Name		( )	Class	

南华中学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2023

Subject : Mathematics
Paper : 4052/01
Level : Secondary Four Express
Date : 17 August 2023
Duration : 2 hours 15 min

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions. The number of marks is given in brackets [ ] at the end of each question or part question.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Gives answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

U	
P	
A	

For Examiner's Use

This paper consists of 286 printed pages and 2 blank pages.

Answer **all** questions.

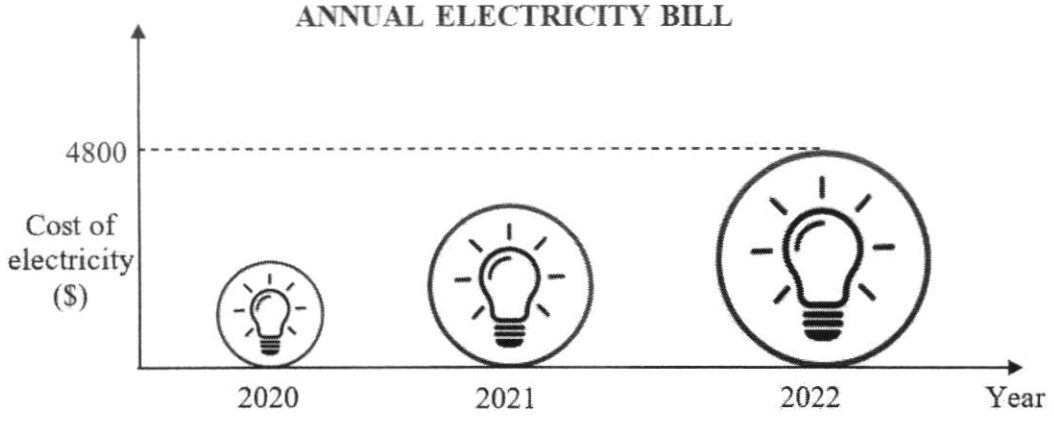
1 $3^x = 3^{11} + 3^{11} + 3^{11}$
Find the value of x.

Answer $x =$ ..... [1]

2 Charles invested with a bank that pays interest of 3.5% per annum compounded half-yearly.
He received \$3500 as interest at the end of 3 years.
Find the original amount that Charles invested.
Give your answer to the nearest dollar.

Answer \$ ..... [2]

3 Ken draws this graph to show his annual electricity cost for his 5-room flat for year 2020, 2021 and 2022.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

[2]

4 The marked price of a 65-inch Ultra HD 4K TV is \$6000.
Mr Lim buys the TV on hire purchase scheme with down payment of 20% on the marked price.
He then pays monthly instalments over 4 years at a simple interest rate of 1.37% per annum.
Calculate the total amount that Mr Lim pays for the television.

Answer \$ ..... [2]

5 The table shows the time, to the nearest hours, taken by 12 students to complete a project.

Time t (hours)	Frequency
$30 \le t < 35$	1
$35 \le t < 40$	2
$40 \le t < 45$	5
$45 \le t < 50$	4

(a) Calculate an estimate for the mean time taken.

Answer ..... hours [1]

(b) Calculate an estimate for the standard deviation of the time taken.

Answer ..... hours [1]

6 The average mass of haemoglobin in a red blood cell is about 31 picograms.

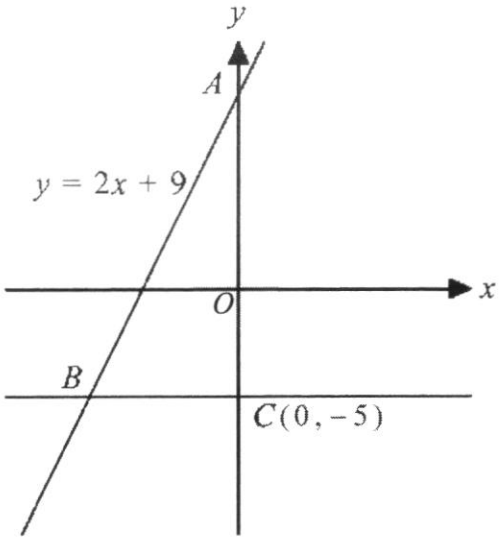
(a) Express 31 picograms in grams, giving your answer in standard form.
[1 picogram = 10^{-12} grams]

Answer ..... g [1]

(b) There are 5×10^{6} red blood cells in 1 cubic millimetre of blood.
Calculate the mass, in grams, of haemoglobin in 1 cubic millimetre of blood.
Give your answer in standard form.

Answer ..... g [1]

7 In the diagram, C is the point $(0,-5)$ and A is a point on the y-axis.
The line AB meets the horizontal line through C at point B.
The equation of line AB is $y=2x+9$.



(a) State the equation of the line parallel to line AB, passing through $(0,7)$.

Answer ..... [1]

(b) Find the coordinates of point A and point B.

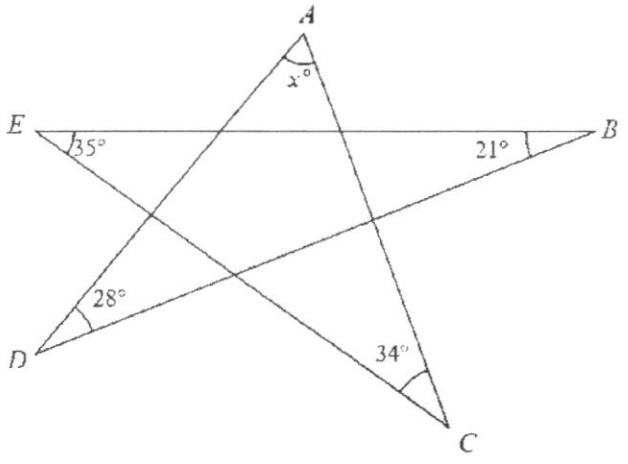
Answer A (..... , ...)

B (..... , ...) [2]

8 The ratio of the number of employees in company P to that in company Q was 3: 4.
8 employees left company Q to join company P.
The number of employees in company P now is 4 more than company Q.
Find the original number of employees in company P.

Answer ..... [3]

9



Straight lines *AC, AD, BE, BD, CE* intersect to form the figure in the diagram.
Find the value of x.

Answer $x =$ ..... [2]

10 **(a)** Factorise completely $3fg+1-g-3f$.

Answer ..... [2]

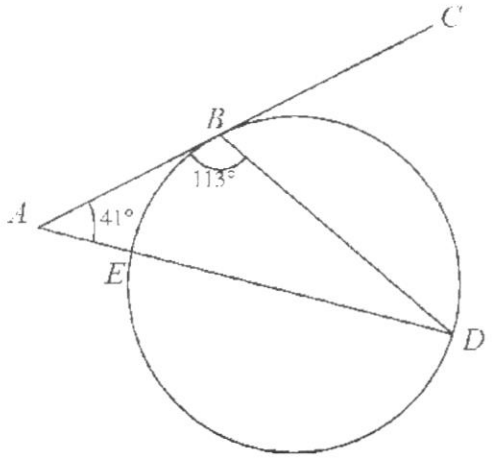
(b) Factorise completely $44-11k^2$.

Answer ..... [2]

11 Make x the subject of the formula $w=\dfrac{2x+y}{3x-y}$.

Answer ..... [3]

12



In the diagram, ABC is a tangent to the circle.
The line AD cuts the circle at E.
Angle $BAD = 41°$ and angle $ABD = 113°$.

Explain why ED is not a diameter of the circle.
Give reasons for each step of your working.

[3]

13 Written as the product of its prime factors, $8316 = 2^2 \times 3^3 \times 7 \times 11$.

(a) Express 840 as the product of its prime factors.

Answer 840 = ... [1]

(b) **(i)** Hence write down the LCM of 8316 and 840, giving your answers as the product of its prime factors.

Answer LCM = ... [1]

(ii) Hence write down the greatest integer that will divide 8316 and 840 exactly.

Answer ..... [1]

14 $\overrightarrow{PQ} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$.

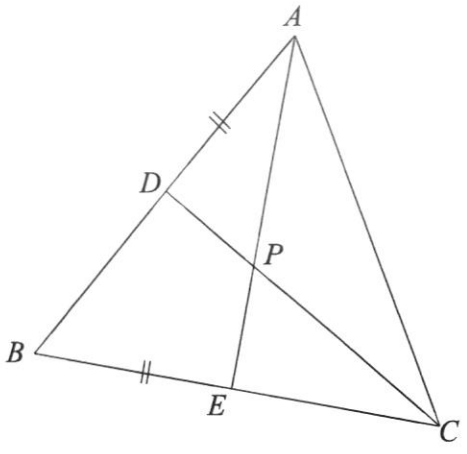
(a) Find $\left| \overrightarrow{PQ} \right|$.

Answer $\left| \overrightarrow{PQ} \right| =$..... [1]

(b) R is the point $(2, -9)$.
$\overrightarrow{SR} = 2\overrightarrow{PQ}$.
Find the coordinates of S.

Answer S (..... , ...) [2]

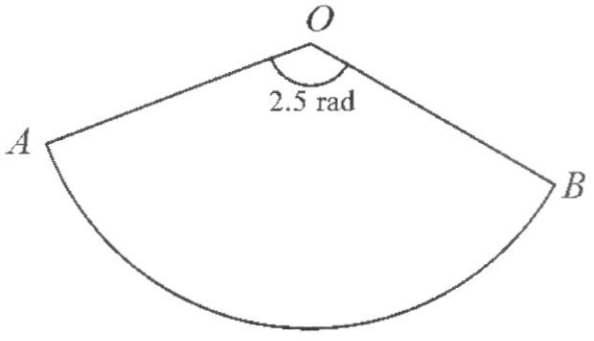
15 The diagram shows an equilateral triangle ABC.
D and E are points on AB and BC respectively such that $AD = BE$.
AE meets CD at P.



