SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

SECONDARY 4
PRELIMINARY EXAMINATION
MATHEMATICS
Paper 1 4048/01

1 September 2020 (Tuesday) **2 hours**

CANDIDATE NAME	SOLUTION
CLASS	
INDEX NUMBER	

READ THESE INSTRUCTIONS FIRST
Do not turn over the page until you are told to do so.
Write your name, class and index number in the spaces above.
Write in dark blue or black pen in the space provided.

You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

INFORMATION FOR CANDIDATES
Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

For Examiner's Use		
Q1	2	
Q2	2	
Q3	2	
Q4	2	
Q5	2	
Q6	3	
Q7	3	
Q8	3	
Q9	3	
Q10	3	
Q11	3	
Q12	4	
Q13	4	
Q14	4	
Q15	4	
Q16	4	
Q17	5	
Q18	5	
Q19	5	
Q20	5	
Q21	6	
Q22	6	
Total	/ 80	

MATHEMATICAL FORMULAE

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

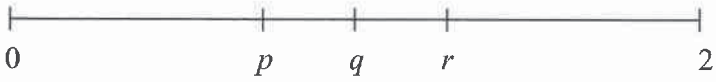
Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 The numbers p, q and r are represented on the number line.



The values of p, q, and r are listed below.

$$\left(-\frac{2}{3}\right)^0 \qquad \frac{\pi}{2} \qquad \frac{5}{8} \qquad \sqrt{5}$$

$= 1$ $\quad = 1.57$ $\quad = 0.625$ $\quad = 2.24$

(a) Find p, q and r.

Answer $p = \frac{5}{8}$ ....

$q = \left(-\frac{2}{3}\right)^0$ ....

$r = \frac{\pi}{2}$ ...... [1]

(b) Write down the irrational number(s).

Answer $\frac{\pi}{2}$ and $\sqrt{5}$ .... [1]

2 A Shinkansen train 273 m long passes through a tunnel 2 km long. The average speed of the train is 240 km/h.

Calculate the time taken for the train to pass through the tunnel completely.
Give your answer in seconds.

$$\text{Time Taken} = \frac{2+\frac{273}{1000}}{240}$$

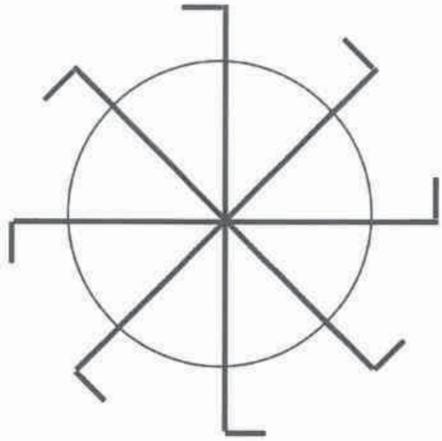
$$= \frac{2+\frac{273}{1000}}{240}\times 60\times 60$$

$$= 34.095 \text{ s}$$

Answer 34.095 seconds [2]

3	It is given that $w = \frac{4a-3b}{b+c}$. Express b in terms of w, a and c.	
	$w = \frac{4a-3b}{b+c}$ $w(b+c) = 4a-3b$ $bw + cw = 4a-3b$ $b(w+3) = 4a-cw$ $b = \frac{4a-cw}{w+3}$	
	Answer $= \frac{4a-cw}{w+3}$	[2]

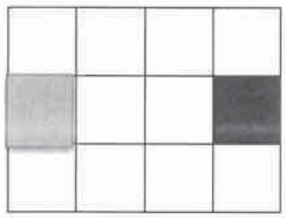
4 The diagram shows a simplified view of a waterwheel.



(a) State the order of rotational symmetry.

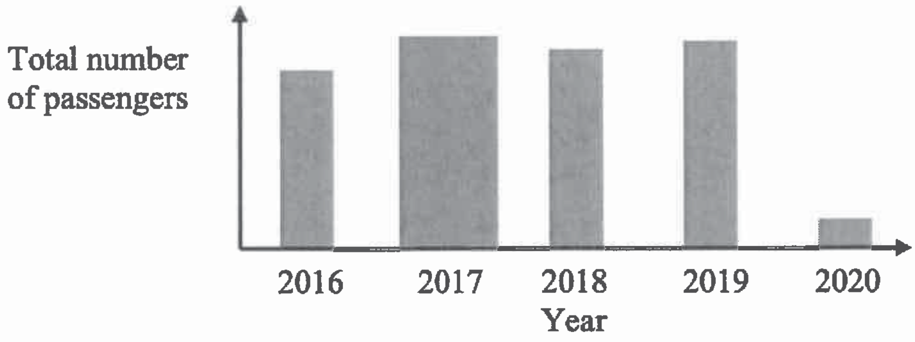
Answer ……………8………………….. [1]

(b) Shade one more square on the diagram, such that the diagram has two lines of symmetry.



[1]

5 The chart shows the total number of passengers at Changi Airport in the years 2016 to February 2020.



State one feature of the chart that may be misleading and explain why.

Features & reason

1. No values or scale on the vertical axis (if it starts from 0 or not) – opened to misinterpretation as to total number of passengers. OR Exaggerates the differences between the years.
2. 2017 has a different width – it is not clear whether the width or the height of the bar should be read as the total number of passenger movement.
3. 2020 bar implies the entire year while information given states up to February 2020 – distortion or misrepresentation of information.

[2]

6 Simplify

(a) $a^2b \times a^{-2}b^5$,

Answer ...... b^6 .... [1]

(b) $\left(\frac{27}{x^{18}}\right)^{\frac{-1}{3}}$.

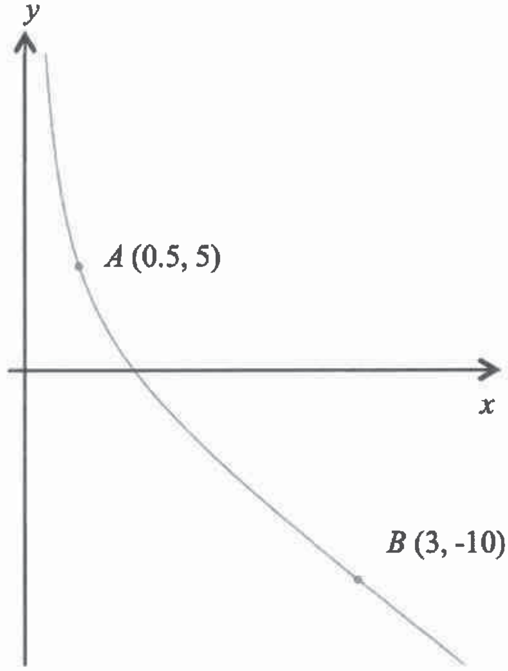
$$\left(\frac{27}{x^{18}}\right)^{\frac{-1}{3}} = \left(\frac{3}{x^6}\right)^{3\times\frac{-1}{3}} = \left(\frac{3}{x^6}\right)^{-1} \text{ or } \frac{x^6}{3}$$

Answer ...... $\left(\frac{3}{x^6}\right)^{-1}$ or $\frac{x^6}{3}$ .... [2]

7 The sketch below shows the graph of $y = \frac{p}{x} + qx + 1$.

The points $A(0.5, 5)$ and $B(3, -10)$ lie on the graph.

Find the values of p and q.



$5 = \frac{p}{0.5} + 0.5q + 1$

$5 = 2p + 0.5q + 1$

$10 = 4p + q + 2$

$4p + q = 8$ ------ (1)

$-10 = \frac{p}{3} + 3q + 1$

$-30 = p + 9q + 3$

$p + 9q = -33$

$p = -33 - 9q$ ----- (2)

Sub (2) into (1)

$4(-33 - 9q) + q = 8$

$-132 - 36q + q = 8$

$-132 - 35q = 8$

$35q = -140$

$q = -4$

Sub $q = -4$ into (2)

$p = -33 - 9(-4)$

$p = 3$

Answer p = ...

q = ... [3]

8 $\xi = \{\text{integers } x : 1 \le x \le 20\}$
$P = \{\text{prime numbers}\}$
$Q = \{\text{multiples of } 3\}$

(a) List the element(s) in $P \cap Q$.

$P = \{2,3,5,7,11,13,17,19\}$
$Q = \{3,6,9,12,15,18\}$

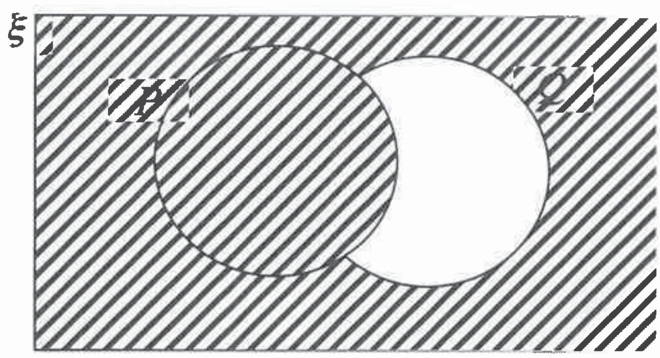
$P \cap Q = \{3\}$

Answer 3 or {3} .... [1]

(b) Circle the correct statement from the list below.

$P \subset Q$ $\quad$ $Q \cap P' \neq \phi.$ $\quad$ $(Q \cup P)' = \{1\}$ [1]

(c) On the Venn diagram, shade the region that represents $P \cup Q'$.



