

Labour costs \$8 per hour, cloth costs \$11 per metre and buttons cost \$3 per row.

It is given that $A=\begin{pmatrix}7 & 5 & 2\\ 4 & 5 & 3\end{pmatrix}$, $B=\begin{pmatrix}8\\ 11\\ 3\end{pmatrix}$ and $C=AB$.

(a) **(i)** Evaluate C.

$$C=\begin{pmatrix}7 & 5 & 2\\ 4 & 5 & 3\end{pmatrix}\begin{pmatrix}8\\ 11\\ 3\end{pmatrix}$$
$$=\begin{pmatrix}56+55+6\\ 32+55+9\end{pmatrix}$$
$$=\begin{pmatrix}117\\ 96\end{pmatrix}$$

Answer $\begin{pmatrix}117\\ 96\end{pmatrix}$ [1]

(ii) Explain what your answer in (a)(i) represents.

Answer
The elements represent the manufacturing cost of a dress (\$117) and a shirt (\$96) respectively.

[1]

(b) In addition, $D=\begin{pmatrix}100 & 150\end{pmatrix}$.

(i) Evaluate DC.

$$DC=\begin{pmatrix}100 & 150\end{pmatrix}\begin{pmatrix}117\\ 96\end{pmatrix}$$
$$=\begin{pmatrix}11700+14400\end{pmatrix}$$
$$=\begin{pmatrix}26100\end{pmatrix}$$

Answer (26100) [1]

(ii) Explain what your answer in (b)(i) represents.

Answer
The total manufacturing cost of 100 dresses and 150 shirts.

[1]

18 Factorise each of the following completely.

(a) $(2a)^2-4ab+28a+(b-7)^2$.

$(2a)^2-4ab+28a+(b-7)^2$

$=(2a)^2-4a(b-7)+(b-7)^2$ M1

$=\left[2a-(b-7)\right]^2$

$=(2a-b+7)^2$ A1

Answer $(2a-b+7)^2$ [2]

(b) $xy^2-9x+3y-y^2$.

$xy^2-9x+3y-y^2$

$=x(y^2-9)-y(y-3)$ M1 – factorise

$=x(y+3)(y-3)-y(y-3)$ M1

$=(y-3)\left[x(y+3)-y\right]$

$=(y-3)(xy+3x-y)$ A1

Answer $(y-3)(xy+3x-y)$ [3]

---

19 Study the number pattern shown in the table below:

Number (n)	Pattern	Sum of Number Pattern (S)	$S + 2$
1	2	2	4
2	2, 3, 2	7	9
3	2, 3, 4, 3, 2	14	16
4	2, 3, 4, 5, 4, 3, 2	23	25

(a) Complete the table. [1]

(b) If $S = 2 + 3 + 4 + \ldots + (k - 1) + k + (k + 1) + k + (k - 1) + \ldots + 4 + 3 + 2 = 98$, find the value of k. A1

$(k+1)^2 = 98 + 2$

$k + 1 = 10 \quad (n + 1 > 0)$

$k = 9$ A1

Answer $k =$ 9 [1]

(c) Write a formula connecting S and n.

$S + 2 = (n+1)^2$ A1

or

$S = (n+1)^2 - 2$

Answer $S = (n+1)^2 - 2$ [1]

(d) Hence, find the sum of 2, 3, 4, …, 98, 99, 98, …, 4, 3, 2.

$S = 99^2 - 2$

$= 9799$ A1

Answer 9799 [1]

20 In a quadratic equation, the coefficients of x and x^2 are m and 5 respectively.
The constant of the quadratic equation is –16.
One solution of the quadratic equation is $-\frac{2}{5}$.

Find m and the other solution of the equation.

$$5\left(-\frac{2}{5}\right)^2+m\left(-\frac{2}{5}\right)-16=0$$

$$-\frac{2}{5}m=\frac{76}{5}$$

$$m=-38 \qquad \text{A1}$$

$5x^2-38x-16=(5x+2)(x-8)$ M1 (need to show factorisation and ensure the coefficient of x^2 will be 5 after multiplication)

$x-8=0$

$x=8$ A1

Answer $m =$ –38

$x =$ 8 [3]

---

21 $\dfrac{3a-bc}{2ac-5b}=\dfrac{1}{2}$

(a) Rearrange the formula to make c the subject.

$6a-2bc=2ac-5b$ M1

$2ac+2bc=6a+5b$

$c(2a+2b)=6a+5b$ M1

$c=\dfrac{6a+5b}{2a+2b}$ A1

Answer $c =$ $\dfrac{6a+5b}{2(a+b)}$ [3]

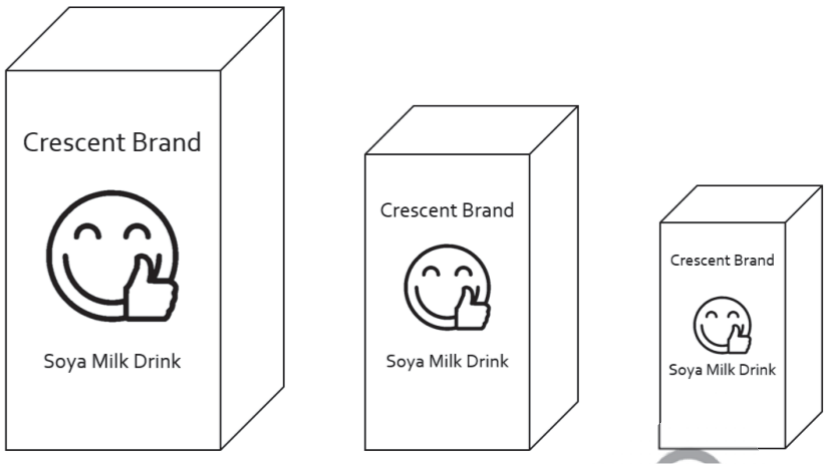
(b) Calculate the value of c when $a=-3$ and $b=2$.

$$c=\frac{6(-3)+5(2)}{2(-3+2)}$$

$=4$ A1

Answer 4 [1]

22 A company manufactures three sizes of the same brand of soya milk.



800 ml
$1.40

375 ml
$0.90

250 ml
$0.53

(a) Show that the cost of the soya milk is not directly proportional to the quantity of soy milk.