Show that triangle ABE is congruent to triangle CAD.
Give a reason for each statement you make.

[3]

16



The diagram shows the sector of a circle, AOB.
The perimeter of sector is 81 cm and angle AOB is 2.5 radian.

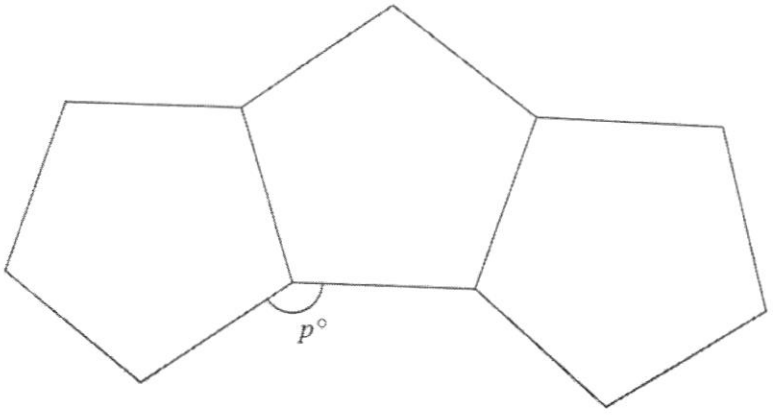
(a) Find the radius of the sector.

Answer ..... cm [2]

(b) The sides, OA and OB, are joined to form a cone.
Find the radius of the cone.

Answer ..... cm [2]

17



The diagram shows three regular pentagons joined together on its edges.

(a) Find the value of p.

Answer $p =$..... [2]

(b) Additional regular pentagons are added to these 3 pentagons to form a closed ring. Find the total number of pentagons that are used to form this ring.

Answer ..... [2]

18 In a sequence, the same number is subtracted each time to obtain the next term.
The first five terms of the sequence are

$$68 \quad a \quad b \quad c \quad 24.$$

(a) Find the values of a, b and c.

Answer $a =$.....

$b =$.....

$c =$..... [2]

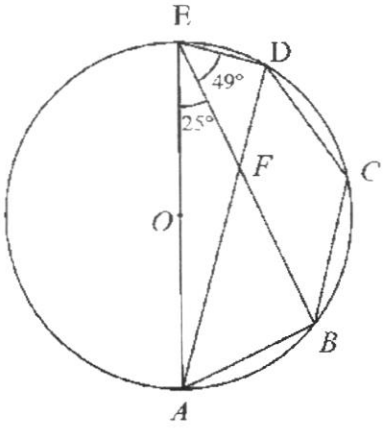
(b) Write down an expression for the nth term of this sequence.

Answer ..... [1]

(c) Explain why -524 is not a term of this sequence.

[1]

19



O is the centre of the circle passing through A, B, C, D and E.
Chords BE and AD cuts at F.
Angle $AEB = 25°$ and angle $BED = 49°$.

(a) Show that triangle ABF is similar to triangle EDF.
Give a reason for each statement you make.

[2]

(b) Find angle BCD.
Give a reason for each step of your answer.

Answer [2]

20 $\mathscr{E} = \{x \text{ is an integer: } 1 \leq x \leq 15\}$
$F = \{\text{prime numbers}\}$
$G = \{x : 5 < x \leq 12\}$

(a) List the elements in F.

Answer ..... [1]

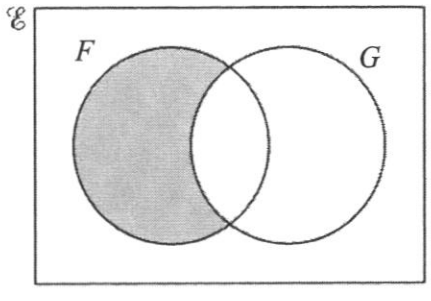
(b) State the value of n(G).

Answer ..... [1]

(c) List all the elements in $(F \cup G)'$.

Answer ..... [1]

(d) On the Venn diagram, shade the region which represents $F \cap G'$.



[1]

21 The ratio of surface area of 2 similar solid spheres is 4: 9.
The radius of the smaller sphere is 6 cm.

(a) Find the radius of the larger sphere.

Answer .....cm [1]

(b) Express the total the volume of the 2 solid spheres in terms of π.

Answer .....cm^3 [2]

(c) The spheres are melted and recast into 2 solid cones, A and B.
The radius and height of cone A is 6 cm and 10 cm respectively.
The radius of cone B is 12 cm.
Showing your working clearly, explain if the 2 cones are similar.

22 The diagram shows the location of 2 towns, A and B.

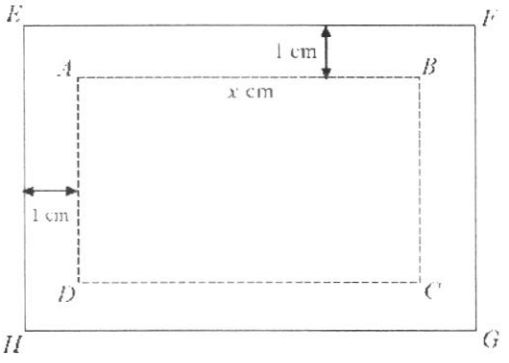
(a) Construct the perpendicular bisector of AB. [1]

(b) Another new town, C, is to be built on a bearing of 210° from town B. [2]
It is equidistant from town A and town B.
Mark the position of town C.

(c) Construct the angle bisector of angle BAC. [1]

(d) A telecommunication tower, T, serving the 3 towns is to be built so that it is [1]
equidistance from AB and AC, and equidistant from town A and B.
Mark the position of T.

23

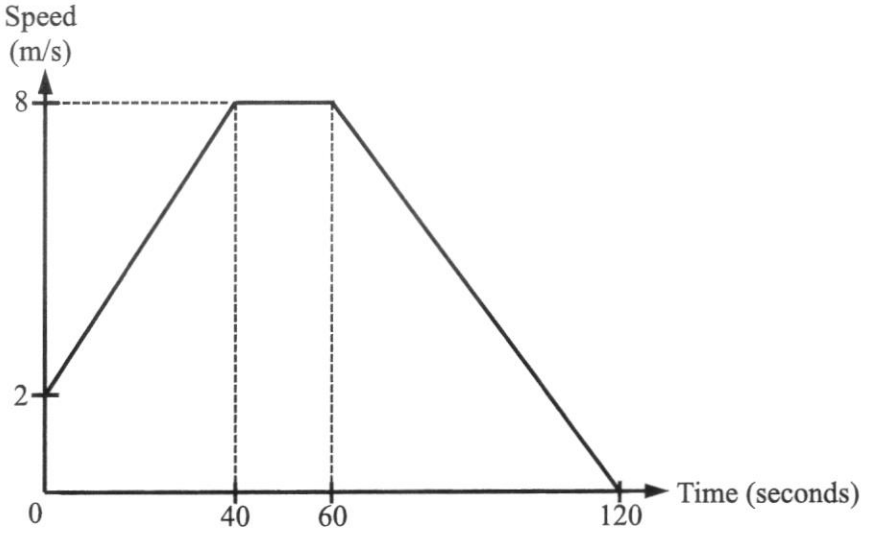


The diagram shows a photograph, *ABCD*, length x cm and perimeter 40 cm.
It is mounted on a piece of rectangular cardboard, *EFGH*.
There is a uniform border of 1cm width around the photograph.

(a) The area of the rectangular cardboard, *EFGH*, is S cm^2. [3]
Show that it can be expressed as $S = -x^2 + 20x + 44$.

(b) Jane claims that the area of the rectangular cardboard, S, can be greater than $150\,\text{cm}^2$.
Explain if the claim is correct. [2]

24 The diagram shows the speed-time graph of a cyclist after passing a fixed point A.



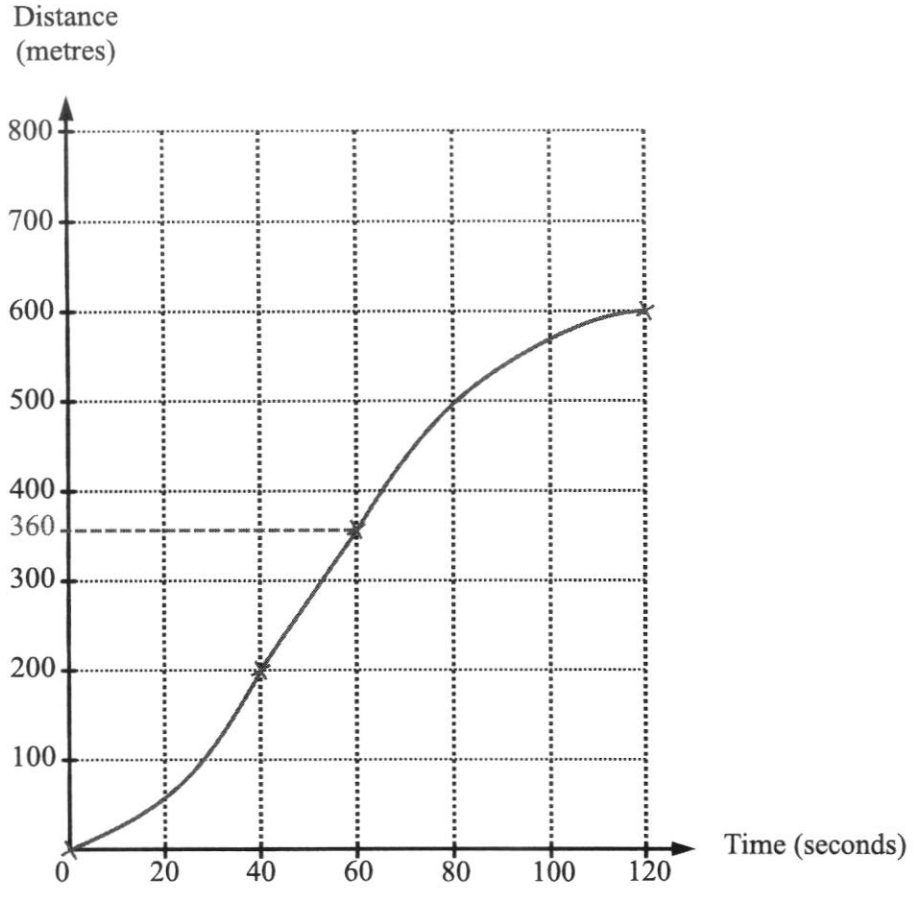
(a) Calculate the acceleration of the cyclist during the first 10 seconds after passing point A.