9 Jen wants to buy a vacuum cleaner costing $480.
Covey Norm departmental store has a payment plan for customers to pay a deposit of $100 and then 24 monthly payments, each of 4% of the original cost of the vacuum cleaner.
How much more than the original cost will Jen pay if she uses the payment plan?

Total cost from offer

$$\$100+\$480\times\frac{4}{100}\times 24=\$560.80$$

Difference

$$\$560.80-\$480=\$80.80$$

Answer $............80.80............ [3]

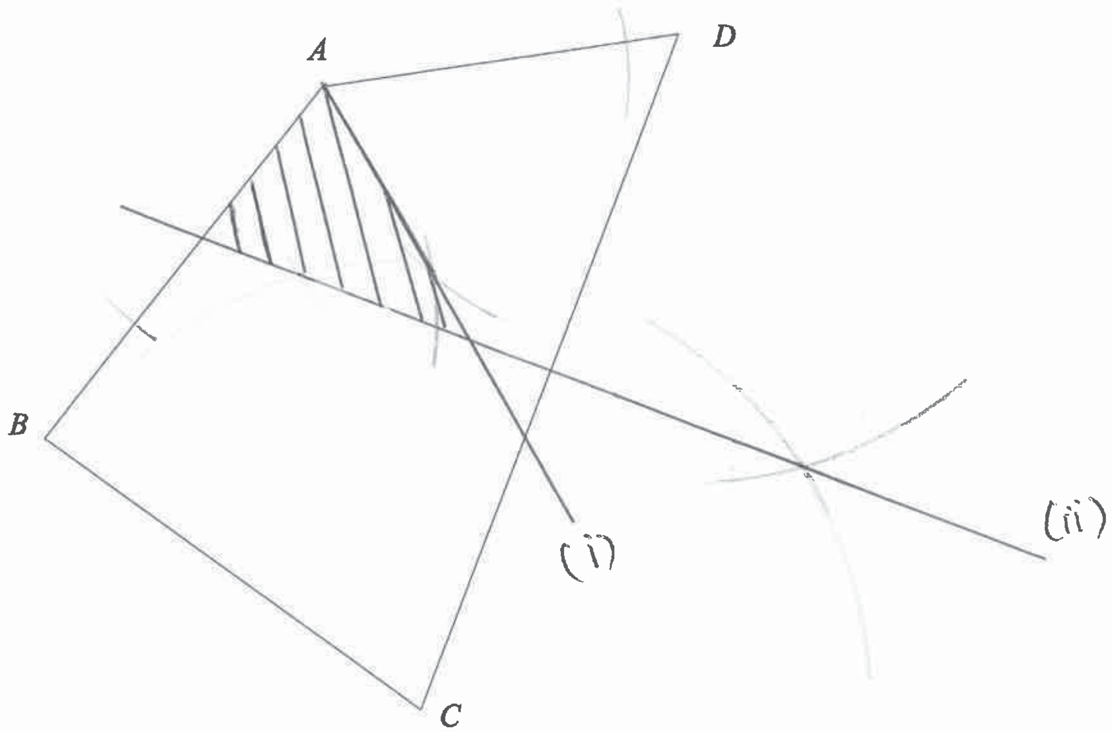
10 The scale drawing in the answer space below shows a piece of land *ABCD*.

The owner of the land decides to use part of the land to build a nursery based on the following requirements:

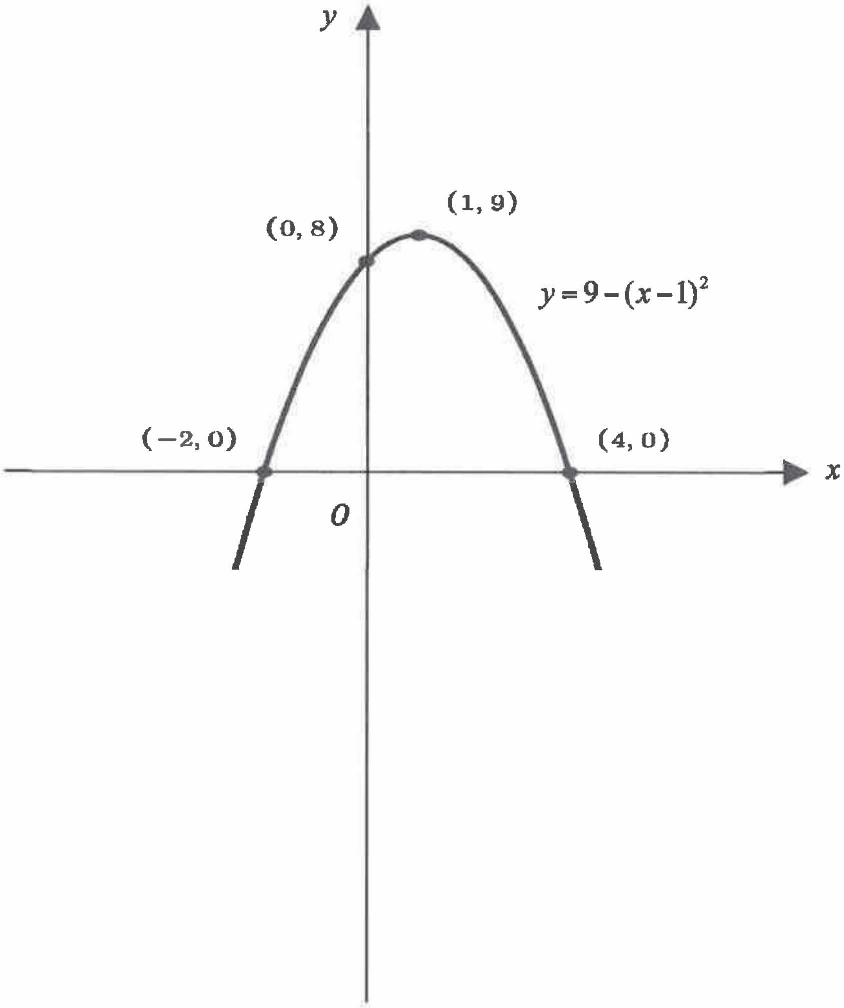
(i) The nursery is closer to the line *AB* than *AD*
(ii) The nursery is closer to the point *D* than *C*

By constructing an angle bisector and a perpendicular bisector, shade the area of the land where the nursery will be built.

Answer [3]



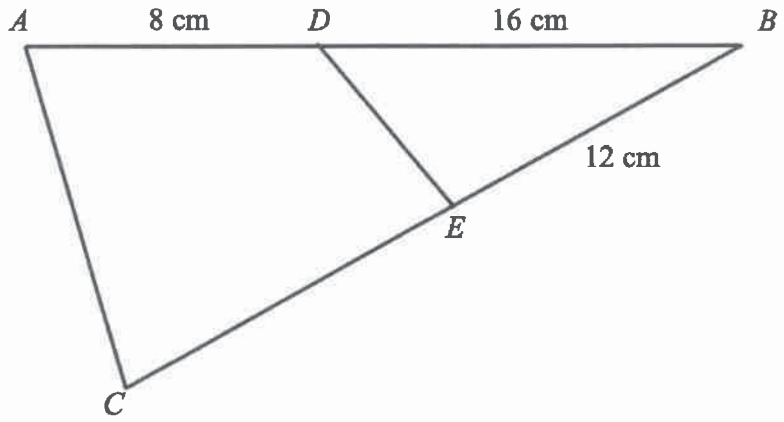
11 **Sketch** the graph of $y = 9 - (x-1)^2$ on the axes below.
Indicate clearly the coordinates of the points where the graph crosses the axes and the maimum point on the curve.
Answer



[3]

12 In the diagram, D and E are points on AB and BC respectively such that angle BDE = angle BCA.

$BD = 16$ cm, $DA = 8$ cm and $BE = 12$ cm.



(a) Show that triangle BDE and triangle BCA are similar.

Answer (*a*) In triangle BDE and triangle BCA

Angle DBE = Angle CBA (is a common angle)
Angle BDE = Angle BCA (given)
Angle BED = Angle BAC (angles sum of triangle)
Therefore, Triangle BDE is similar to triangle BCA (AA test)

[2]

(b) Hence, find EC.

$$\frac{BC}{BD} = \frac{BA}{BE}$$

$$\frac{12 + EC}{16} = \frac{24}{12}$$

$$EC = 20$$

Answer EC = ……….20…….cm [2]

[Turn over

13 **(a)** **(i)** Express 84 as the product of its prime factors.

Answer ……$2^2 \times 3 \times 7$.. [1]

(ii) The number $\frac{84h}{g}$ is a perfect square.

h and g are prime numbers such that $h < g$.
Find the value of h and the value of g.

Answer $h =$ ………3………………
$g =$ ………7……………… [1]

(b) Find two numbers, except 360 and 24, that have a lowest common multiple of 360 and a highest common factor of 24.

$360 = 2^3 \times 3^2 \times 5$

$24 = 2^3 \times 3$

$2^3 \times 3^2 \times 5^0 = 72$

$2^3 \times 3 \times 5 = 120$

Answer ………72………………,
……120………………... [2]

14 Factorise completely

(a) $2ax - x - a + 2x^2$.

Method 1

$2ax - x - a + 2x^2$

$= 2ax - a - x + 2x^2$

$= a(2x-1) + x(-1+2x)$

$= (2x-1)(a+x)$

Method 2

$2ax - x - a + 2x^2$

$= 2ax + 2x^2 - x - a$

$= 2x(a+x) - (x+a)$

$= (2x-1)(a+x)$

Answer $(2x-1)(a+x)$ .... [2]

(b) $4y^3 - 16yx^2$.