Answer

$$\text{Cost per ml for 800 ml packet} = \frac{1.45}{800} = \$0.00175$$

$$\text{Cost per ml for 375 ml packet} = \frac{0.90}{375} = \$0.0024$$

$$\text{Cost per ml for 250 ml packet} = \frac{0.53}{250} = \$0.00212$$

M1 for any 2

Since the cost per ml for the 800 ml packet is different from that of the 375 ml packet and the 250 ml packet, the cost of the soya milk is not directly proportional to the quantity of soy milk.

A1

[2]

(b) The packets are geometrically similar.
The height of the 250 ml packet is 11 cm.

Calculate the height of the 800 ml packet.

$$\frac{250}{800} = \left(\frac{11}{\text{height of 800 ml packet}}\right)^3 \quad \text{M1}$$

$$\text{height of 800 ml packet} = \frac{11}{\sqrt[3]{\frac{250}{800}}} \quad \text{M1}$$

$$= 16.2097$$

$$= 16.2 \text{ cm (3sf)} \quad \text{A1}$$

Answer 16.2 cm [3]

END OF PAPER

Answer **all** the questions.

1 **(a)** Solve the inequality $\frac{3-x}{2} \le 2-\frac{x}{3}$. [2]

$\frac{3-x}{2} \le 2-\frac{x}{3}$

$\frac{3-x}{2} \le \frac{6-x}{3}$ (M1)

$9-3x \le 12-2x$

$x \ge -3$ (A1)

(b) Express as a single fraction in its simplest form $\frac{3y}{(3-2y)^2}+\frac{y}{(2y-3)}$. [2]

$\frac{3y}{(3-2y)^2}+\frac{y}{(2y-3)} = \frac{3y}{(3-2y)^2}-\frac{y(3-2y)}{(3-2y)^2}$ (M1)

$= \frac{3y-3y+2y^2}{(3-2y)^2}$

$= \frac{2y^2}{(3-2y)^2}$ (A1)

(c) Simplify $\frac{6p^2q}{15r^3} \div \frac{3p^3q^2}{5r} \times \frac{p^4}{q^2}$, leave your answer in positive indices. [2]

$\frac{6p^2q}{15r^3} \div \frac{3p^3q^2}{5r} \times \frac{p^4}{q^2} = \frac{6p^2q}{15r^3} \times \frac{5r}{3p^3q^2} \times \frac{p^4}{q^2}$ (M1)

$= \frac{2}{3} p^{2+4-3} q^{1-2-2} r^{1-3}$

$= \frac{2}{3} p^3 q^{-3} r^{-2}$

$= \frac{2p^3}{3q^3r^2}$ (A1)

(d) Simplify $\left(\frac{v^{12}}{27t^9}\right)^{-\frac{2}{3}}$, leave your answer in positive indices. [2]

$$\left(\frac{v^{12}}{27t^9}\right)^{-\frac{2}{3}} = \left(\frac{27t^9}{v^{12}}\right)^{\frac{2}{3}} \quad \text{(M1)}$$

$$= \left(\frac{3^3 t^9}{v^{12}}\right)^{\frac{2}{3}}$$

$$= \frac{9t^6}{v^8} \quad \text{(A1)}$$

(e) Solve the equation $\frac{8}{x} - \frac{3}{x+1} = 3$. [3]

$$\frac{8}{x} - \frac{3}{x+1} = 3$$

$$\frac{8(x+1) - 3x}{x(x+1)} = 3 \quad \textbf{(M1)}$$

$$\frac{5x+8}{x^2+x} = 3$$

$$5x + 8 = 3x^2 + 3x$$

$$3x^2 - 2x - 8 = 0$$

$$(3x+4)(x-2) = 0 \quad \text{(M1)}$$

$$x = 2 \text{ or } -1\frac{1}{3} \quad \text{(A1)}$$

2 The following table shows the world population by region in 2020.

Region	Population
Asia	4.64×10^9
Africa	1.34×10^9
Europe	7.48×10^8
Southern America	6.54×10^8
Northern America	3.69×10^8
Oceania	42,677,813

(a) Express the population of Oceania in standard form, correct to 3 significant figures. [1]

4.27×10^7 (A1)