Answer Acceleration = ... m/s^2 [1]

(b) Calculate the average speed, in m/s, of the cyclist in the last 80 seconds.

Answer Average speed = ... m/s [2]

(c) On the grid below, sketch the distance-time graph for the journey.



25 **(a)** The table shows the choice of CCA for Secondary One students in a school.

	Sports	**Clubs and Society**	**Performing Arts**	**Total**
Males	70	26	72	168
Females	38	a	c	d
Total	108	b	141	316

(a) Find the values of a, b, c and d in the table.

Answer $a =$.....

$b =$ .....

$c =$ .....

$d =$ ..... [2]

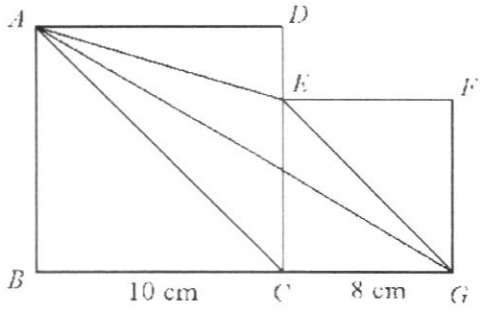
(b) A pie chart is to be drawn showing the data for **males**.
Calculate the angle representing the males who choose Sports.

Answer ..... [1]

(c) In which group, males or females, did a greater percentage choose Performing Arts.
Explain.

[2]

26



The diagram shows two squares, $ABCD$ and $CGFE$, of lengths 10 cm and 8 cm respectively.

(a) Find the length of AG.

Answer ..... cm [1]

(b) Show that AC is parallel to EG.

[2]

(c) Find the area of triangle AGE.

Answer ..... cm^2 [3]

---------- End of Paper ----------

Name		( )	Class	

南华中学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2023

Subject : **Mathematics**
Paper : **4052/02**
Level : **Secondary Four Express**
Date : **21 August 2023**
Duration : **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

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For π, use either your calculator value or 3.142.

For Examiner's Use

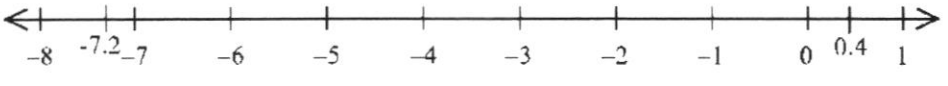
Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	U		TOTAL
									P		
									A		

This paper consists of **22** printed pages and **2** blank pages.

1 (a) (i) Solve the inequalities $\frac{4x-1}{2} \leq \frac{3x}{4} \leq \frac{5x+9}{5}$.

Answer ..... [2]

(ii) Represent the solutions of the inequalities on the number line below.



[1]

(iii) State the smallest integer which satisfies the inequalities.

Answer ..... [1]

(b) Simplify $\frac{1}{ab^{-2}} \div \left(\frac{2a}{\sqrt[3]{b^5}}\right)^{-3} \times \frac{ab^0}{10}$, leaving your answer in positive index form.

Answer ..... [3]

(c) Express $\frac{x+10}{3x^2-21x+30} + \frac{1}{2-x}$ as a single fraction in its simplest form.

Answer ..... [3]

(d) (i) y is inversely proportional to the square of x.

Which of these diagrams represents the graph of y against x?

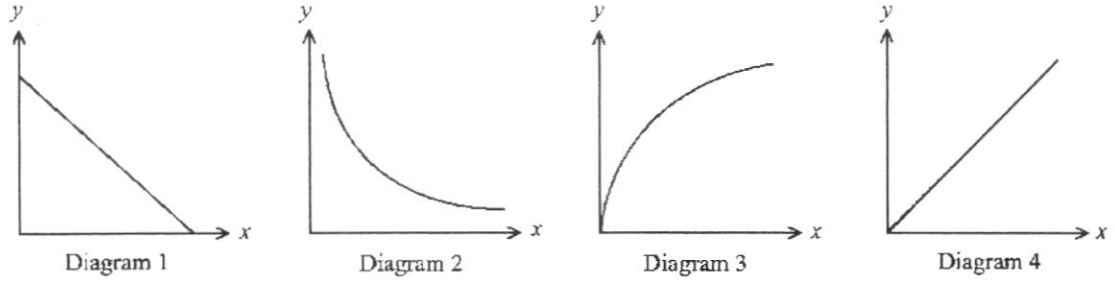


Diagram 1 Diagram 2 Diagram 3 Diagram 4

Answer Diagram ..... [1]

(ii) Find the percentage change in y when x is increased by 150%.

Answer .....% [3]

2 At a store, 20 red apples cost $\$x$.

The same number of green apples in the store cost \$4 more.

(a) Write an expression, in terms of x, for the number of red apples that can be bought with \$2.40.

Answer ..... [1]

(b) Write an expression, in terms of x, for the number of green apples that can be bought with \$2.40.

Answer ..... [1]

(c) \$4.80 can buy a total of 10 apples at the store.

Write down an equation in x to represent this information and show that it reduces to $5x^2 - 28x - 96 = 0$.

[3]

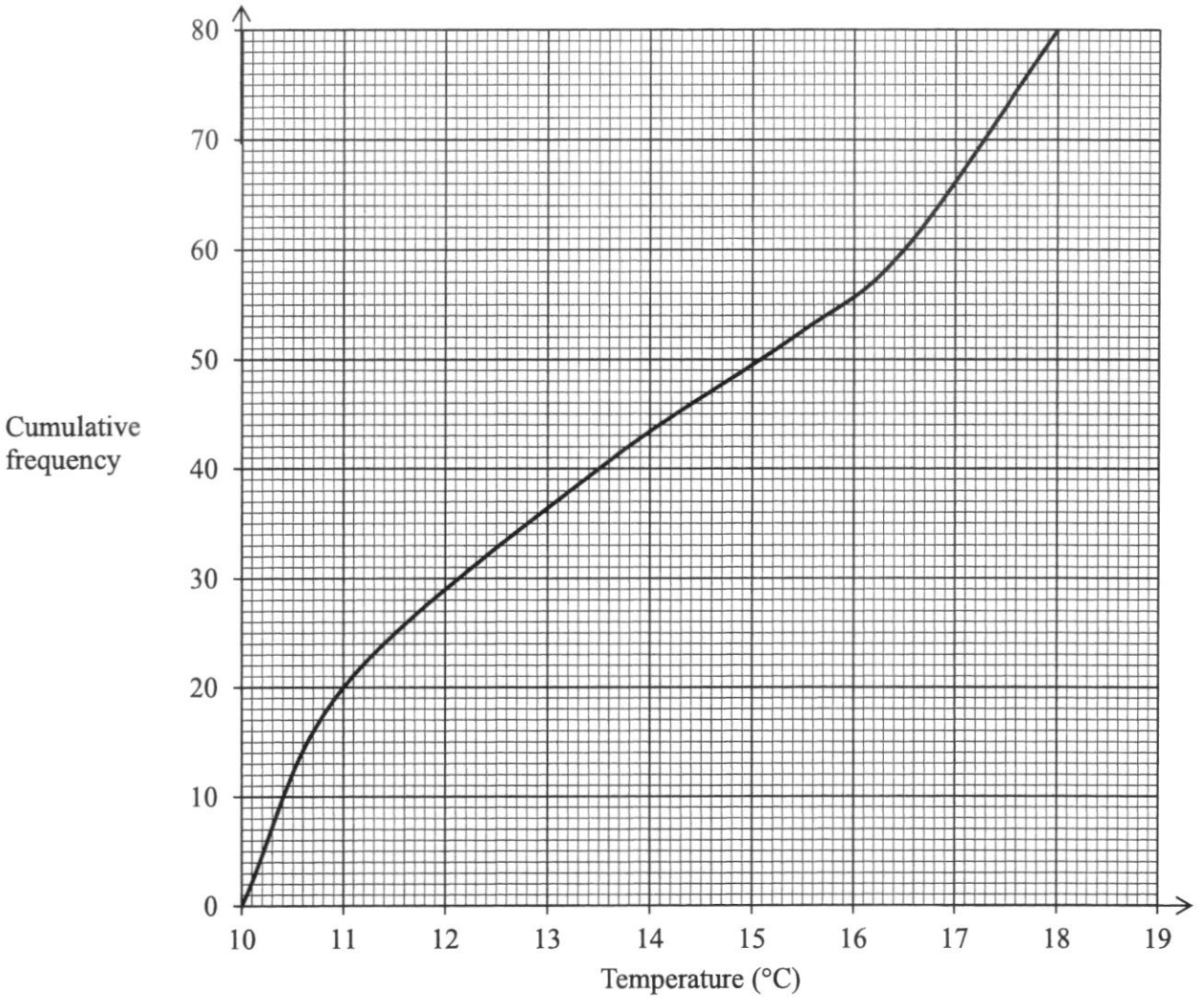
(d) Solve the equation $5x^2 - 28x - 96 = 0$.