$4y^3 - 16yx^2$

$= 4y(y^2 - 4x^2)$

$= 4y(y-2x)(y+2x)$

Answer ... $4y(y-2x)(y+2x)$ .... [2]

15 A polygon has n sides. Two of its exterior angles are $21°$ and $54°$.
The remaining exterior angles are $15°$ each.
Find

(a) the value of n,

Sum of exterior angles = $360°$

$21° + 54° + (n-2)15 = 360°$

$n = 21$

Answer $n =$ ....21.............. [2]

(b) the sum of interior angles of the polygon.

sum of interior angles

$= (21-2)180°$

$= 3420°$

Answer ....3420°.................. [2]

16 **(a)** 4 skilled workers can complete a job in 5 days. 5 semi-skilled workers can complete the same job in 6 days. How long does it take 1 skilled worker and 1 semi-skilled worker to complete the same job if they work together?

Skilled	Semi-skilled	no. of days	Rate	
4	-	5	1/20	inverse relation
	5	6	1/30	inverse relation
1	1	$\left(\frac{1}{20}+\frac{1}{30}\right)^{-1}=12$		

Answer ....12...........days [2]

(b) It is given that y is directly proportional to the square of x and $y = p$ for a particular value of x. Express y in terms of p when this value of x is halved.

$y = kx^2$

$p = kx^2$ -

$y = k\left(\frac{x}{2}\right)^2$

$y = \frac{kx^2}{4}$

$= \frac{1}{4}p$

Answer $y =$ ..... $\frac{1}{4}p$ .... [2]

[Turn over

17 The ratio of the base areas of two geometrically similar cylinders is 9:25.

(a) If the surface area of the smaller cylinder is 480 cm^2, what is the surface area of the larger cylinder?

A_s surface area of smaller cylinder
A_L surface area of larger cylinder

$$\frac{480}{x} = \frac{9}{25}$$

$$x = 1333\frac{1}{3}$$

$$= 1330 \text{ cm}^2 \text{ (3 s.f)}$$

Answer ....1330................ cm^2 [2]

(b) Find the ratio of the heights of the two cylinders.

h_s height of smaller cylinder
h_L height of larger cylinder

$$\frac{h_s}{h_L} = \sqrt{\frac{A_s}{A_L}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

or

$$\frac{h_L}{h_s} = \sqrt{\frac{A_L}{A_s}} = \sqrt{\frac{25}{9}} = \frac{5}{3}$$

Answer 3 : 5 or 5 : 3 .... [1]

(c) Both cylinders are filled with silica. The mass of the silica in the larger cylinder is 36 kg. Find the mass of silica in the smaller cylinder.

$$\frac{m_S}{m_L} = \left(\frac{3}{5}\right)^3$$

$$m_s = \left(\frac{3}{5}\right)^3 \times 36$$

$$= 7.776 \text{ kg}$$

Answer ....7.776.....................kg [2]

[Turn over

18 A is the point (3 , 2) and B is the point (9 , -1).

(a) Find the length of the line AB.

Length AB = $\sqrt{(9-3)^2+(-1-2)^2}$

$= \sqrt{45}$

$= 6.7082 = 6.71$ units (3 s.f)

Answer ....6.71............units [2]

(b) Find the equation of the straight line that is parallel to AB, and passes through point $C(-4, 0)$.

$Gradient_{AB} = \frac{-1-2}{9-3}$

$= \frac{-3}{6} = -\frac{1}{2}$

$y = -\frac{1}{2}x + c$

Sub C (-4 , 0)

$0 = -\frac{1}{2}(-4) + c$

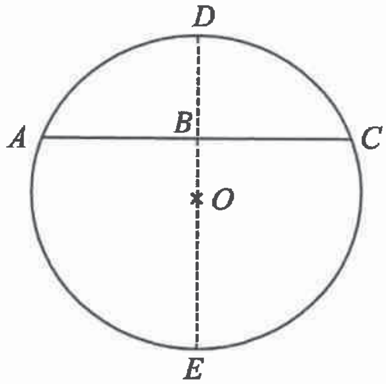
$c = -2$

$y = -\frac{1}{2}x - 2$

Answer ... $y = -\frac{1}{2}x - 2$ [3]

....

19 The diagram shows a circle, centre O, radius 16 cm.
B is the mid-point of the chord AC, DE is a diameter and $BE = 25$ cm.



(a) Calculate angle AOC in radian.

$$\cos B\hat{O}C = \frac{9}{16}$$

$$B\hat{O}C = \cos^{-1}\frac{9}{16}$$

$$A\hat{O}C = 2\cos^{-1}\frac{9}{16}$$
$$= 1.94678$$
$$= 1.95 \text{ (3 s.f)}$$

Answer ....1.95............... [2]

(b) Hence, find the area of the segment $ABCE$.

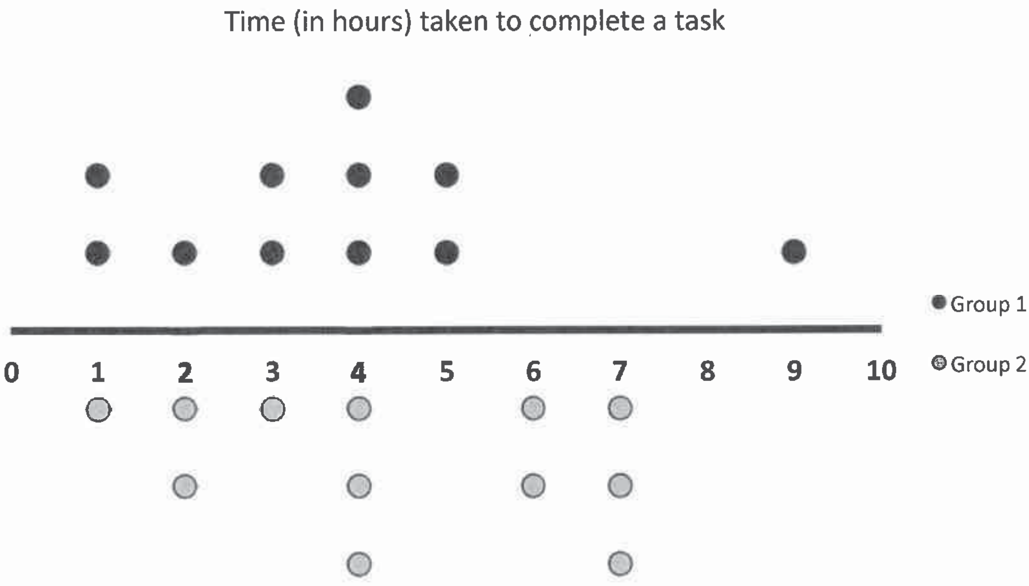
Reflex angle $AOC = 2\pi - 2\cos^{-1}\frac{9}{16}$
$= 4.336405$

Area of segment $ABCE$
$$= \frac{1}{2}\left(16^2\right)\left(4.336405 - \sin 4.336405\right)$$

$$= 674.1186$$
$$= 674 \text{ cm}^2 \text{ (3 s.f)}$$

Answer ....674............ cm^2 [3]

20 The time taken to complete a task by two groups of students, Group 1 and Group 2, were recorded.
The results are shown in the dot diagram.



(a) **Write down the median time** taken by Group 1.

Answer ....4........... hours [1]

(b) **Write down the range of the** time taken by Group 2.

Answer ....6............ hours [1]

(c) Alice claims that Group 2 took a shorter time to complete the task as the range of the time taken by Group 2 is shorter than that of Group 1. Do you agree? Justify your answer.

I ....disagree... **because Group 1 has an extreme value which will affect** the range of Group 1. **Hence, it is not fair/ accurate to claim that Group 2 took a shorter** time based on the **range**.... [2]