(b) Calculate the percentage of population in Asia compared to the world population. [2]

$$\text{Percentage of population in Asia} = \frac{464\times10^7}{(464+134+74.8+65.4+36.9+4.27)\times10^7}\times100\% \quad \text{(M1)}$$

$$= \frac{464\times10^7}{779.37\times10^7}\times100\%$$

$$= 59.5\% \quad \text{(A1)}$$

(c) From 2019 to 2020, the total population of Japan decreased by 1.3%. The population of Japan in 2020 is 1.261×10^8. Find the population of Japan in 2019. Give your answer in standard form. [2]

$$\text{Population of Japan} = \frac{1.261\times10^8}{0.987} \quad \text{(M1)}$$

$$= 1.2776\times10^8$$

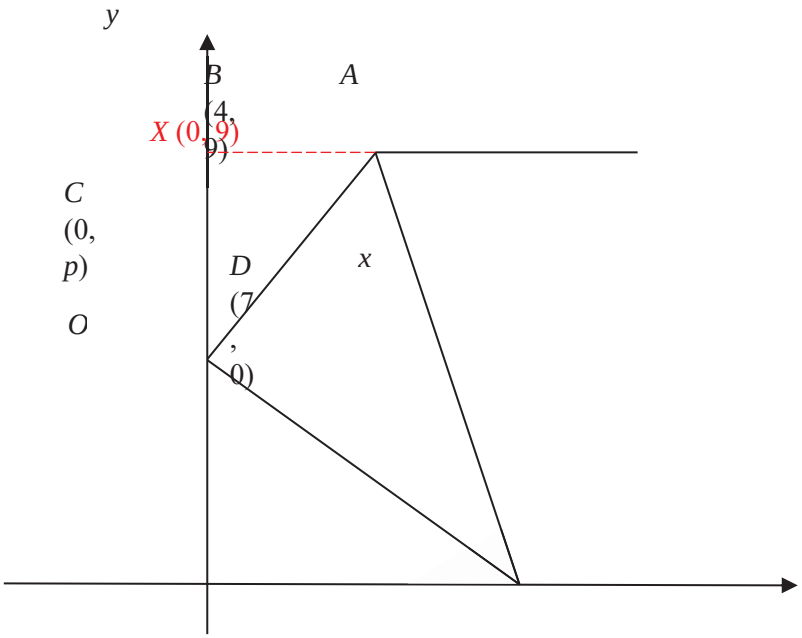
$$= 1.28\times10^8 \quad \text{(A1)}$$

(d) The population of Africa grew by 2.5% every year for two consecutive years from 2018 to 2020. Calculate the population of Africa in 2018. Give your answer in standard form. [2]

$$\text{Population of Africa} = \frac{1.34\times10^9}{(1.025)^2} \quad \text{(M1)}$$

$$= 1.28\times10^9 \quad \text{(A1)}$$

3 The diagram, not drawn to scale, shows a triangle with vertices $B(4, 9)$, $C(0, p)$ and $D(7, 0)$.



(a) Given that AB is parallel to the x axis and $AB = BC = 5$ units.
State the coordinates of A. [1]

$A(9, 9)$ **(B1)**

(b) Find the value of p. [2]

$$\sqrt{(4-0)^2+(9-p)^2} = 5 \qquad \text{(M1)}$$

$$16+(9-p)^2 = 25$$

$$9-p = \sqrt{9} \text{ or } -\sqrt{9}$$

$$-p = 3-9 \text{ or } -3-9$$

$$p = 6 \text{ (shown) or } 12 \text{ (rejected)} \qquad \text{(A1)}$$

(c) Find the equation of the line BD. [2]

$$\text{Gradient of } BD = \frac{0-9}{7-4}$$

$$= -3$$

(M1)

To find c, the y intercept of line BD:

$$0 = -3(7) + c$$

$$c = 21$$

The equation of BD is $y = -3x + 21$. (A1)

(d) The point E lies on the y-axis such that D, B and E lie on a straight line. Find the coordinates of point E. [1]

$E(0, 21)$ (B1)

(e) Find the area of triangle BCD. [3]

$$\text{Area of trapezium } OXBD = \frac{1}{2}(4+7)(9)$$
$$= 49.5 \text{ units}^2 \quad \text{(M1)}$$
$$\text{Area of } \Delta BCX = \frac{1}{2}(3)(4)$$
$$= 6 \text{ units}^2$$
$$\text{Area of } \Delta OCD = \frac{1}{2}(7)(6) \quad \text{(M1 for either area)}$$
$$= 21 \text{ units}^2$$
$$\text{Area of } \Delta BCD = 49.5 - 6 - 21$$
$$= 22.5 \text{ units}^2 \quad \text{(A1)}$$

4 The variables x and y are connected by the equation $y = \frac{x^2}{5} + \frac{12}{x} - 5$. Some corresponding values of x and y, correct to one decimal place, are given in the table below.

x	1	1.5	2	3	4	5	6	7
y	7.2	3.5	1.8	p	1.2	2.4	4.2	6.5

(a) Calculate the value of p. [1]

$p = 0.8$ (B1)

(b) On the grid opposite, draw the graph of $y = \frac{x^2}{5} + \frac{12}{x} - 5$ for $1 \le x \le 7$. [3]

1 mark for all points correctly drawn
2 marks for smooth and labelled curve

(c) Use your graph to find the values of x in the range $1 \le x \le 7$ for which $x^3 - 35x = -60$.