Answer $x =$ ..... or ..... [2]

(e) Find the cost of 30 green apples in store A.

Answer $..... [2]

3 The cumulative frequency curve shows the distribution of the daily temperature of city X in the first 80 days of the year 2021.



(i) Use the curve to estimate

(a) the median temperature,

Answer ..... °C [1]

(b) the interquartile range of the temperatures,

Answer ..... °C [2]

(c) the 65th percentile,

Answer ..... °C [1]

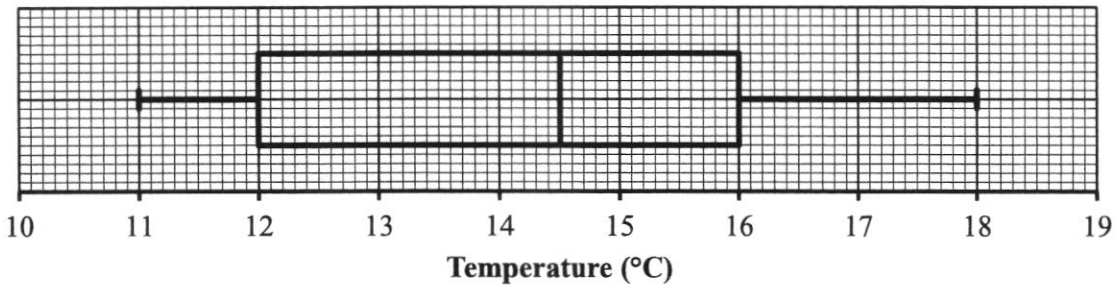
(d) the percentage of days where the temperature was at least 17°C.

Answer ..... % [1]

(ii) The temperatures of the first 80 days of City X for the year 2022 were also recorded.
The median temperature for the year 2022 is smaller than that of the year 2021.
The interquartile range of the temperatures for the year 2022 is the same as that of the year 2021.
Describe how the cumulative frequency curve for the year 2022 may differ from the curve in the year 2021.

[1]

(iii) The temperatures of the first 80 days of City Y in the year 2021 were also recorded.
The box-and-whisker plot shows the distribution of the temperatures.



Make two comments comparing the temperatures of both cities.

[2]

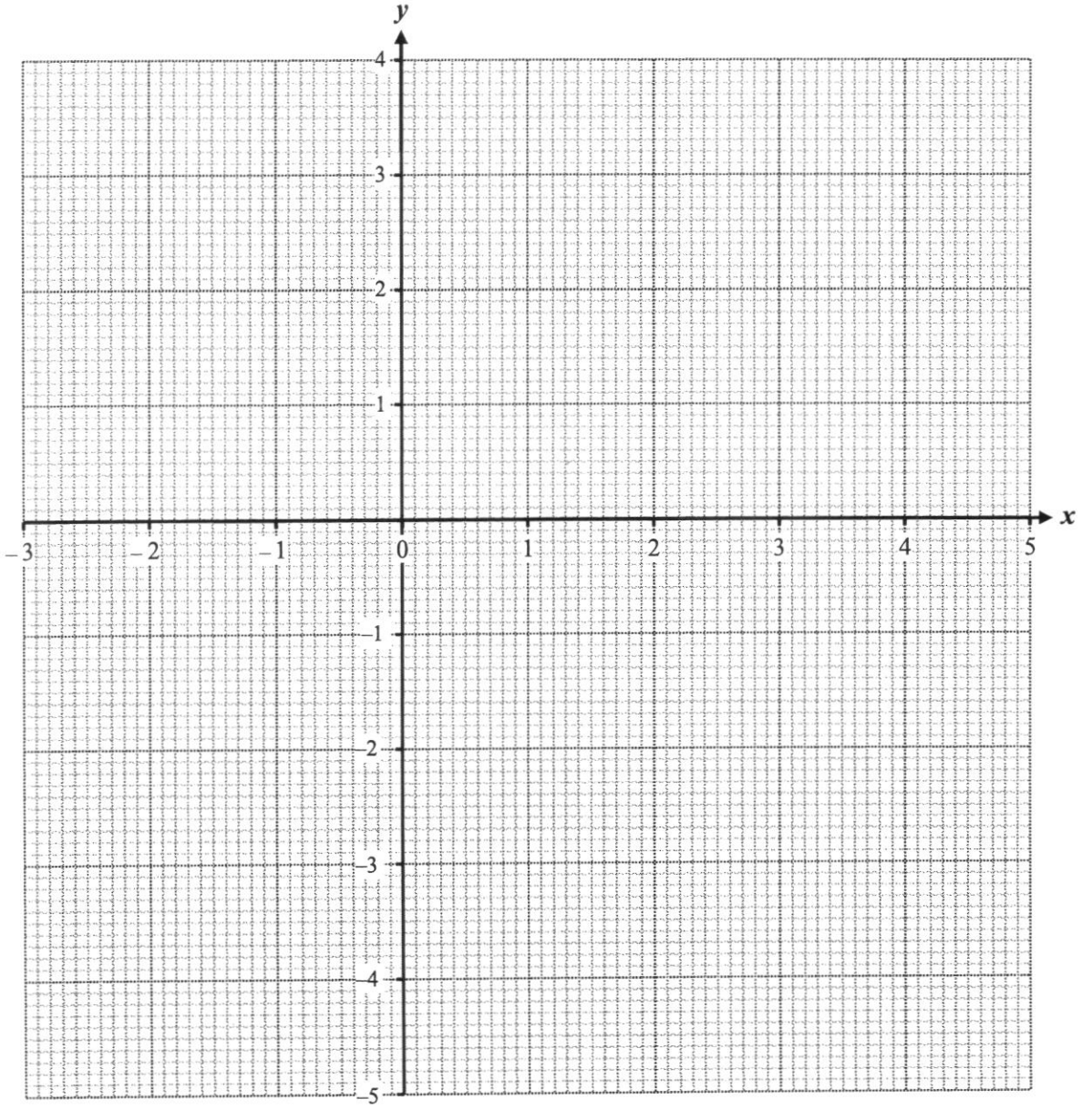
4 **(a)** Complete the table of values for $y = \frac{(x-1)^3}{7} - \frac{4x}{3} + 1$.

Give your answer correct to 1 decimal place.

x	-3	-2	-1	0	1	2	3	4	5
y	-4.1	-0.2	1.2	0.9	-0.3	-1.5	-1.9		3.5

[1]

(b) On the grid, draw the graph of $y = \frac{(x-1)^3}{7} - \frac{4x}{3} + 1$ for $-3 \le x \le 5$.



[3]

(c) The equation $\frac{(x-1)^3}{7}-\frac{4x}{3}=1$ has only one solution.

Explain how this can be seen from your graph.

[2]

(d) By drawing a tangent, find the gradient of the curve at $(-2,-0.2)$.

Answer ..... [2]

(e) **(i)** On the same axes, draw the line $2y+x=5$ for $-3\le x\le 5$. [1]

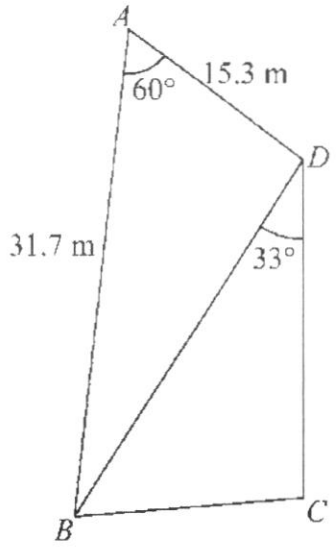
(ii) Write down the x-coordinate of the point where this line intersects the curve.

Answer $x=$ ..... [1]

(iii) This value of x is the solution of the equation $6(x-1)^3-Ax-B=0$.
Find the value of A and the value of B.

[2]

5 $ABCD$ represents a plot of land.



D is due north of C.
The bearing of B from C is 257°.
$AB = 31.7$ m and $AD = 15.3$ m.
Angle $BAD = 60°$ and angle $BDC = 33°$.

(a) Find the length of BD.

Answer ..... m [2]

(b) Find the length of CD.

Answer ..... m [3]

(c) Find the bearing of A from B.

Answer ..... [2]

(d) Alex walks from point B to point D.

(i) Calculate his distance from D when Alex is due west of C.

Answer ..... m [2]

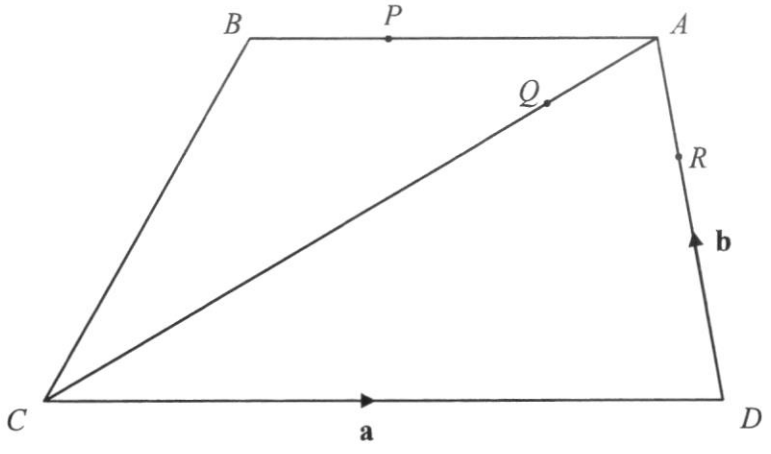
(ii) A lamp post of height 11 m is at point C.
Calculate the greatest possible angle of elevation of the top of the lamp post from Alex during his walk.