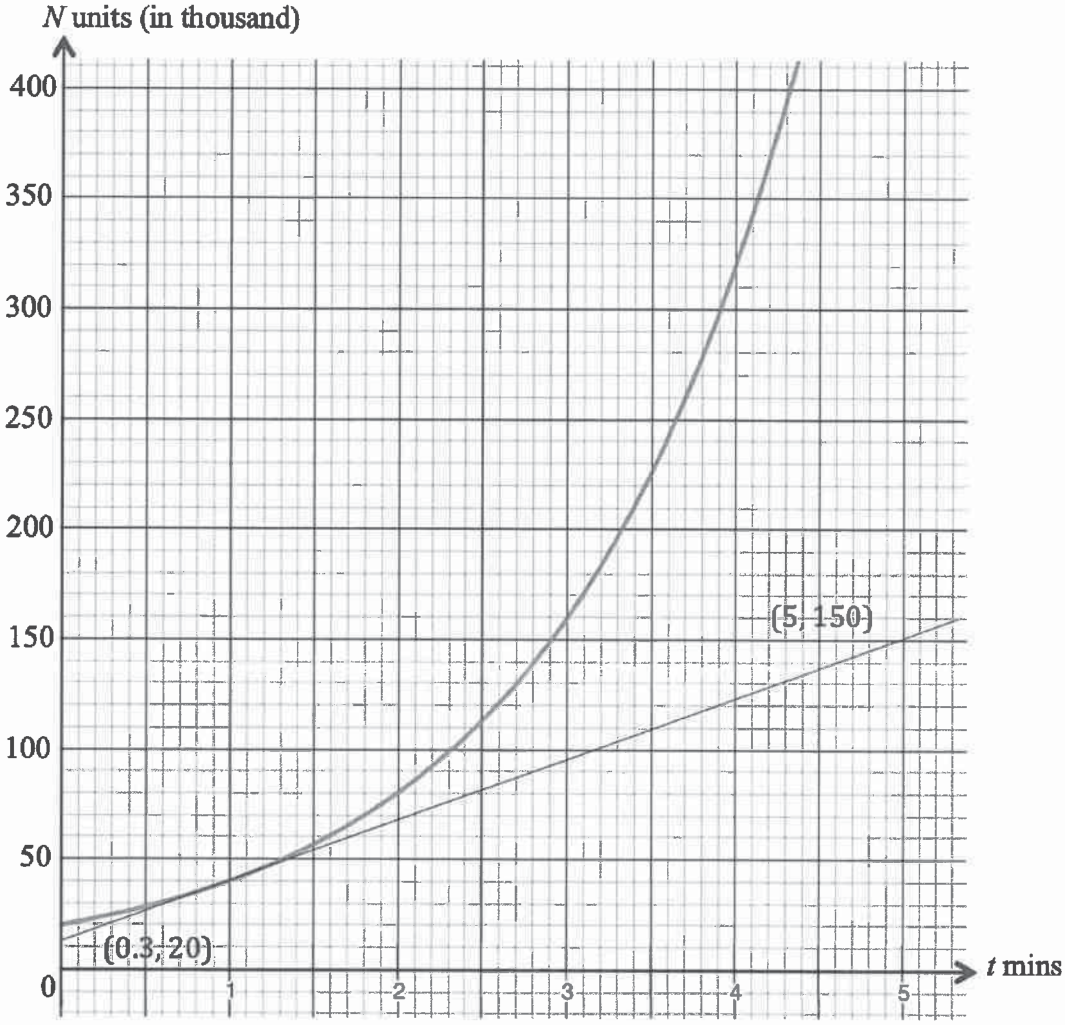
(d) Which group's timing was more consistent? Justify your answer.

Answer (d) Group ....1..'s timing was more consistent **because** the interquartile range of Group 1, 3 hours, is less than that of Group 2, 4 hours.

Alternative solution:
Group 2's timing was more consistent because the standard deviation of Group 2 is 2.06 hours (3 s.f) which is less than that of Group 1's, 2.14 hours. [1]

21 The number of bacteria, N units (in thousand), in a food item after t minutes, are connected by the equation $N = 40(2^{t-1})$.

The graph $N = 40(2^{t-1})$ is drawn on the grid below.



(a) Use the graph to estimate the range of values of t such that $80 < N < 320$.

Answer ....$2 < t < 4$.............. [1]

(b) **(i)** By drawing a tangent, find the gradient of the curve at (1, 40).

$$\text{Gradient} = \frac{150-20}{5-0.3}$$
$$= 27.7 \ (\pm 0.5 \text{ i.e } 27.2 \le gradient \le 28.2)$$

Answer ....27.7.................... [2]

[Turn over

(ii) Hence, state what the tangent at (1, 40) represents.

Answer The tangent represents the rate of change/ growth/ increase of the number of bacteria in the food after 1 mins or after/ at $t = 1$min... [1]

(iii) The food item is not safe to be consumed when the number of bacteria is at least 800% from its original amount.

Using the graph, determine after how long the food item is no longer safe to be consumed.

At $t = 0,\ N = 20$ (can be read from graph)

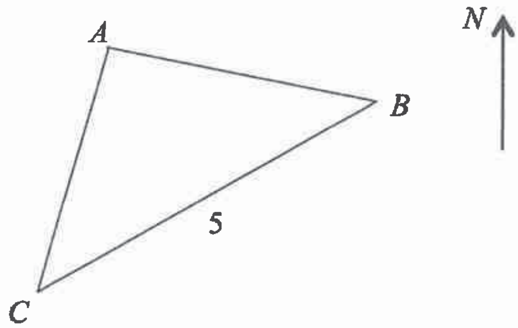
$$800\% \times 20 = \frac{800}{100} \times 20 = 160$$

time = 3 min

Solving of *t*-values using algebraic method is not acceptable.

Answer ....3...........min [2]

22 In the diagram, ABC represents a horizontal triangular garden.
$CB = 5$ m.
Angle $ACB = 64^\circ$ and the bearing of A from B is 280°.



(a) Calculate the bearing of B from A,

Bearing $= 280^\circ - 180^\circ = 100^\circ$
OR
Bearing $= 180^\circ - \left(360^\circ - 280^\circ\right) = 100^\circ$

Answer ......100......................... ° [1]

(b) A vertical tree, BT, has its base at B.
The angle of depression of the point C when viewed from the top of the tree is 24°.

(i) Find the height of the tree.

Height of cliff $= 5\tan 24^\circ$
$= 2.22614$
$= 2.23$ m

Answer ....2.23......................m [2]

(ii) David measured the largest angle of elevation of the top of the tree as seen from the path AC.
Calculate this angle of elevation.

$$\sin 64^\circ = \frac{shortest\ distance}{5}$$

shortest distance $= 5\sin 64^\circ$
$= 4.49397$

Let the largest angle of elevation be θ.

$$\tan\theta = \frac{5\tan 24^\circ}{5\sin 64^\circ}$$

$\theta = 26.35208$
$= 26.4^\circ$ (1 $d.p$)

Answer ....26.4° .... [3]

End of Paper

[Turn over

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

Solutions

MATHEMATICS 4048/2

PAPER 2
SECONDARY 4
2020 PRELIMINARY EXAMINATION

2 September 2020 (Wednesday) **2 hours 30 minutes**

CANDIDATE NAME	

CLASS		INDEX NUMBER		

READ THESE INSTRUCTIONS FIRST
Do not turn over the page until you are told to do so.
Write your name, class and index number in the spaces above.
Write in dark blue or black pen in the space provided for each question.

You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

INFORMATION FOR CANDIDATES
Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

For Examiner's Use		
Q1	10	
Q2	4	
Q3	5	
Q4	6	
Q5	9	
Q6	8	
Q7	10	
Q8	8	
Q9	9	
Q10	10	
Q11	11	
Q12	10	
Total		/100

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all questions.

1 **(a)** Solve the equation $\frac{a^2}{7} = \frac{a}{3}$. [3]

$\frac{a^2}{7} = \frac{a}{3}$

$3a^2 = 7a$

$3a^2 - 7a = 0$

$a(3a - 7) = 0$

$a = 0 \quad or \quad a = 2\frac{1}{3}$

(b) Solve the inequalities $-2 < \frac{7x+3}{2} \le 3 - x$. [3]

$-2 < \frac{7x+3}{2} \le 3 - x$

$-2 < \frac{7x+3}{2} \quad and \quad \frac{7x+3}{2} \le 3 - x$

$-4 < 7x + 3 \quad and \quad 7x + 3 \le 2(3 - x)$

$-4 < 7x + 3 \quad and \quad 7x + 3 \le 6 - 2x$

$-7x < 7 \quad and \quad 9x + 3 \le 6$

$-x < 1 \quad and \quad 9x \le 3$

$x > -1 \quad and \quad x \le \frac{1}{3}$

$Ans: \quad -1 < x \le \frac{1}{3}$

(c) Express as a single fraction in its simplest form

$$\frac{6}{18d^2-30d+8}+\frac{3}{16-9d^2}.$$ [4]

$$\frac{6}{18d^2-30d+8}+\frac{3}{16-9d^2}$$

$$=\frac{6}{2(3d-1)(3d-4)}+\frac{3}{(4-3d)(4+3d)}$$

$$=\frac{3}{(3d-1)(3d-4)}+\frac{3}{(4-3d)(4+3d)}$$

$$=\frac{3}{(3d-1)(3d-4)}-\frac{3}{(3d-4)(4+3d)}$$

$$=\frac{3(4+3d)-3(3d-1)}{(3d-1)(3d-4)(4+3d)}$$

$$=\frac{12+9d-9d+3}{(3d-1)(3d-4)(4+3d)}$$

$$=\frac{15}{(3d-1)(3d-4)(4+3d)}$$

or

$$=\frac{-15}{(4-3d)(3d-1)(4+3d)}$$

Alternative solution

$$\frac{6}{18d^2-30d+8}+\frac{3}{16-9d^2}$$

$$=\frac{6(16-9d^2)+3(18d^2-30d+8)}{(16-9d^2)(18d^2-30d+8)}$$

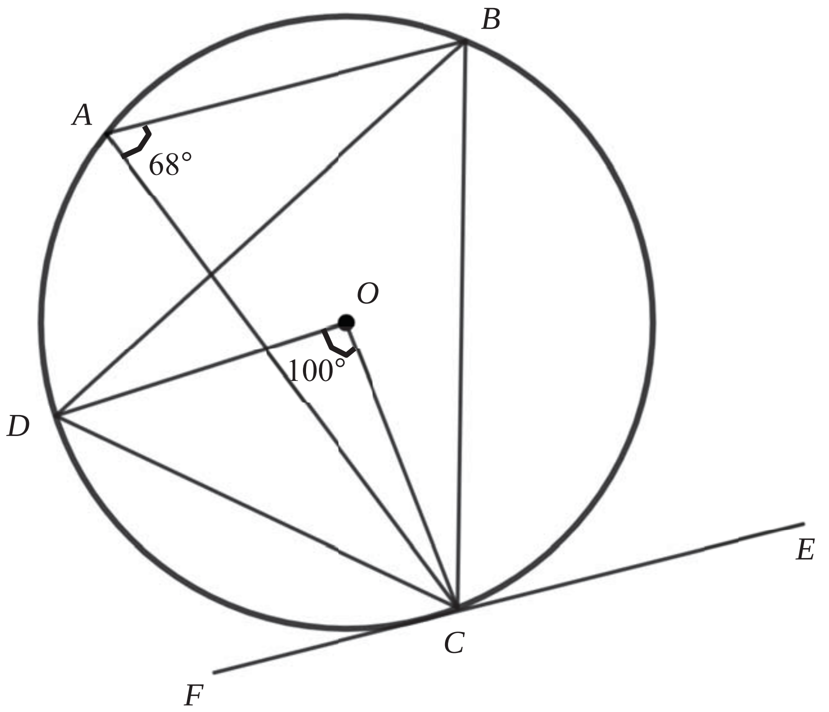
$$=\frac{6(4-3d)(4+3d)+6(3d-1)(3d-4)}{2(4-3d)(4+3d)(3d-1)(3d-4)}$$

$$=\frac{6(4-3d)(4+3d)-6(3d-1)(4-3d)}{2(4-3d)(4+3d)(3d-1)(3d-4)}$$

$$=\frac{6(4-3d)\left[(4+3d)-(3d-1)\right]}{2(4-3d)(4+3d)(3d-1)(3d-4)}$$

$$=\frac{15}{(3d-1)(3d-4)(4+3d)}$$ [*A*1]