$x^3 - 35x = -60$

$\frac{x^3}{5x} - \frac{35x}{5x} + \frac{60}{5x} = 0$

$x^2 - 7 + \frac{12}{5x} = 0$

$x^2 + \frac{12}{x} - 5 = 2$ (B1)

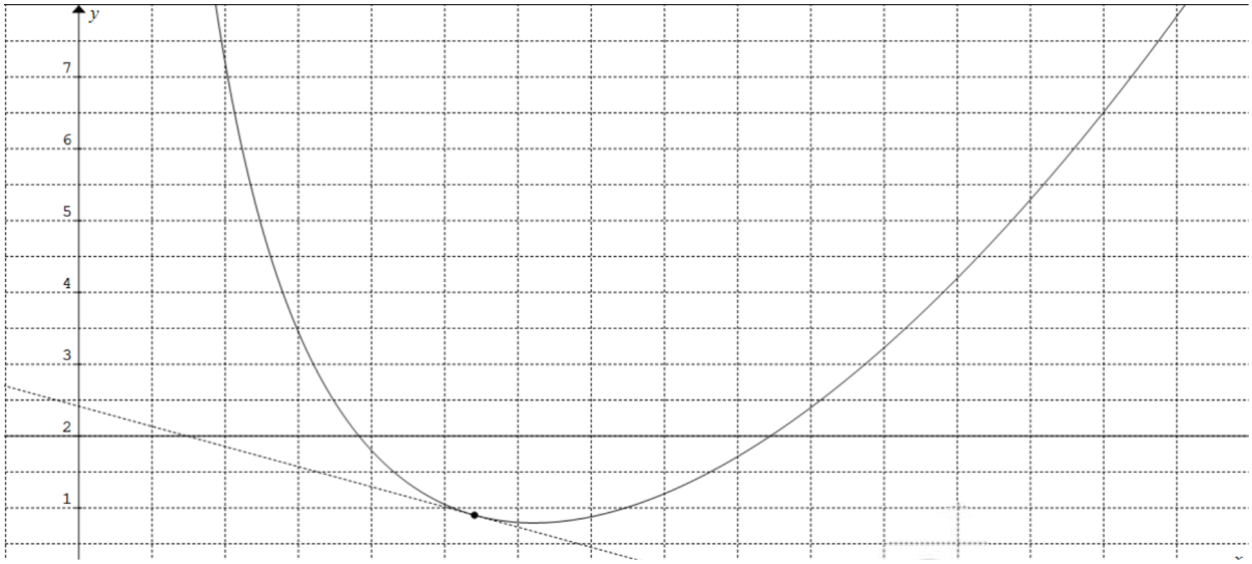
Draw $y = 2$

$x = 1.9$ (1.85 to 1.95) or 4.75 (4.7 to 4.8) (B1 for each correct answer)

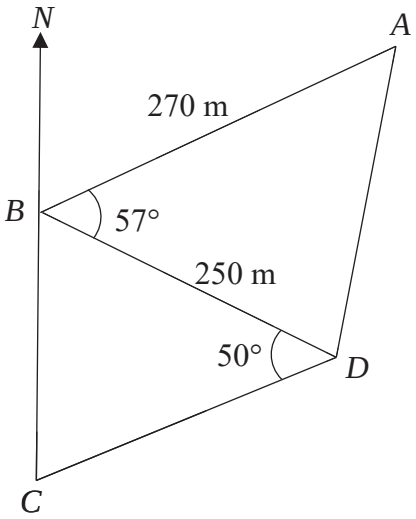
(d) Use your graph to find the x-coordinate of a point on the curve $y = \frac{x^2}{5} + \frac{12}{x} - 5$ at which the gradient of the tangent is equal to -0.5.

1 mark for drawing parallel lines

x-coordinate = 2.75 (acceptable if between 2.7 to 2.8) (B1)


y
7
6
5
4
3
2
1

5 The diagram represents two fields ABD and BCD in a horizontal plane.
$AB = 270$ m, $BD = 250$ m, $\angle ABD = 57°$ and $\angle CDB = 50°$.
B is due north of C.
The bearing of B from D is $293°$.



(a) Find

(i) the bearing of D from C, [2]

$\angle NDB = 360° - 293°$
$= 67°$ (M1)

Bearing of D from $C = 180° - 50° - 67°$
$= 063°$ (A1)

(ii) the bearing of A from B. [1]

Bearing $= 63° + 50° - 57°$
$= 056°$ (B1)

(b) Calculate, correct to the nearest metre, the length of

(i) AD, [2]

$AD^2 = 270^2 + 250^2 - 2(270)(250)\cos 57°$ (M1)
$AD = \sqrt{61874}$
$= 248.74$
$= 249$ m (A1)

(ii) BC. [2]

$\frac{BC}{\sin 50°} = \frac{250}{\sin 63°}$ (M1)
$BC = 214.94$
$= 215$ m (A1)

(c) A vertical tower with a height of 80 m is directly above D.

(i) A man is walking along AB.
Calculate the shortest distance between the man and point D. [2]

Let X be the position that is nearest to D.

$$\sin 57° = \frac{XD}{250} \qquad \text{(M1)}$$

$$XD = 209.67$$

$$= 210 \text{ m} \qquad \text{(A1)}$$

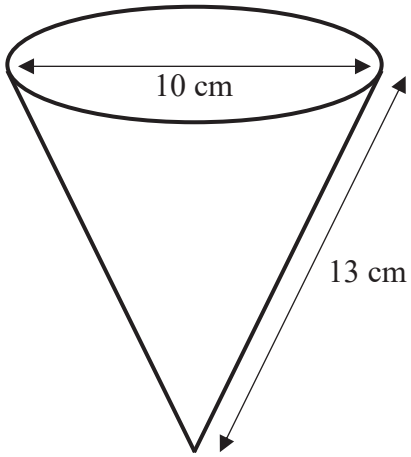
(ii) Find the angle of depression of the man from the top of the tower when he is nearest to the tower. [2]

$$\tan\theta = \frac{80}{209.67} \qquad \text{(M1)}$$

$$\theta = 20.884°$$

$$= 20.9° \qquad \text{(A1)}$$

6 A disposable cup used for water dispensers has a slant length of 13 cm and a diameter of 10 cm.



(a) Show that the volume of the cup is 100π cm^3. [2]

$$\text{height of cup} = \sqrt{13^2 - 5^2}$$
$$= 12 \text{ cm} \qquad \text{(M1)}$$

$$\text{Volume of cup} = \frac{1}{3}\pi(5)^2(12)$$
$$= 100\pi \text{ cm}^3 \quad \textbf{(shown)} \qquad \text{(A1)}$$