Answer ..... [2]

6 $ABCD$ is a trapezium where BA is parallel to CD and $3AB = 2CD$.

$AP = 3BP$, $2DA = 3DR$, and $QA = \frac{1}{4}CQ$.

$\overrightarrow{CD} = \mathbf{a}$ and $\overrightarrow{DA} = \mathbf{b}$.



(a) **(i)** Express $\overrightarrow{QA}$ in terms of **a** and **b**, as simply as possible.

Answer $\overrightarrow{QA} =$ ..... [1]

(ii) Express $\overrightarrow{QR}$ in terms of **a** and **b**, as simply as possible.

Answer $\overrightarrow{QR} =$ ..... [1]

(iii) Express $\overrightarrow{PR}$ in terms of **a** and **b**, as simply as possible.

Answer $\overrightarrow{PR} =$ ..... [2]

(b) Explain why P, Q and R are collinear.

[2]

(c) Given that the area of triangle $ABC = 13$ cm^2, find the area of trapezium $ABCD$.

Answer ..... cm^2 [2]

(d) The point S is the midpoint of CD. Determine if CA is parallel to SR.

.....

..... [2]

7 A café offers three set meals, A, B and C, during lunch and dinner on weekdays.

The matrix **M** shows the number of orders for set meals on a particular day.

$$\mathbf{M} = \begin{matrix} \\ \text{lunch} \\ \text{dinner} \end{matrix} \begin{matrix} \begin{matrix} \text{Set } A & \text{Set } B & \text{Set } C \end{matrix} \\ \begin{pmatrix} 19 & 23 & 32 \\ 25 & 12 & 29 \end{pmatrix} \end{matrix}$$

(a) The café charges \$20 for each set lunch, and \$30 for each set dinner.

Represent the prices in a 1×2 matrix **P**.

Answer **P** = [1]

(b) Evaluate the matrix $\mathbf{N} = 5\mathbf{M}$.

Answer **N** = [1]

(c) Evaluate the matrix $\mathbf{T} = \mathbf{PN}$.

Answer **T** = [2]

(d) State what each of the elements of **T** represents.

[1]

(e) Using matrix multiplication only, evaluate the total number of set meals sold on this particular day.

Answer ..... [2]

8 Bag A contains 4 red discs, 5 blue discs and 8 yellow discs.

Bag B contains 5 red discs and 8 blue discs.

A disc is chosen at random from Bag A and placed into Bag B.

A disc is then chosen at random from Bag B.

(a) Draw a tree diagram to show the possible outcomes and their probabilities.

[2]

(b) **(i)** Find, as a fraction in its simplest form, the probability that both discs chosen are the same colour.

Answer ..... [2]

(ii) Find, as a fraction in its simplest form, the probability that one disc is red, and the other disc is blue.

Answer ..... [2]

(iii) Find, as a fraction in its simplest form, the probability that the disc chosen from bag A is not blue.

Answer ..... [1]

(iv) Find, as a fraction in its simplest form, the probability that at least one disc chosen is blue.

Answer ..... [2]

9 Yan is intending to travel from Singapore to Kuala Lumpur with his family.

He is considering between the following three different modes of transportation.

Option 1: Direct flight from Singapore to Kuala Lumpur

Option 2: Direct drive from Singapore to Kuala Lumpur

Option 3: Trains following this route from Singapore to Kuala Lumpur

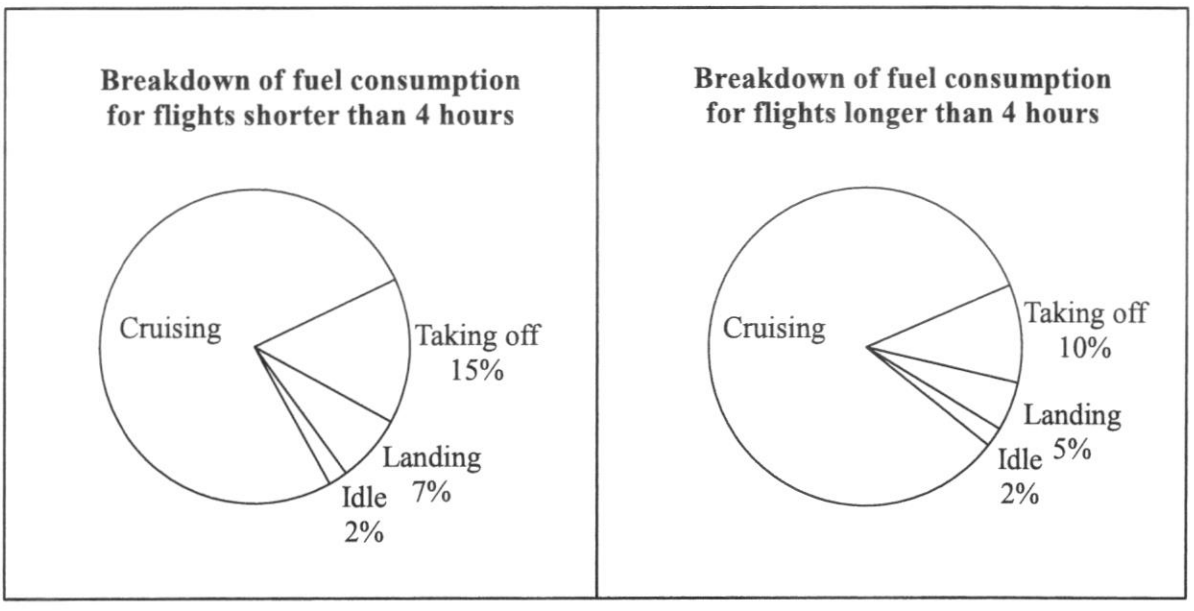
Yan finds the information below.

Mode of transportation	**Plane**	**Train (from Woodlands to JB Sentral)**	**Train (from JB Sentral to Gemas)**	**Train (from Gemas to KL Sentral)**	**Car**
Passenger capacity	300	320	150	125	5
Travel duration	60 – 65 minutes	5 minutes	4 hours 45 minutes	2 hours 35 minutes	6 – 7 hours
Distance covered (km)	310	5	205	130	370
Amount of fuel		6 gallons	200 gallons	130 gallons	45 litres

Other useful information:

- Fuel efficiency is a measure of how much energy is produced by an engine in relation to the amount of fuel that it uses. In other words, it shows how far your vehicle can travel with a certain amount of fuel. It is normally measured in litres per kilometre.
- 1 gallon = 3.785 litres

(a) The pie charts below show the breakdown of the total fuel consumption for a flight.



The amount of fuel needed for cruising during any flight is 5760 *l* / 1000 km.
In case of emergencies, it is mandatory for all flights to carry an additional 20% of the total fuel needed to complete the flight.

Find the amount of fuel, in litres, required for the plane to fly from Singapore to Kuala Lumpur.

Answer ..... *l* [3]

(b) An environmentalist, Yan is looking to travel using the mode that is the most fuel efficient per passenger.

Suggest a suitable mode of transport for Yan and his family to travel to Kuala Lumpur. Justify the decision you made and show your calculations clearly.

[6]

(c) State one assumption you have made in the above calculations.

Name	SOLUTION ( )	Class	

南华中学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2023

Subject : **Mathematics**
Paper : **4052/01**
Level : **Secondary Four Express**
Date : **17 August 2023**
Duration : **2 hours 15 min**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions. The number of marks is given in brackets [ ] at the end of each question or part question.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Gives answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use

U	
P	
A	

This paper consists of 286 printed pages and 2 blank pages.

Answer **all** questions.

1 $3^x = 3^{11} + 3^{11} + 3^{11}$

Find the value of x.

$$\begin{aligned} 3^x &= 3^{11} + 3^{11} + 3^{11} \\ &= 3(3^{11}) \\ &= 3^{1+11} \\ &= 3^{12} \\ \therefore x &= 12 \end{aligned}$$

Answer $x =$ ..... [1]

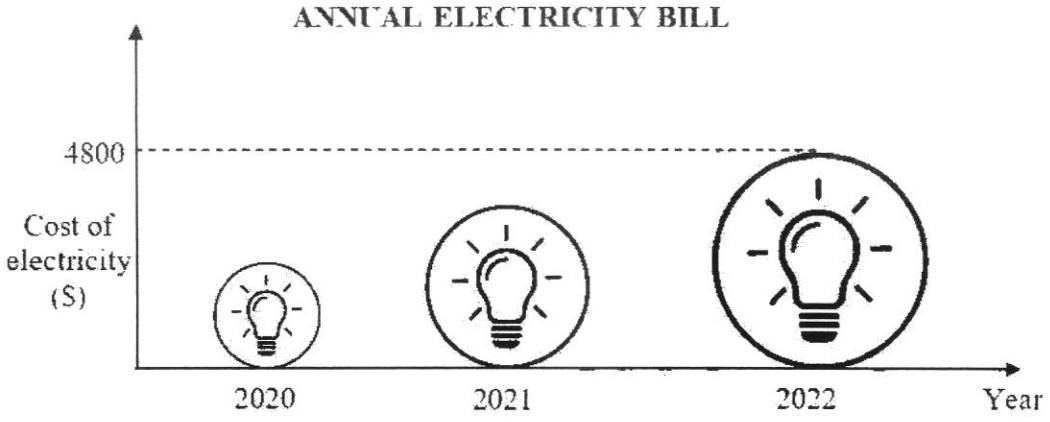
2 Charles invested with a bank that pays interest of 3.5% per annum compounded half-yearly.
He received \$3500 as interest at the end of 3 years.
Find the original amount that Charles invested.
Give your answer to the nearest dollar.