2 In the diagram below, A, B, C and D are points on the circumference of a circle of centre O. FCE is a tangent to the circle at C.
Angle $BAC = 68°$ and angle $DOC = 100°$



Find, giving reasons for each answer,
(i) angle DBC,

Angle DBC
$= \frac{1}{2}(\text{angle } DOC)$
$= 50°$ [Angle at the centre is twice the angle at the circumference]

(ii) angle DCF

angle DCF = angle DBC = 50° (angles in alternate segment are equal)

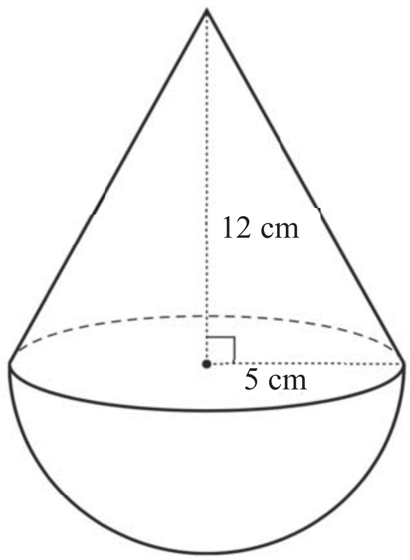
(iii) angle BDO,

Angle ODC
$= \frac{180° - 100°}{2}$
$= 40°$

Angle BDC = Angle BAC = 68° (angles in the same segment are equal

Angle BDO
= Angle DBC – Angle PDC
= 68° - 40°
= 28°

3 The diagram shows a solid made up of a cone and a hemisphere.
The hemisphere has a radius of 5 cm.
The cone has a base radius of 5 cm and a height of 12 cm.



(a) Calculate the surface area of the solid. [3]

Slant height

$= \sqrt{12^2 + 5^2}$

$= 13\,cm$

Surface area of solid

$= \left[\pi(5)(13)\right] + \left[\frac{1}{2} \times 4\pi(5)^2\right]$

$= 65\pi + 50\pi$

$= 115\pi\ cm^2$

$= 361\,cm^2\ (3sf)$

(b) Calculate the volume of the solid. [2]

Volume of the solid

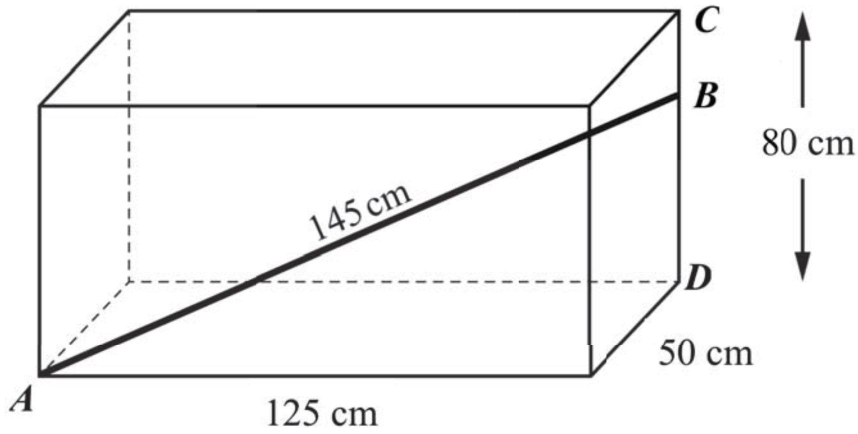
$$=\left[\frac{1}{3}\pi(5)^2(12)\right]+\left[\left(\frac{1}{2}\right)\left(\frac{4}{3}\right)\pi\left(5\right)^3\right]$$

$$=100\pi+83\frac{1}{3}\pi$$

$$=183\frac{1}{3}\pi \quad or \quad \frac{550}{3}\pi\ cm^3$$

$$=576cm^3(3sf)$$

4



A piece of steel rod *AB* is placed in a box as shown above. *AB* is 145 cm.
One end of the steel rod, *A*, is in the bottom corner of the box and the other end of the steel rod, *B*, is below the top corner of the box.

It is given that the box has a height of 80 cm, length of 125 cm and width of 50 cm.

(a) Find the length of *BD*. [3]

By Pythagoras' theorem,

$(AD)^2 = (DX)^2 + (AX)^2$

$(AD)^2 = 50^2 + 125^2$

$AD = \sqrt{2500 + 15625}$

$AD = \sqrt{18125}\ cm$

$(BD)^2 = (AB)^2 - (AD)^2$

$(BD)^2 = 145^2 - \left(\sqrt{18125}\right)^2$

$BD = \sqrt{21025 - 18125}$

$BD = 53.8516$

$BD = 53.9\ cm\ (3sf)$

(b) Find the angle *ABC*. [3]

$$\cos\angle ABD = \frac{BD}{AB}$$
$$\cos\angle ABD = \frac{\sqrt{2900}}{145}$$
$$\angle ABD = \cos^{-1}\left(\frac{\sqrt{2900}}{145}\right)$$
$$\angle ABD = 68.199^\circ\ (3dp) \quad [M1]$$

$$or\ \sin\angle ABD = \frac{\sqrt{18125}}{145}$$
$$\angle ABD = 68.199^\circ\ (3dp)$$

$$or\ \tan\angle ABD = \frac{\sqrt{18125}}{\sqrt{2900}}$$
$$\angle ABD = 68.199^\circ\ (3dp)$$

$$\angle ABC = 180^\circ - \angle ABD$$
$$\angle ABC = 180^\circ - 68.199^\circ$$
$$\angle ABC = 111.8^\circ\ (1dp)$$

<u>Alternative solution</u>

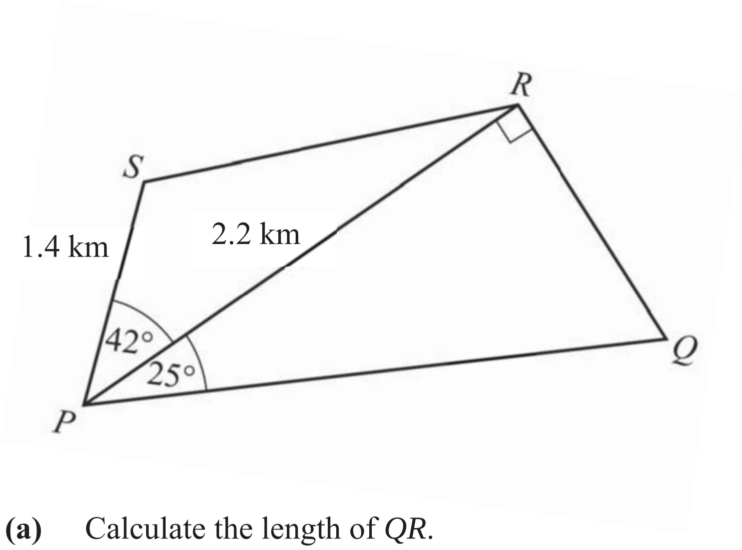
By Pythagoras' theorem,

$$(AC)^2 = (CD)^2 + (AD)^2$$
$$(AC)^2 = 80^2 + \left(\sqrt{18125}\right)^2$$
$$AC = \sqrt{24525}\ cm$$

Using cosine rule,

$$(AC)^2 = (AB)^2 + (BC)^2 - 2(AB)(BC)\cos\angle ABC$$
$$\left(\sqrt{24525}\right)^2 = (145)^2 + \left(80-\sqrt{2900}\right)^2 - 2(145)\left(80-\sqrt{2900}\right)\cos\angle ABC$$
$$\cos\angle ABC = \frac{(145)^2 + \left(80-\sqrt{2900}\right)^2 - \left(\sqrt{24525}\right)^2}{2(145)\left(80-\sqrt{2900}\right)}$$
$$\cos\angle ABC = -0.37139$$
$$\angle ABC = 111.8^\circ\ (1dp)$$

5 The diagram shows a plot of land that is in the shape of a quadrilateral $PQRS$ where $PR = 2.2$ km and $PS = 1.4$ km. Angle $PRQ = 90°$, angle $RPQ = 25°$ and angle SPR = 42°.