(b) **(i)** The cup is filled with water from a small tap flowing at a constant rate of x cm^3/s.
Express, in terms of π and x, the time taken in seconds to fill up the cup with the small tap. [1]

$$\text{Time taken by small tap} = \frac{100\pi}{x} \text{ seconds} \qquad \text{(B1)}$$

(ii) If a big tap is used, the rate increases to $(2x+1)$ cm^3/s.
Express, in terms of π and x, the time taken in seconds to fill up the cup with the big tap. [1]

$$\text{Time taken by big tap} = \frac{100\pi}{2x+1} \text{ seconds} \qquad \text{(B1)}$$

(c) When the big tap is used, the time taken to fill up the cup will be reduced by one second. Write an equation, in terms of π and x, and show that it can be simplified to $2x^2+(1-100\pi)x-100\pi=0$. [3]

$$\frac{100\pi}{x}-\frac{100\pi}{2x+1}=1 \qquad \text{(M1)}$$

$$\frac{(2x+1)-x}{x(2x+1)}=\frac{1}{100\pi} \qquad \text{(M1)}$$

$$\frac{x+1}{2x^2+x}=\frac{1}{100\pi}$$

$$100\pi x+100\pi=2x^2+x$$

$$2x^2+(1-100\pi)x-100\pi=0 \qquad \text{(A1)}$$

(d) Solve the equation $2x^2+(1-100\pi)x-100\pi=0$, giving your answers correct to 2 decimal places. [3]

$$2x^2+(1-100\pi)x-100\pi=0$$

$$x=\frac{-(1-100\pi)\pm\sqrt{(1-100\pi)^2-4(2)(-100\pi)}}{2(2)} \qquad \text{(M2)}$$

$$x=-0.99685 \text{ or } 157.58$$

$$x=-1.00 \text{ or } 157.58 \qquad \text{(A1)}$$

(e) Hence find the time taken to fill up the cup using the small tap. [2]

$$\text{Time taken by small tap}=\frac{320\pi}{157.58} \qquad \text{(M1)}$$

$$=1.99 \text{ seconds} \qquad \text{(A1)}$$

7 In the diagram, *ABCDE* is a regular pentagon (Figure 1).

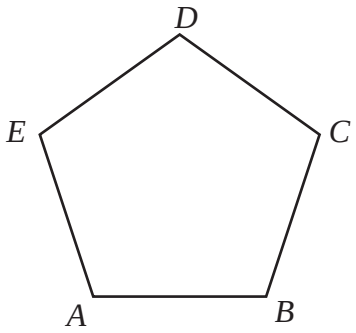


Figure 1

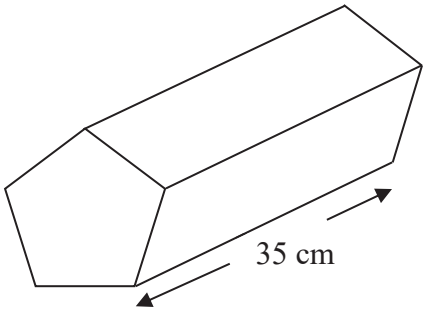


Figure 2

(a) Calculate

(i) $\angle BAE$, [1]

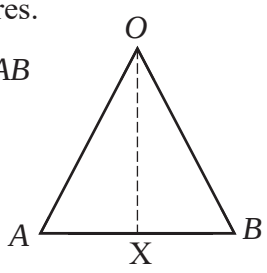
$$\angle BAE = \frac{180°(5-2)}{5}$$
$$= 108° \quad \text{(B1)}$$

(ii) $\angle BEC$. [1]

$$\angle AEB = \frac{180° - 108°}{2}$$
$$= 36°$$
$$\angle BEC = 108° - 2(36°)$$
$$= 36° \quad \text{(B1)}$$

(b) Given that the length of the side of the pentagon is 8 cm, show that the area of the pentagon *ABCDE* is 110.11 cm^2, correct to 5 significant figures. [4]

Let *O* be the centre of the pentagon. *OX* is perpendicular to *AB*



$$\angle AOB = \frac{360°}{5}$$
$$= 72°$$
$$\angle OAX = \frac{180° - 72°}{2}$$
$$= 54° \quad \text{(M1)}$$

$$\tan 54° = \frac{OX}{AX}$$
$$OX = 4\tan 54°$$
$$= 5.5055 \quad \text{(M1)}$$

or

$$\tan 36° = \frac{AX}{OX}$$
$$OX = \frac{4}{\tan 36°}$$
$$= 5.5055 \quad \text{(M1)}$$

$$\text{Area} = 5 \times \frac{1}{2}(8)(5.5055) \quad \text{(M1)}$$
$$= 110.11 \text{ cm}^2 \quad \textbf{(A1)}$$

(c) Pentagon *ABCDE* forms the base of a solid prism (Figure 2) with length 35 cm.