Let P be the original amount.

$$\begin{aligned} 3500 + P &= P\left(1 + \frac{3.5}{200}\right)^6 \\ &= 1.0175^6 P \\ 1.0175^6 P - P &= 3500 \\ P(1.0175^6 - 1) &= 3500 \\ P &= \frac{3500}{1.0175^6 - 1} \\ &= 31905 \end{aligned}$$

Answer \$ ..... [2]

3 Ken draws this graph to show his annual electricity cost for his 5-room flat for year 2020, 2021 and 2022.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

Comparing scale
The vertical axis is not labelled at the start **OR** Start value not indicated on the vertical axis.

It is not possible to determine the exact costs for the three years or that it is not possible to find the increase in costs as it is not possible to compare the years without a scale.

OR

Comparing height or area
It is not clear as to whether the height of the lightbulb or the area of the circle should be used to determine the annual electricity bill. Or
The size of the picture not only increases with height, but also in the width.

When using area, the cost looks 4 times more in 2022 as compared to 2020. If using the height of the lightbulbs, the cost in 2022 looks twice as much in 2020.

[2]

4 The marked price of a 65-inch Ultra HD 4K TV is \$6000.
Mr Lim buys the TV on hire purchase scheme with down payment of 20% on the marked price.
He then pays monthly instalments over 4 years at a simple interest rate of 1.37% per annum.
Calculate the total amount that Mr Lim pays for the television.

$$\text{Interest} = \frac{4800 \times 1.37 \times 4}{100}$$
$$= \$263.04$$
$$\text{Total amount} = 263.04 + 6000$$
$$= \$6263.04$$

Answer \$ ..... [2]

5 The table shows the time, to the nearest hours, taken by 12 students to complete a project.

Time t (hours)	Frequency
$30 \le t < 35$	1
$35 \le t < 40$	2
$40 \le t < 45$	5
$45 \le t < 50$	4

(a) Calculate an estimate for the mean time taken.

$$\text{Mean time} = \frac{1 \times 32.5 + 2 \times 37.5 + 5 \times 42.5 + 4 \times 47.5}{12}$$
$$= \frac{510}{12}$$
$$= 42.5 \text{ hours}$$

Answer ..... hours [1]

(b) Calculate an estimate for the standard deviation of the time taken.

Standard deviation
$$= \sqrt{\frac{1 \times 32.5^2 + 2 \times 37.5^2 + 5 \times 42.5^2 + 4 \times 47.5^2}{12} - \left(\frac{510}{12}\right)^2}$$
$$= 4.5644$$
$$= 4.56 \text{ hours (3 s.f.)}$$

Answer ..... hours [1]

6 The average mass of haemoglobin in a red blood cell is about 31 picograms.

(a) Express 31 picograms in grams, giving your answer in standard form.
[1 picogram = 10^{-12} grams]

30 picograms $= 31\times10^{-12}$ grams
$= 3.1\times10^{-11}$ grams

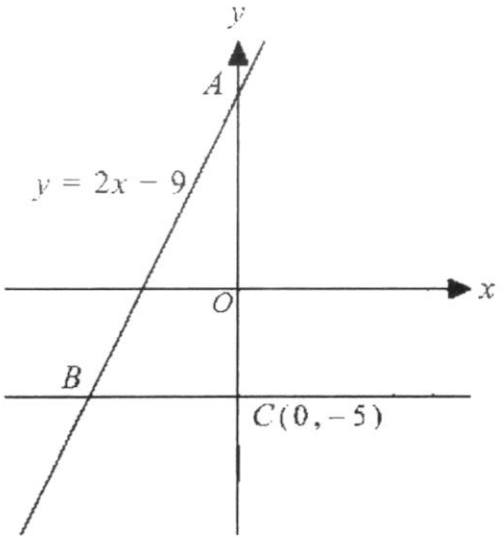
Answer ..... g [1]

(b) There are 5×10^{6} red blood cells in 1 cubic millimetre of blood.
Calculate the mass, in grams, of haemoglobin in 1 cubic millimetre of blood.
Give your answer in standard form.

Average amount $= 3.1\times10^{-11}\times5\times10^{6}$
$= 15.5\times10^{-5}$
$= 1.55\times10^{-4}$ g

Answer ..... g [1]

7 In the diagram, C is the point $(0,-5)$ and A is a point on the y-axis.
The line AB meets the horizontal line through C at point B.
The equation of line AB is $y=2x+9$.



(a) State the equation of the line parallel to line AB, passing through $(0,7)$.

$y=2x+7$

Answer ..... [1]

(b) Find the coordinates of point A and point B.

Coordinates of $A=(0,9)$.

When $y=-5$,
$-5=2x+9$
$2x=-14$
$x=-7$
Coordinates of $B=(-7,-5)$.

Answer A (..... , ...)

B (..... , ...) [2]

8 The ratio of the number of employees in company P to that in company Q was 3: 4.
8 employees left company Q to join company P.
The number of employees in company P now is 4 more than company Q.
Find the original number of employees in company P.

Let the number of employees in P be $3x$

Let the number of employees in Q be $4x$

$3x+8=(4x-8)+4$

$x=12$

Original number in $P=36$

Alternative solution:

Let no of employees in each company be P and Q

Ratio of $Q:P=4:3$

$Q=\frac{4}{3}P$

$P+8=(Q-8)+4$

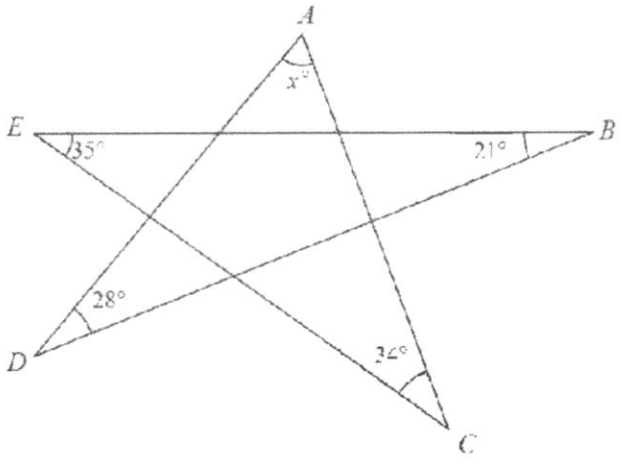
$P+8=\frac{4}{3}P-8+4$

$\frac{P}{3}=12$

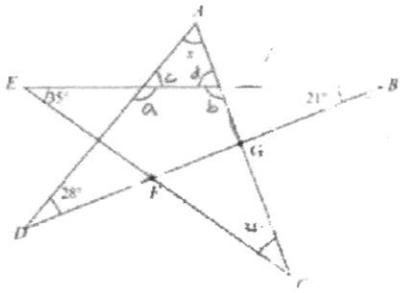
$P=36$

Answer ..... [3]

9



Straight lines AC, AD, BE, BD, CE intersect to form the figure in the diagram.
Find the value of x.



$\angle GFC = 35° + 21°$ (Exterior $\angle$ of Δ)

$= 56°$

$\angle FGC = x° + 28°$ (Exterior $\angle$ of Δ)

$x° + 28° + 56° + 34° = 180°$ ($\angle$ sum of Δ)

$x = 62$

Alternative:

$\angle a = 180° - 28° - 21°$ ($\angle$ sum of Δ)

$= 131°$

$\angle c = 180° - 131°$ (adjacent $\angle$ on a straight line)

$= 49°$

$\angle b = 180° - 35° - 34°$ ($\angle$ sum of Δ)

$= 111°$

$\angle d = 180° - 111°$ (adjacent $\angle$ on a straight line)

$= 69°$

$x° = 180° - 69° - 49°$ ($\angle$ sum of Δ)

$= 62°$

Answer $x =$ ..... [2]

10 **(a)** Factorise completely $3fg+1-g-3f$.

$$\begin{aligned}3fg+1-g-3f &= 3fg-g+1-3f\\ &= g(3f-1)+(1-3f)\\ &= g(3f-1)-(3f-1)\\ &= (g-1)(3f-1)\end{aligned}$$

Answer ..... [2]

(b) Factorise completely $44-11k^2$.

$$\begin{aligned}44-11k^2 &= 11(4-k^2)\\ &= 11(2-k)(2+k)\end{aligned}$$

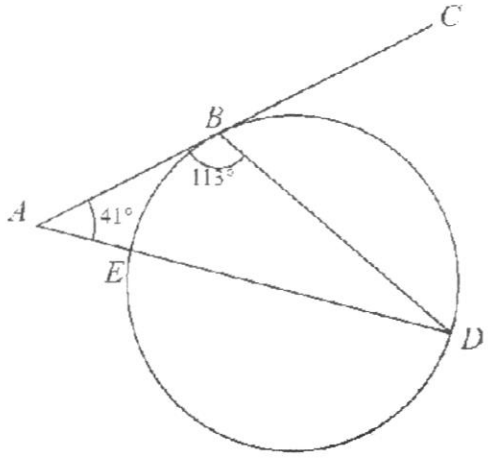
Answer ..... [2]

11 Make x the subject of the formula $w=\dfrac{2x+y}{3x-y}$.

$$\begin{aligned}w &= \frac{2x+y}{3x-y}\\ w(3x-y) &= 2x+y\\ 3wx-wy &= 2x+y\\ 3wx-2x &= y+wy\\ x(3w-2) &= y(1+w)\\ x &= \frac{y(1+w)}{3w-2} \text{ or } \frac{y+wy}{3w-2}\end{aligned}$$

Answer ..... [3]

12



In the diagram, ABC is a tangent to the circle.
The line AD cuts the circle at E.
Angle $BAD = 41°$ and angle $ABD = 113°$.