(a) Calculate the length of QR. [2]

$$\tan 25° = \frac{QR}{2.2}$$
$$QR = 2.2 \tan 25°$$
$$QR = 1.03\ km\ (3s.f.)$$

*Alternative method: Using Sine Rule

(b) Calculate the area of the plot of land. [3]

Area of the plot of land

$$= \left[\frac{1}{2}(2.2)(2.2 \tan 25°)\right] + \left[\frac{1}{2}(1.4)(2.2 \sin 42°)\right]$$

$[M1, M1 - subtitution\ of\ values\ into\ formula\ for\ area]$

$$= 1.12846 + 1.03046$$
$$= 2.16\ km^2 \quad [A1]$$

(c) Determine whether it is possible to build a circular fence such that points P, Q, R and S lie on the circumference of a circle. **[4]**
Justify your decision and show your calculations clearly.

Using cosine rule,

$$(SR)^2 = (PS)^2 + (PR)^2 - 2(PS)(PR)\cos\angle SPR$$

$$(SR)^2 = (1.4)^2 + (2.2)^2 - 2(1.4)(2.2)\cos 42^\circ$$

$$(SR)^2 = 2.2222\ (5sf)$$

$$SR = 1.4907\ km\ (5sf)$$

Using sine rule,

$$\frac{\sin\angle PSR}{PR} = \frac{\sin\angle SPR}{SR}$$

$$\frac{\sin\angle PSR}{2.2} = \frac{\sin 42^\circ}{1.4907}$$

$$\sin\angle PSR = \frac{\sin 42^\circ}{1.4907} \times 2.2$$

$$\angle PSR = 80.936^\circ\ (3dp)$$

$$\angle RQP = 180^\circ - \angle PRQ - \angle RPQ$$

$$\angle RQP = 180^\circ - 90^\circ - 25^\circ$$

$$\angle RQP = 65^\circ$$

$$\angle PSR + \angle RQP = 80.936^\circ + 65^\circ$$

$$\angle PSR + \angle RQP = 145.936^\circ$$

Since $\angle PSR + \angle RQP \neq 180^\circ$, points P, Q, R and S do not lie on the circumference of the circle

(By the converse of the property that angles in opposite segments are supplementary or opposite angles in a cyclic quadrilateral add up to 180°).

6 Ben has just relocated back to Singapore from United Kingdom.

(a) Ben converted his savings of £50 000 to Singapore dollars when the exchange rate between pounds (£) and Singapore dollars (S$) was £1 = S$1.78. [1]
Calculate the amount of money in Singapore dollars Ben received after the conversion.

£1 = S$1.78
£50 000 = S1.78 x 50 000
£50 000 = S$89 000
Ben had S$89 000 after conversion.

(b) Ben is intending to invest this sum of money in a fixed deposit called the "*Multiply Plan"* for a period of one year. The interest earned from the "*Multiply Plan*" is compounded monthly based on the interest rates listed below. [3]

Multiply Plan	
Interest rate for first six-month period	2% per annum
Interest rate for next six-month period	2.5% per annum (based on total amount at the end of the first six months)

Calculate the total amount of money in Singapore dollars that Ben would have at the end of one year, correct to the nearest cent.

Amount of money Ben has after the first 6 months

$$=(89000)\left(1+\frac{\left(\frac{2}{12}\right)}{100}\right)^6$$

$$=(89000)\left(1+\frac{1}{600}\right)^6$$

$$=\$89\,893.7166\ (4dp)$$

Amount of money Ben has after the next 6 months

$$=(89\,893.7166)\left(1+\frac{\left(\frac{2.5}{12}\right)}{100}\right)^6$$

$$=(89\,893.7166)\left(1+\frac{1}{480}\right)^6$$

$$=\$91\,023.2568\ (4dp)$$

$$=\$91\,023.26\ (2dp)$$

(c) Prior to returning to Singapore, Ben worked out a projected monthly expenditure for living in Singapore.

Rental of a HDB flat	S$2200
Food	S$400
Transport	S$100
Leisure	S$500

However, he realised that his actual monthly expenditure was different from his projected amount. He has to pay 25% more for rental of a flat, 40% more for food and 30% more for transport than what he had projected.

In order to reduce his monthly expenditure, he decided to cut back his monthly expenses for leisure by 50%. [2]

(i) Calculate Ben's actual monthly expenditure.

Ben's actual monthly expenditure

$$=\left[\frac{125}{100}\times 2200\right]+\left[\frac{140}{100}\times 400\right]+\left[\frac{130}{100}\times 100\right]+\left[\frac{50}{100}\times 500\right]$$

$$=2750+560+130+250$$

$$=S\$3\,690$$

(ii) Calculate the percentage increase in Ben's actual expenditure compared to his projected monthly expenses, correcting your answer to 3 significant figures. [2]

Total projected expenditure
= S$2200 + S$400 + S$100 + S$500
=S$3200

Percentage increase in his actual expenditure compared to his projected monthly expenses

$$=\frac{3690-3200}{3200}\times 100$$

$$=15.3\% \quad (3s.f.) \quad [A1]$$

7 A local restaurant has 3 outlets in Bishan, Clementi and Orchard Road.

Each outlet offers 4 different set meals at the following prices.

Set A	Set B	Set C	Set D
$10	$12	$14	$20

The table below shows the number of set meals sold at the different outlets for the month of January.

Set / Outlet	Bishan	Clementi	Orchard Road
Set A	250	100	300
Set B	100	80	150
Set C	50	30	120
Set D	80	20	100

(a) Represent the information given in two matrices, a 1 × 4 matrix, **P,** and a 4 × 3 matrix, **Q**. [2]

$$P = \begin{bmatrix} 10 & 12 & 14 & 20 \end{bmatrix}$$

$$Q = \begin{bmatrix} 250 & 100 & 300 \\ 100 & 80 & 150 \\ 50 & 30 & 120 \\ 80 & 20 & 100 \end{bmatrix}$$

(b) Evaluate **PQ.** [2]

$$PQ = \begin{bmatrix} 10 & 12 & 14 & 20 \end{bmatrix} \begin{bmatrix} 250 & 100 & 300 \\ 100 & 80 & 150 \\ 50 & 30 & 120 \\ 80 & 20 & 100 \end{bmatrix}$$

$$PQ = \begin{bmatrix} 2500+1200+700+1600 & 1000+960+420+400 & 3000+1800+1680+2000 \end{bmatrix}$$

$$PQ = \begin{bmatrix} 6000 & 2780 & 8480 \end{bmatrix} \quad [A1]$$

(c) Explain what each element in **PQ** represents. [1]

6000 represents the total amount of money (in dollars) collected from the sales of set meals by the Bishan outlet of the restaurant in the month of January.

2780 represents the total amount of money (in dollars) collected from the sales of set meals by the Clementi outlet of the restaurant in the month of January.

8480 represents the total amount of money (in dollars) collected from the sales of set meals by the Orchard Road outlet of the restaurant in the month of January.

In view of the Circuit Breaker which took place in the month of April, the restaurant started a delivery service in place of dine-in and offered the different set meals at the same price.

In the month of April, there was a 20% increase for each type of set meal sold at the outlet in Bishan and a decrease of 40% for each type of set meal sold at the outlet in Orchard Road. The number for each type of set meal sold at the Clementi outlet remained the same in the month of April.

The matrix **R**, a 3 × 1 matrix, is such **PQR** gives the total amount of money collected by the three outlets in the month of April.

(d) Write down matrix *R*. [1]

$$R = \begin{bmatrix} 1.2 \\ 1 \\ 0.6 \end{bmatrix}$$

(e) Evaluate *PQR*. [2]

$$PQR = \begin{bmatrix} 6000 & 2780 & 8480 \end{bmatrix} \begin{bmatrix} 1.2 \\ 1 \\ 0.6 \end{bmatrix}$$

$$PQR = \begin{bmatrix} 7200 + 2780 + 5088 \end{bmatrix}$$

$$PQR = [15068]$$

(f) Calculate the difference in the total amount of money collected by the restaurant in April compared to January.

[2]

Was there an increase or decrease in the total amount of money collected by the restaurant in April compared to January?

Total amount collected in January
= \$6000 + \$2780 + \$8480
=\$17 260

Total amount collected in April
= \$15068

Difference
= \$17 260 - \$15068
= \$2 192

There was a **decrease** in the total amount collected by the restaurant in April.

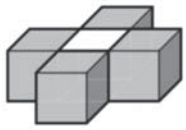
[Turn over

8 A tower is made of layers of grey and white cubes.

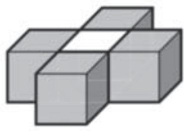
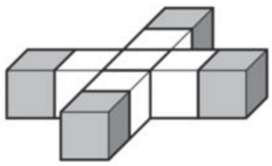
The first layer of the tower is made with one grey cube.

Layer 1	

A tower with two layers is made with five grey cubes and one white cube as shown below.

Layer 1	
Layer 2	

A tower with three layers is made with nine grey cubes and six white cubes as shown below.