Calculate

(i) the volume of the prism, [1]

$$\text{Volume} = 110.11 \times 35$$
$$= 3853.85$$
$$= 3850 \text{ cm}^3 \quad \text{(B1)}$$

(ii) the total surface area of the prism. [2]

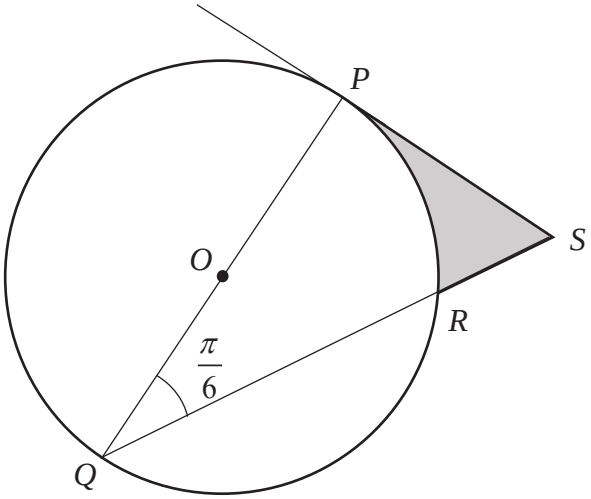
$$\text{Surface area} = 2(110.11) + 5(8)(35) \quad \text{(M1)}$$
$$= 1620.22$$
$$= 1620 \text{ cm}^2 \quad \text{(A1)}$$

(d) The outer surfaces of the prism are to be painted. The paint is sold in tins. Each tin of paint costs \$3.15 before GST and can cover 250 cm^2. The prevailing GST is 7%. Find the cost of painting the prism. [2]

$$\text{No. of tins} = \frac{1620.22}{250}$$
$$= 6.48$$
$$= 7 \text{ (round up)} \quad \text{(M1)}$$

$$\text{Cost} = 7 \times 3.15 \times 1.07$$
$$= \$23.59 \quad \text{(A1)}$$

8 In the diagram, the circle, centre O, of radius 8 cm, passes through points P, Q and R and angle $PQR = \frac{\pi}{6}$ radians .The tangent at P meets QR produced at S.



(a) Show that $RS = 4.619$ cm, correct to 4 significant figures. [3]

$$\cos\frac{\pi}{6} = \frac{16}{QS}$$
$$QS = 18.475 \quad \text{(M1)}$$
$$\cos\frac{\pi}{6} = \frac{QR}{16}$$
$$QR = 13.856 \quad \text{(M1)}$$
$$RS = 18.475 - 13.856$$
$$= 4.619 \text{ cm} \quad \text{(A1)}$$

(b) Calculate

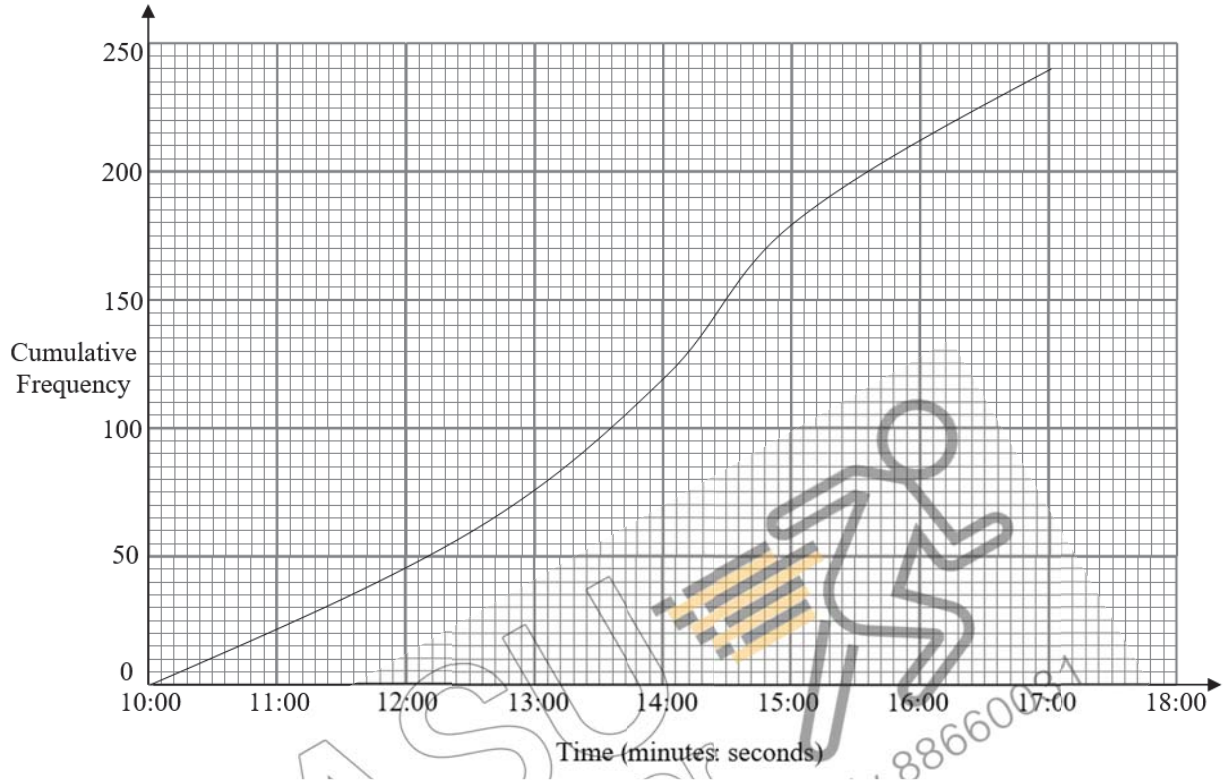
(i) the perimeter of the shaded region, [3]

$$\tan\frac{\pi}{6} = \frac{PS}{16}$$
$$PS = 9.2376 \quad \text{(M1)}$$
$$PR = 8\left(\frac{\pi}{3}\right) \quad \text{(M1)}$$
$$= 8.3776$$
$$\text{Perimeter} = 9.2376 + 8.3776 + 4.619$$
$$= 22.2 \text{ cm (3sf)} \quad \text{(A1)}$$