Explain why ED is not a diameter of the circle.
Give reasons for each step of your working.

$\angle BDA$

$= 180° - 113° - 41°$ ($\angle$ sum of Δ)

$= 26°$

If ED is the diameter, and centre of circle is O,

then $\angle ABO = 90°$(tangent $\perp$ radius)

$\angle OBD$

$= 113° - 90°$

$= 23°$

Since $\angle BDA \neq \angle OBD$, $OB \neq OD$

O does not lie on ED and hence ED is not diameter.

Alternative:

$\angle ODB = 180° - 41° - 113°$($\angle$sum of Δ)

$= 26°$

$= \angle OBD$ (base $\angle$of isosceles Δ)

$\angle OBA = 113° - 26°$

$= 87°$

$\neq 90°$

O does not lie on ED and hence ED is not diameter.

[3]

13 Written as the product of its prime factors, $8316 = 2^2 \times 3^3 \times 7 \times 11$.

(a) Express 840 as the product of its prime factors.

$840 = 2^3 \times 3 \times 5 \times 7$

Answer 840 = ... [1]

(b) **(i)** Hence write down the LCM of 8316 and 840, giving your answers as the product of its prime factors.

LCM of 8316 and 840
$= 2^3 \times 3^3 \times 5 \times 7 \times 11$

Answer LCM = ... [1]

(ii) Hence write down the greatest integer that will divide 8316 and 840 exactly.

the greatest integer that will divide 8316
and 840 exactly $= 2^2 \times 3 \times 7 = 84$

Answer ..... [1]

14 $\overrightarrow{PQ} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$.

(a) Find $\left|\overrightarrow{PQ}\right|$.

$$\left|\overrightarrow{PQ}\right| = \sqrt{6^2 + (-8)^2}$$

$$= 10 \text{ units}$$

Answer $\left|\overrightarrow{PQ}\right| =$ ..... [1]

(b) R is the point $(2, -9)$.

$\overrightarrow{SR} = 2\overrightarrow{PQ}$.

Find the coordinates of S.

$$\overrightarrow{SR} = 2\overrightarrow{PQ}$$

$$\overrightarrow{OR} - \overrightarrow{OS} = 2\overrightarrow{PQ}$$

$$\begin{pmatrix} 2 \\ -9 \end{pmatrix} - \overrightarrow{OS} = 2\begin{pmatrix} 6 \\ -8 \end{pmatrix}$$

$$\overrightarrow{OS} = \begin{pmatrix} 2 \\ -9 \end{pmatrix} - 2\begin{pmatrix} 6 \\ -8 \end{pmatrix}$$

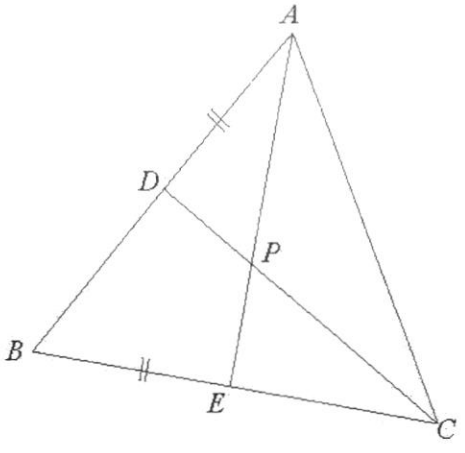
$$= \begin{pmatrix} -10 \\ 7 \end{pmatrix}$$

Coordinates of $S = (-10, 7)$.

Answer S (..... , ...) [2]

Note: minus 1 from question for not using vector presentation throughtout

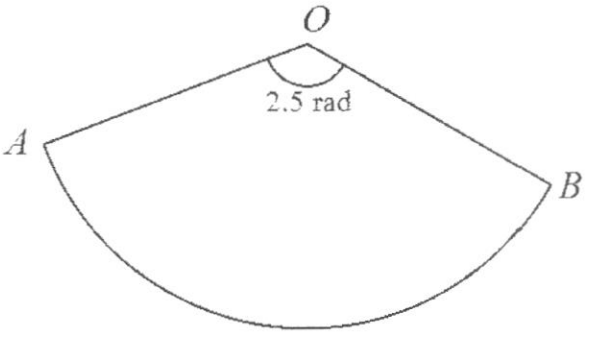
15 The diagram shows an equilateral triangle ABC.
D and E are points on AB and BC respectively such that $AD = BE$.
AE meets CD at P.



Show that triangle ABE is congruent to triangle CAD.
Give a reason for each statement you make.

$BE = AD$ (given)
$AB = AC$ (sides of equilateral Δs)
$\angle ABE = \angle CAD = 60°$ (interior $\angle$s of equilateral Δs)
$\therefore \Delta ABE \equiv \Delta CAD$ (SAS) [3]

16



The diagram shows the sector of a circle, AOB.
The perimeter of sector is 81 cm and angle AOB is 2.5 radian.

(a) Find the radius of the sector.

Let radius of the sector be r cm.

$r + r + 2.5r = 81$
$r = 18$ cm

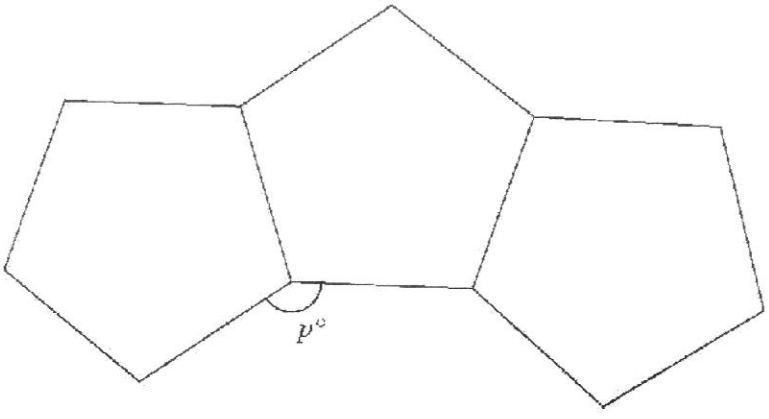
Answer ..... cm [2]

(b) The sides, OA and OB, are joined to form a cone.
Find the radius of the cone.

Let the radius of the base of the cone
be s cm
$2\pi s = 2.5(18)$
$s = 7.1620$
$= 7.16$ cm (3 s.f.)

Answer ..... cm [2]

17



The diagram shows three regular pentagons joined together on its edges.

(a) Find the value of p.

$$\text{Size of interior angle} = \frac{3\times 180°}{5}$$
$$= 108°$$

$$p = 360 - 2(108)$$
$$= 144$$

Answer $p =$..... [2]

(b) Additional regular pentagons are added to these 3 pentagons to form a closed ring. Find the total number of pentagons that are used to form this ring.

$$\frac{(n-2)180}{n} = 144$$
$$180n - 360 = 144n$$
$$36n = 360$$
$$n = 10$$

Alternative Solution:

$$\text{Ext}\angle = 180° - 144°$$
$$= 36°$$
$$36n = 360$$
$$n = 10$$

Answer ..... [2]

18 In a sequence, the same number is subtracted each time to obtain the next term.
The first five terms of the sequence are

$$68 \quad a \quad b \quad c \quad 24.$$

(a) Find the values of a, b and c.

$68+4d=24$

$4d=24-68$

$d=-11$

$a=57,\ b=46,\ c=35$

Answer $a=$.....

$b=$.....

$c=$..... [2]

(b) Write down an expression for the nth term of this sequence.

$68+(n-1)(-11)$

$=79-11n$ or $68-11(n-1)$

Answer ..... [1]

(c) Explain why -524 is not a term of this sequence.

$79-11n=-524$

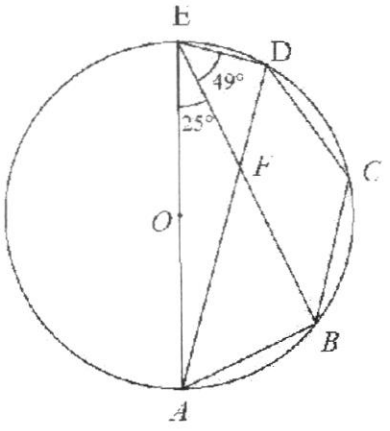
$-11n=-603$

$n=54.8$

Since n has to be a positive integer, then
-524 is not a term of this sequence

[1]

19



O is the centre of the circle passing through A, B, C, D and E.
Chords BE and AD cuts at F.
Angle $AEB = 25°$ and angle $BED = 49°$.

(a) Show that triangle ABF is similar to triangle EDF.
Give a reason for each statement you make.

Answer

$\angle FDE = \angle FBA = 90°$ ($\angle$ in a semi-circle / $\angle$s in the same segment)
$\angle EFD = \angle AFB$ (Vertically opposite $\angle$s)
$\angle BAF = \angle DEF = 49°$ ($\angle s$ in the same segment)
ΔABF is similar to ΔEDF (AA similarity)

[2]

(b) Find angle BCD.
Give a reason for each step of your answer.