Layer 1	
Layer 2	
Layer 3	

(a) Complete the table below. [3]

Number of layers	**1**	**2**	**3**	**4**	**5**		**n**
Total number of white cubes	0	1	6	15	28		$(n-1)(2n-3)$
Total number of grey cubes	1	5	9	13	17		$4n - 3$
Total number of cubes	1	6	15	28	45		$2n^2 - n$

(b) Find, in terms of *n*, the total number of grey cubes in a tower with *n* layers. [1]

For n = 1, no. of grey cubes = 1
For n = 2, no. of grey cubes = 5 = 1 + 4
For n = 3, no. of grey cubes = 9 = 1+ 4+ 4
For n = 4, no. of grey cubes = 13 = 1+ 4 + 4 + 4
For n = 5, no. of grey cubes = 17 = 1 + 4 + 4 + 4+ 4+ 4

Total no of grey cubes in a tower with n layers
= 1 + (n – 1)(4)
= 1 + 4n – 4
or
= 4n – 3

(c) Find, in terms of *n*, the total number of cubes in a tower with *n* layers. [1]

For n = 1, total no. cubes = 1 = 1 x 1
For n = 2, total no. cubes = 6 = 2 x 3
For n = 3, total no. cubes = 15 = 3 x 5
For n = 4, total no. cubes = 28 = 4 x 7
For n = 5, total no. cubes = 45 = 5 x 9

Total no of grey cubes in a tower with n layers
= n x (2n – 1) [B1]
or
= $2n^2$- n [B1]

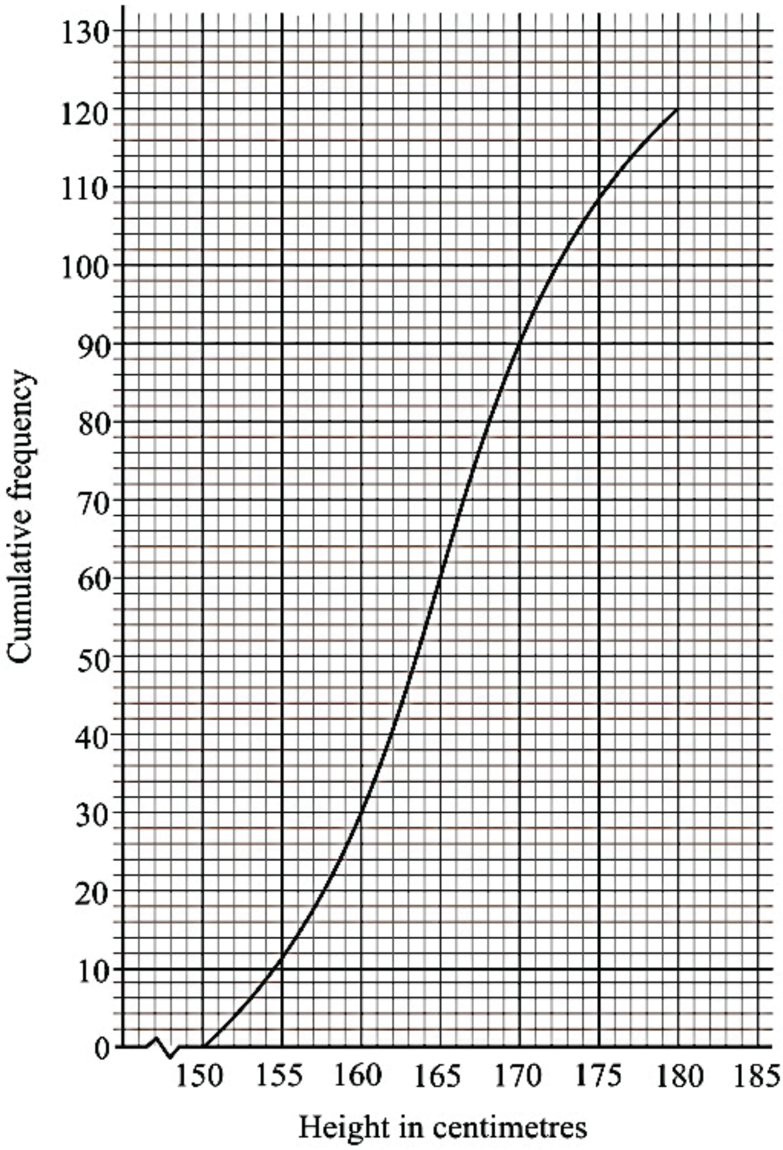
(d) Sarah used 97 grey cubes to build a tower. [3]
Calculate the number of white cubes she used.

4n – 3 = 97
4n = 100
n = 25

Total no of cubes to build a tower with 25 layers
= 25 (50 - 1)
= 25(49)
=1225

No. of white cubes used
= 1225 – 97
= 1128

9 The graph below shows the heights of 120 Secondary Four students in a school.



(a) Using the graph,
(i) write down the median, [1]

Median = 165 cm

(ii) find the interquartile range [1]

Interquartile range
= 170 – 160
= 10 cm

(b) Given that 40% of the students are taller than *x* cm, find the value of *x*. [1]

60% of students
= 0.6 x 120
= 72
x = 167

(c) The minimum height requirement to join the school's basketball team is 175 cm. If two students are randomly selected, calculate the probability that they meet the height requirement for the school's basketball team. [2]

Probability

$= \frac{11}{120} \times \frac{10}{119}$

$= \frac{11}{1428}$

OR

$= \frac{12}{120} \times \frac{11}{119}$

$= \frac{11}{1190}$

(d) (i) Complete the frequency table below. [2]

Height (cm)	Frequency	Frequency (Alternative Solution)	Frequency (Alternative Solution)	Frequency (Alternative Solution)
$150 < x \leq 155$	11	11	12	12
$155 < x \leq 160$	19	19	18	18
$160 < x \leq 165$	30	30	30	30
$165 < x \leq 170$	30	30	30	30
$170 < x \leq 175$	19	18	19	18
$175 < x \leq 180$	11	12	11	12
(e) Mean	165 cm (3 sf)	165 cm (3 sf)	165 cm (3 sf)	165 cm (3 sf)
(f) SD	7.10 cm (3 sf)	7.16 cm (3sf)	7.04 cm (3sf)	7.10 cm (3sf)

(ii) Estimate the mean height of the students. [1]

Mean height = 165 cm (3 s.f.)

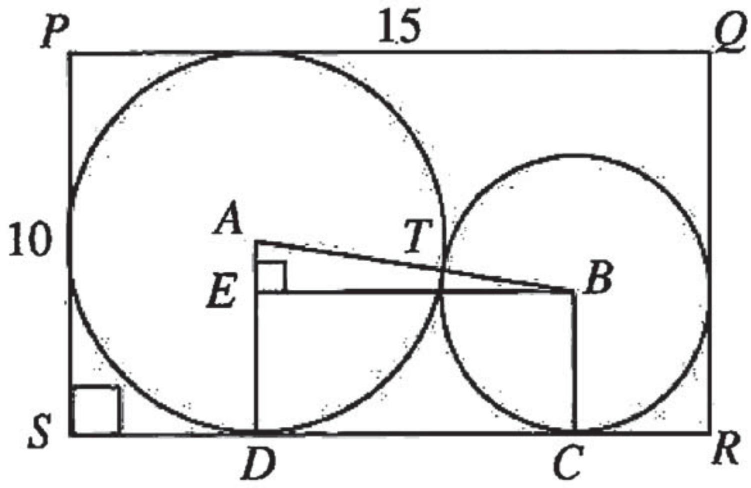
(iii) Estimate the standard deviation of the height of students. [1]

Standard deviation = 7.04 cm or 7.10 cm

[Turn over

10 The diagram shows a rectangle *PQRS*, where *PQ* = 15 cm and *PS* = 10 cm.

The large circle has a centre *A* and touches three sides of the rectangle.
The small circle has a centre *B* and touches two sides of the rectangle.
The small circle touches the large circle at point *T*.
The large and small circles touch the side *SR* at *D* and *C* respectively.
The point *E* is the foot of the perpendicular from *B* to *AD*.



It is given that the radius of the large circle is 5 cm and the radius of the small circle is *x* cm.

(a) Write down, in terms of *x*, an expression for the lengths of [3]

(i) AB = 5 + x

(ii)
AE = AD – ED
AE = 5 – x

(iii)
EB = DC
EB = SR – SD – CR
EB = 15 – 5 – x
EB = 10 – x
DC = 10 – x

(b) Form an equation in x and show that it simplifies to $x^2 - 40x + 100 = 0$. [3]

$$(AB)^2 = (AE)^2 + (EB)^2$$
$$(5+x)^2 = (5-x)^2 + (10-x)^2$$
$$25 + x^2 + 10x = 25 + x^2 - 10x + 100 + x^2 - 20x$$
$$25 + x^2 - 10x + 100 + x^2 - 20x - 25 - x^2 - 10x = 0$$
$$x^2 - 40x + 100 = 0 \ (shown)$$

(c) Solve $x^2 - 40x + 100 = 0$, giving your answer correct to 2 decimal places. [3]

$$x^2 - 40x + 100 = 0$$
$$x = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(1)(100)}}{2(1)} \quad [M1]$$
$$x = 37.3205 \quad or \quad x = 2.67949$$
$$x = 37.32\ (2dp) \quad or \quad x = 2.68\ (2dp) \quad [A1, A1]$$

(d) Hence, find the radius of the small circle, correct to two decimal places. [1]

Radius of the small circle = 2.68 cm (Reject $x = 37.32$ as as $x < 5$)

11 The variables x and y are connected by the equation $y = 2x + \frac{4}{x^2} - 5$.

Some corresponding values of x and y, correct to 2 decimal places, are given in the table below.