(ii) the area of the shaded region. [4]

$$\text{Area of } \Delta QPS = \frac{1}{2}(16)(9.2376)$$
$$= 73.901 \text{ cm}^2 \quad \text{(M1)}$$
$$\text{Area of } \Delta OQR = \frac{1}{2}(8)^2 \sin\frac{2\pi}{3}$$
$$= 27.713 \text{ cm}^2 \quad \text{(M1)}$$
$$\text{Area of sector } OPS = \frac{1}{2}(8)^2(\frac{\pi}{3})$$
$$= 33.510 \text{ cm}^2 \quad \text{(M1)}$$
$$\text{Shared area} = 73.901 - 27.713 - 33.510$$
$$= 12.7 \text{ cm}^2 \text{(3sf)} \quad \text{(A1)}$$

9 **(a)** The results of the 2.4 km Run-walk segment of the NAPFA test by each of the 240 Secondary Four girls in School *X* were measured. The cumulative frequency curve below shows the timings clocked by the girls.



(i) Use the curve to estimate

(a) the median timing, [1]

14 minutes (B1)

(b) the interquartile range of the timings, [2]

Interquartile range = 15 minutes – 12 minutes 30 seconds (M1)

= 2 minutes and 30 seconds (A1)

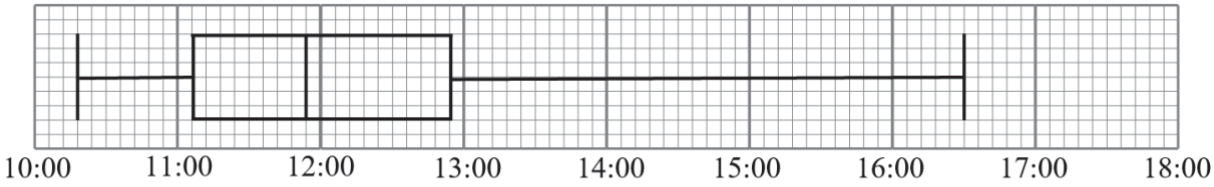
(ii) The timing for a B grade is between 14:00 to 15:00 minutes.

Estimate the percentage of the girls who achieved this grade. [2]

$$\text{Percentage} = \frac{180-120}{240} \quad \text{(M1)}$$

$$= 25\% \quad \text{(A1)}$$

(iii) The results of the 2.4 km Run-walk segment of the NAPFA test by each of the 240 Secondary Four girls in School *Y* were also measured. The box-and-whisker plot below shows the distribution of their results.



Make two comments comparing the timing of the girls for the 2.4 km Run-walk segment of the two schools. [2]

Median for School *X* is 14 minutes while median for School *Y* is 11 minutes 54 seconds. Generally, girls from School *Y* performed better as the median is lower. (B1)

The interquartile range of School *X* is 2 minutes 30 seconds while interquartile range for School *Y* is 1 minutes 48 seconds. Therefore, the timing for the girls from School *Y* has less variation. (B1)

(b) The table below summarises the statistics for number of inclined pull-ups completed for the same group of 240 Secondary Four girls from School *Y*.

Grade	**A**	**B**	**C**	**D**	**E**
Number of inclined pull-ups	>17	14-17	11-13	7-10	3-6
Number of students	24	38	92	71	15

(i) One student is selected at random.
Find the probability that she achieved a C grade. [1]

$$\text{Probability} = \frac{92}{240}$$
$$= \frac{23}{60} \quad \text{(B1)}$$

(ii) Two students are selected at random.
Find, as a fraction in its simplest form, the probability that

(a) they both scored a C grade, [2]

$$\text{Probability} = \frac{92}{240} \times \frac{91}{239} \quad \text{(M1)}$$
$$= \frac{2093}{14340} \quad \text{(A1)}$$

(b) one scored an A and the other scored a D grade. [2]

$$\text{Probability} = 2\left(\frac{24}{240} \times \frac{71}{239}\right) \quad \text{(M1)}$$
$$= \frac{71}{1195} \quad \text{(A1)}$$

10 Since the outbreak of the Coronavirus Disease 2019 (COVID-19) early this year, the virus has spread rapidly across the world, and more than 5 million people in over 200 countries have been infected to-date. In order to control the spread of the virus, many countries have implemented strict public health measures, including lockdowns and border closures to limit the movement of people. This has led to significant disruptions in global economic activity.

(a) All primary, secondary, Pre-University and IHL students, including students from Special Education (SPED) schools, shifted to full Home-Based Learning (HBL) from 8 April 2020 till (HBL) till 4 May 2020.

School *X* and *Y* conducted separate surveys on 20 of their students to find out how much time they spent on HBL a week. The results are presented in a stem-and-leaf diagram as shown below.