x	0.75	1	1.5	2	3	4	5	6
y	3.61	1	p	0	1.44	3.25	5.16	7.11

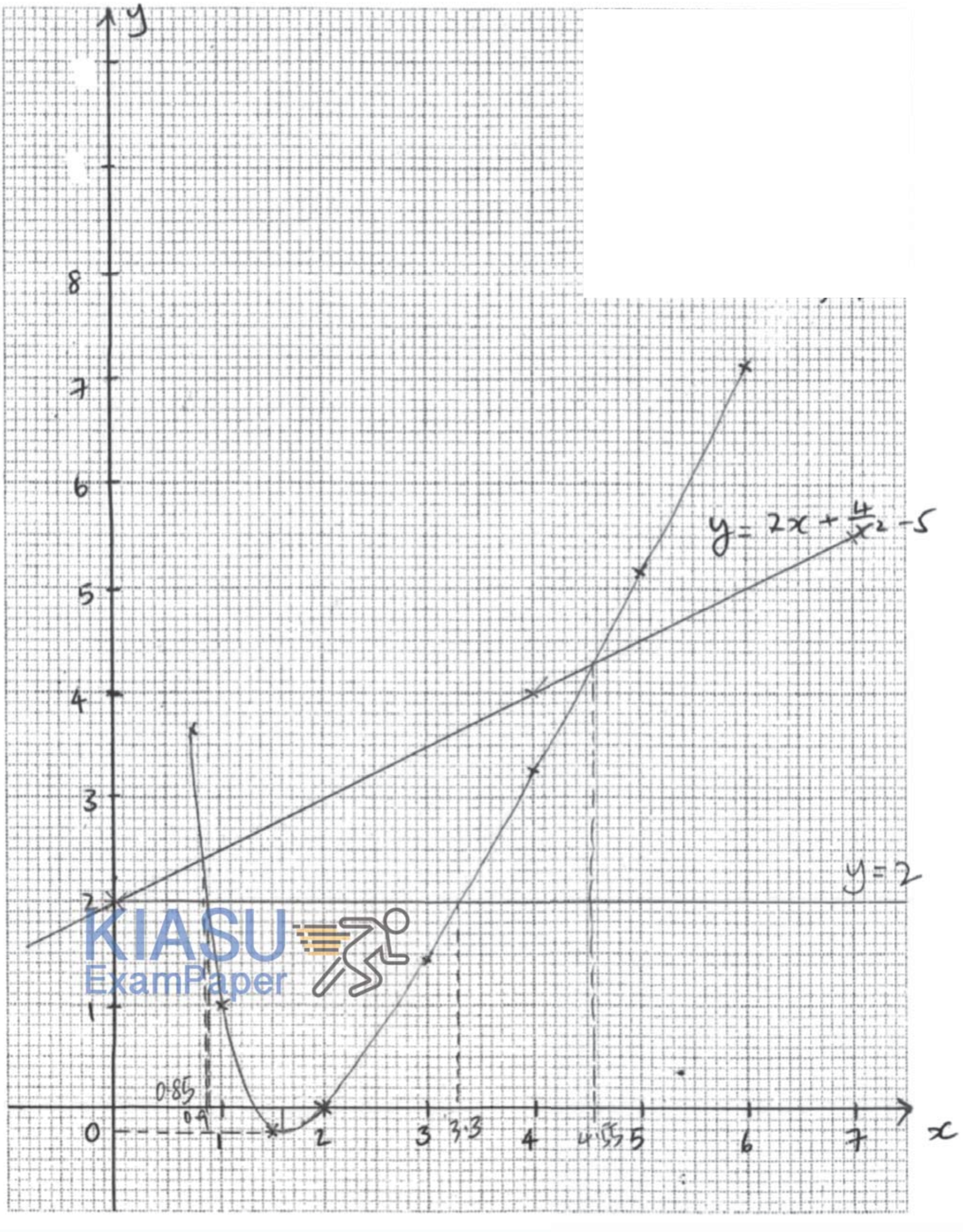
(a) Find the value of p, giving your answer to 2 decimal places. [1]

$sub\ x = 1.5,$

$p = 2(1.5) + \frac{4}{(1.5)^2} - 5$

$p = -0.22\ (2dp)$

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \le x \le 7$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $-1 \le y \le 8$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]



(c) Use your graph to find

(i) the least value of y, [1]

Least value of y = - 0.24 (± 0.1)

(ii) the range of values of x for which y is less than 2, [2]

Graph of y = 2 drawn

$0.9\ (\pm 0.1) < x < 3.3\ (\pm 0.1)$

(iii) the solutions of the equation $3x^3 = 14x^2 - 8$ by drawing a suitable straight line. [4]

$$3x^3 = 14x^2 - 8$$

$$\frac{3x^3}{x^2} = \frac{14x^2 - 8}{x^2}$$

$$3x = 14 - \frac{8}{x^2}$$

$$\frac{3x}{2} = 7 - \frac{4}{x^2}$$

$$\frac{3x}{2} + \frac{4}{x^2} = 7$$

$$\frac{3x}{2} + \frac{4}{x^2} - 5 = 7 - 5$$

$$\frac{3x}{2} + \frac{4}{x^2} - 5 = 2$$

$$\frac{x}{2} + \frac{3x}{2} + \frac{4}{x^2} - 5 = 2 + \frac{x}{2}$$

$$2x + \frac{4}{x^2} - 5 = \frac{1}{2}x + 2$$

Line of $y = \frac{1}{2}x + 2$ drawn

x	0	4	7
y	2	4	5.5

x = 0.85 (± 0.1) or x = 4.55 (± 0.1)

12 The most popular ride at the Outer Space amusement park is the Ferris wheel.
The diagram below shows a model of the Ferris wheel at the park.

Each day, the Ferris wheel rides run from 10:30 a.m. to 8:30 p.m. with an hour break for maintenance work at 3 p.m.

The Ferris wheel has 16 passenger cabins. There are seats for 3 passengers in each cabin. Due to recent rules on social distancing, it is mandatory to have an empty seat in between two passengers in each cabin.

Before each ride, passengers take about 5 minutes to be seated and undergo safety checks. At the end of each ride, passengers take about 2 minutes to disembark.

The table below shows the ticket prices for one ride on the Ferris wheel for weekdays and weekends. Tickets must be used on the same day of purchase.

Weekday Ticket	$18
Weekend Ticket	$23

(a) Given that the Ferris wheel makes 1 revolution every two minutes and makes 4 revolutions per ride, calculate the total number of rides each cabin on the Ferris wheel makes in one day. [2]

Time taken for 1 ride
= 4 revolutions x 2 minutes
= 8 minutes

No. of rides per day

$= \dfrac{60}{8+2+5} \times 9 \quad [M1]$

$= 36$

(b) Calculate the maximum number of passengers that the Ferris wheel can take in one day. [1]

Maximum number of passengers that the Ferris wheel can take a day
= 2 passengers × 16 cabins × 36 rides
= 1152

(c) If all the seats are taken up for every ride, calculate the total amount of money that the park can collect from the sale of tickets for Ferris wheel rides on a weekday. [1]

Total amount of money the park can collect from the sale of tickets for rides on the Ferris wheel on a weekday.
$= 1152 \times \$18$
$= \$20\ 736$

(d) To commemorate its tenth birthday, the amusement park is planning to give out free tickets for its Ferris wheel rides. [6]

Propose a sensible number of tickets to be given free such that the amount collected from the sale of tickets sales for the day covers its operating cost of $10 000. It is also estimated that 40% to 60% of the total possible seats will be taken for each ride.

Explain your proposal clearly and state any assumptions made.

Marking Scheme

| Stating reasons for decision to give out weekend or weekday tickets.
Eg. Identifying that maximum possible free tickets would be on a **weekend** due to greater possible free tickets due to higher ticket prices for paid tickets

Stating assumptions related to range of occupancy between 40% to 60%.
Eg. 50% occupancy rate as it is the mean between 40% and 60%.
Eg. 60% occupancy rate for weekend is a high possibility

Showing calculation for number of paid tickets required to cover operating cost of $10 000. Value should be rounded up.

Showing calculation for number of free tickets based on assumption made on occupancy rate (subtraction made for the right values)

[M1] Overall clear explanation and conclusion

Weekday Tickets		
Max no of paid tickets to cover operating cost $=\frac{\$10000}{18}$ $=555.6$ $=556$ (rounded up to nearest whole no.)		
Assumption: 40% occupancy	Assumption: 50% occupancy	Assumption: 60% occupancy
No. of free tickets $=\left(\frac{40}{100}\times 1152\right)-556$ $=460-556$ $=-96$ [Not possible] (460.8 is rounded up to nearest whole no.)	No. of free tickets $=\left(\frac{50}{100}\times 1152\right)-556$ $=576-556$ $=20$	No. of free tickets $=\left(\frac{60}{100}\times 1152\right)-556$ $=691.2-556$ $=691-556$ $=135$ (691.2 is rounded down to nearest whole no.)

Weekend Tickets		
Max no of paid tickets to cover operating cost $=\frac{\$10000}{23}$ $=434.8$ $=435$ (rounded up to nearest whole no.)		
Assumption: 40% occupancy	Assumption: 50% occupancy	Assumption: 60% occupancy
No. of free tickets $=\left(\frac{40}{100}\times 1152\right)-435$ $=460.8-435$ $=461-435$ $=26$ (460.8 is rounded up to nearest whole no.)	No. of free tickets $=\left(\frac{50}{100}\times 1152\right)-435$ $=576-435$ $=141$	No. of free tickets $=\left(\frac{60}{100}\times 1152\right)-435$ $=691.2-435$ $=691-435$ $=256$ (691.2 is rounded down to nearest whole no.)

End of Paper