School *X*								School *Y*						
			8	7	7	6	0							
8	6	6	6	3	0	0	1	1	1	2	4	8		
			7	6	4	3	2	2	4	4	6	8	8	8
			4	4	3	0	3	2	*k*	*k*	*k*	7		
							4	0	2	3				
						2	5							

Key: 4 | 2 | 6 means 24 hours for School *X* and 26 hours for School *Y*

(i) One student from School *X* indicated 52 hours of HBL in the survey. Would you prefer to use range or interquartile range to calculate the spread of data? Explain. [1]

Interquartile range is preferred as it is not affected by outliers like 52 hour (B1)

(ii) There are 3 unknown values *k* in the diagram. "In general, students in School *Y* spent more time on HBL." Do you agree? Substantiate your answer. [2]

Yes.
The mean/ median for School *Y* is higher. – 1 mark
Generally, students in School *Y* spent more time on HBL – 1 mark

(b) Since the start of the Circuit Breaker period, there has been an increase in the number of people who order meals through food delivery service providers. Below are the charges from 3 different food delivery service providers.

Food delivery service provider	Food delivery fee	Minimum Order Value	Discount (if any)/ Promotion
A	\$3	Depends on the restaurant	- \$1 off
B	\$3	\$12	- 5% off on food items only - \$5 voucher. Can only be used for subsequent purchase
C	\$1.99 on top of each item	None	- 40% off for the lowest priced item (excluding food delivery fee)

Mrs Tan would like to order food delivery for her family from a restaurant which can be delivered by any of the 3 food delivery service providers.
Below are the prices of the items listed.

Food delivery service provider	Chicken Rice	Vegetable	Drink
A	\$4	\$5.50	\$2.50
B	\$4.20	\$4.80	\$2.80
C	\$3.50	\$4	\$2

Mrs Tan would like to order 4 portions of chicken rice, 1 portion of vegetable and 2 drinks. Which food deliver service provider would you recommend her to order from? Show all relevant calculations to support your recommendation. [5]

$$\begin{aligned}\text{Total charges for A} &= (\$4\times 4)+\$5.50+(\$2.50\times 2)+\$3)-\$1\\ &= \$28.50\\ \text{Total charges for B} &= ((\$4.20\times 4)+\$4.80)\times 0.95+(\$2.80\times 2)+\$3\\ &= \$29.12\\ \text{Total charges for C} &= (\$3.50+\$1.99)\times 4+(\$4+\$1.99)+(\$2+\$1.99)+((\$2\times 0.60)+\$1.99)\\ &= \$35.13\end{aligned}$$

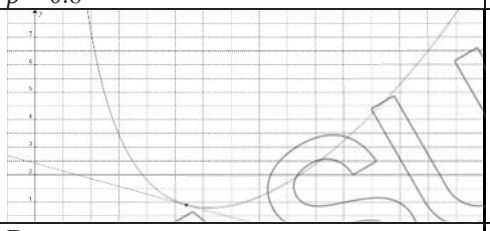
1 mark for each correction calculation.

Acceptable answers
1) I would recommend A. Total cost is lowest.
2) I would recommend B. The value of the voucher is more than the price difference between food delivery service providers A and B. Thus, it is more worthwhile buying from food delivery service provider B.

1 mark for correct recommendation. 1 mark for reasonable justification.

End Of Paper

Answer Key

1(a)	$x \geq -3$	**7(a)(i)**	108°
(b)	$\frac{2y^2}{(3-2y)^2}$	**(a)(ii)**	36°
(c)	$\frac{2p^3}{3q^3r^2}$	**(b)**	110.11 cm^2
(d)	$\frac{9t^6}{v^8}$	**(c)(i)**	3850 cm^3
(e)	$x = 2$ or $-1\frac{1}{3}$	**(c)(ii)**	1620 cm^2
2(a)	4.27×10^7	**(d)**	$23.59
(b)	59.5%	**8(a)**	4.619 cm
(c)	1.28×10^8	**(b)(i)**	22.2 cm
(d)	1.28×10^9	**(b)(ii)**	12.7 cm^3
3(a)	$A(9,9)$	**9(a)(i)(a)**	14 minutes
(b)	p = 6 or 12 (rejected)	**(a)(i)(b)**	2 minutes 30 seconds
(c)	$y = -3x+21$	**(a)(ii)**	25%
(d)	$E(0.\ 21)$	**(a)(iii)**	Median for School *X* is 14 minutes while median for School *Y* is 11 minutes 54 seconds. Generally, girls from School *Y* performed better as the median is lower. The interquartile range of School *X* is 2 minutes 30 seconds while interquartile range for School *Y* is 1 minutes 48 seconds. Therefore, the timing for the girls from School *Y* has less variation.
(e)	22.5 $units^2$		
4(a)	p = 0.8		
(b)			
(c)	Draw $y = 2$ x = 1.9 or 4.75	**(b)(i)**	$\frac{23}{60}$
(d)	2.75	**(b)(ii)(a)**	$\frac{2093}{14340}$
5(a)(i)	063°	**(b)(ii)(b)**	$\frac{71}{1195}$
(a)(ii)	056°	**10(a)(i)**	Interquartile range is preferred as it is not affected by outliers like 52 hours
(b)(i)	249 m	**(a)(ii)**	Yes. The mean/ median for School *Y* is higher. – 1 mark Generally, students in School *Y* spent more time on HBL – 1 mark
(b)(ii)	215 m	**(b)**	A - $28.50 B - $29.12 C - $35.13 1) I would recommend A. Total cost is lowest. 2) I would recommend B. The value of the voucher is more than the price difference between food delivery service providers A and B. Thus, it is more worthwhile buying from food delivery service provider B.
(c)(i)	210 m		
(c)(ii)	20.9°		
6(a)	100π cm^3		
(b)(i)	$\frac{100\pi}{x}$ seconds		
(b)(ii)	$\frac{100\pi}{2x+1}$ seconds		
(c)	$2x^2+(1-100\pi)x-100\pi = 0$		
(d)	-1.00 or 157.58		
(e)	1.99 seconds		