$\angle BCD = 180° - 49°$ ($\angle$s in opposite segment)
$= 131°$

Answer [2]

20 $\mathscr{E} = \{x \text{ is an integer: } 1 \leq x \leq 15\}$
$F = \{\text{prime numbers}\}$
$G = \{x : 5 < x \leq 12\}$

(a) List the elements in F.

$F = \{2, 3, 5, 7, 11, 13\}$

Answer ..... [1]

(b) State the value of $n(G)$.

$G = \{6, 7, 8, 9, 10, 11, 12\}$
$n(G) = 7$

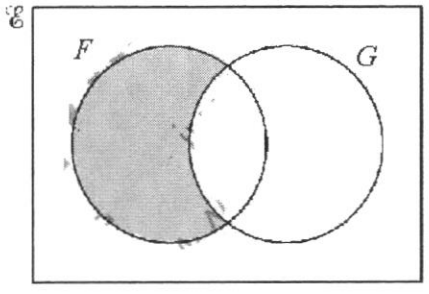
Answer ..... [1]

(c) List all the elements in $(F \cup G)'$.

$(F \cup G)' = \{1, 4, 14, 15\}$

Answer ..... [1]

(d) On the Venn diagram, shade the region which represents $F \cap G'$.



[1]

21 The ratio of surface area of 2 similar solid spheres is 4: 9.
The radius of the smaller sphere is 6 cm.

(a) Find the radius of the larger sphere.

$$\left(\frac{r}{6}\right)^2 = \frac{9}{4}$$
$$r^2 = \frac{9}{4} \times 36$$
$$r = 9$$

Answer .....cm [1]

(b) Express the total the volume of the 2 solid spheres in terms of π.

$$\text{Vol} = \frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(9)^3$$
$$= 1260\pi$$

Answer .....cm^3 [2]

(c) The spheres are melted and recast into 2 solid cones, A and B.
The radius and height of cone A is 6 cm and 10 cm respectively.
The radius of cone B is 12 cm.
Showing your working clearly, explain if the 2 cones are similar.

Answer

$$\text{Volume of cone } A = \frac{1}{3}\pi(6)^2(10)$$
$$= 120\pi$$
$$\text{Volume of cone } B = 1260\pi - 120\pi$$
$$= 1140\pi$$
$$\text{Ratio of radius} = \frac{6}{12}$$
$$= \frac{1}{2}$$
$$\text{Ratio of volume} = \frac{120\pi}{1140\pi}$$
$$= \frac{2}{19}$$
$$\neq \left(\frac{1}{2}\right)^3$$

Since ratio of volume not equal to ratio of the cube of corresponding sides, the cones are not similar.

Alternative:

Let height of cone B be h.

$$\frac{1}{3}\pi(6)^2(10)+\frac{1}{3}\pi(12)^2(h)=1260\pi$$

$$120+48h=1260$$

$$h=23.75$$

$$\text{Ratio of radius} = \frac{6}{12}$$

$$= \frac{1}{2}$$

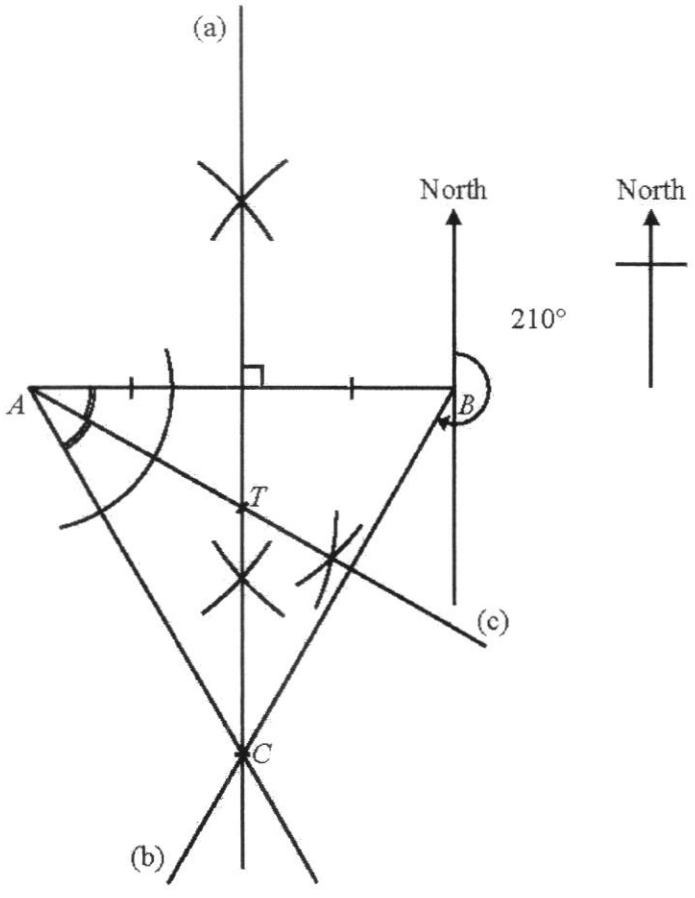
$$\text{Ratio of height} = \frac{10}{23.75}$$

$$\neq \frac{1}{2}$$

Since ratio of corresponding sides are not equal, the cones are not similar.

[2]

22 The diagram shows the location of 2 towns, A and B.



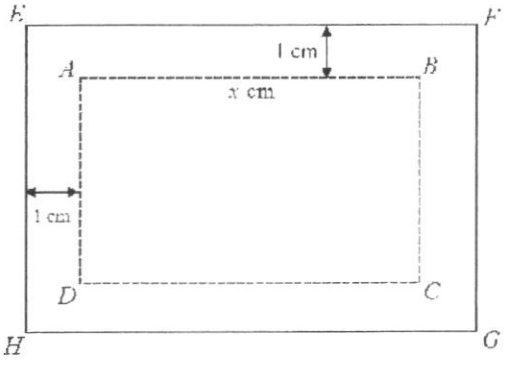
(a) Construct the perpendicular bisector of AB. [1]

(b) Another new town, C, is to be built on a bearing of 210° from town B. [2]
It is equidistant from town A and town B.
Mark the position of town C.

(c) Construct the angle bisector of angle BAC. [1]

(d) A telecommunication tower, T, serving the 3 towns is to be built so that it is [1]
equidistance from AB and AC, and equidistant from town A and B.
Mark the position of T.

23



The diagram shows a photograph, $ABCD$, length x cm and perimeter 40 cm.
It is mounted on a piece of rectangular cardboard, $EFGH$.
There is a uniform border of 1cm width around the photograph.

(a) The area of the rectangular cardboard, $EFGH$, is S cm^2. [3]
Show that it can be expressed as $S = -x^2 + 20x + 44$.

Breadth of photo

$= \dfrac{40 - 2x}{2}$

$= 20 - x$

$S = (x+2)(20-x+2)$

$= (x+2)(22-x)$

$= 22x - x^2 - 2x + 44$

$= -x^2 + 20x + 44$

(b) Jane claims that the area of the rectangular cardboard, S, can be greater than 150 cm^2.
Explain if the claim is correct. [2]

$S = -x^2 + 20x + 44$

$= -(x^2 - 20x) + 44$

$= -(x-10)^2 + 44 + 100$

$= -(x-10)^2 + 144$

For all values of x,

$(x-10)^2 \geq 0$

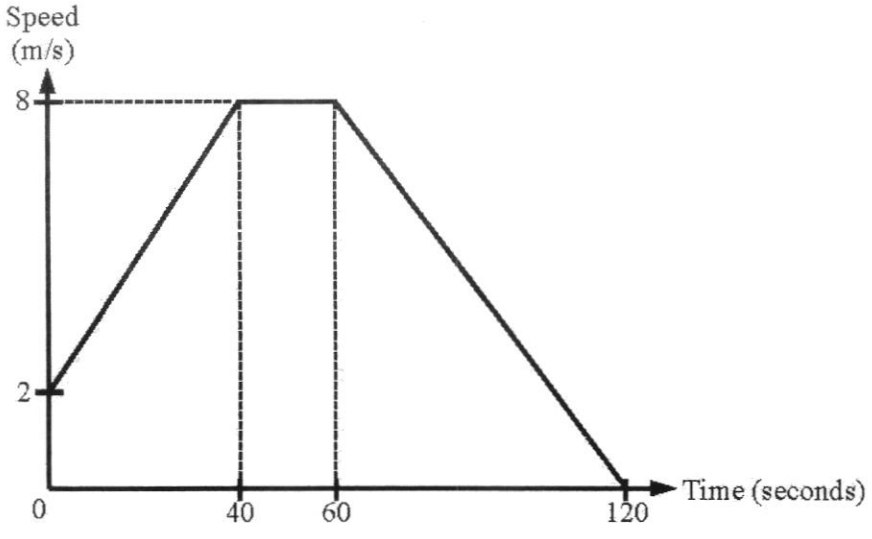
$-(x-10)^2 \leq 0$

$-(x-10)^2 + 144 \leq 144$

$S \leq 144$

Hence Jane is wrong.

24 The diagram shows the speed-time graph of a cyclist after passing a fixed point A.



(a) Calculate the acceleration of the cyclist during the first 10 seconds after passing point A.

Acceleration

$$= \frac{8-2}{40}$$

$$= 0.15 \text{ m/s}^2$$

Answer Acceleration = ... m/s^2 [1]

(b) Calculate the average speed, in m/s, of the cyclist in the last 80 seconds.

Distance travelled in the last 80s

$$= \frac{1}{2}(80+20)(8)$$

$$= 400 \text{ m}$$

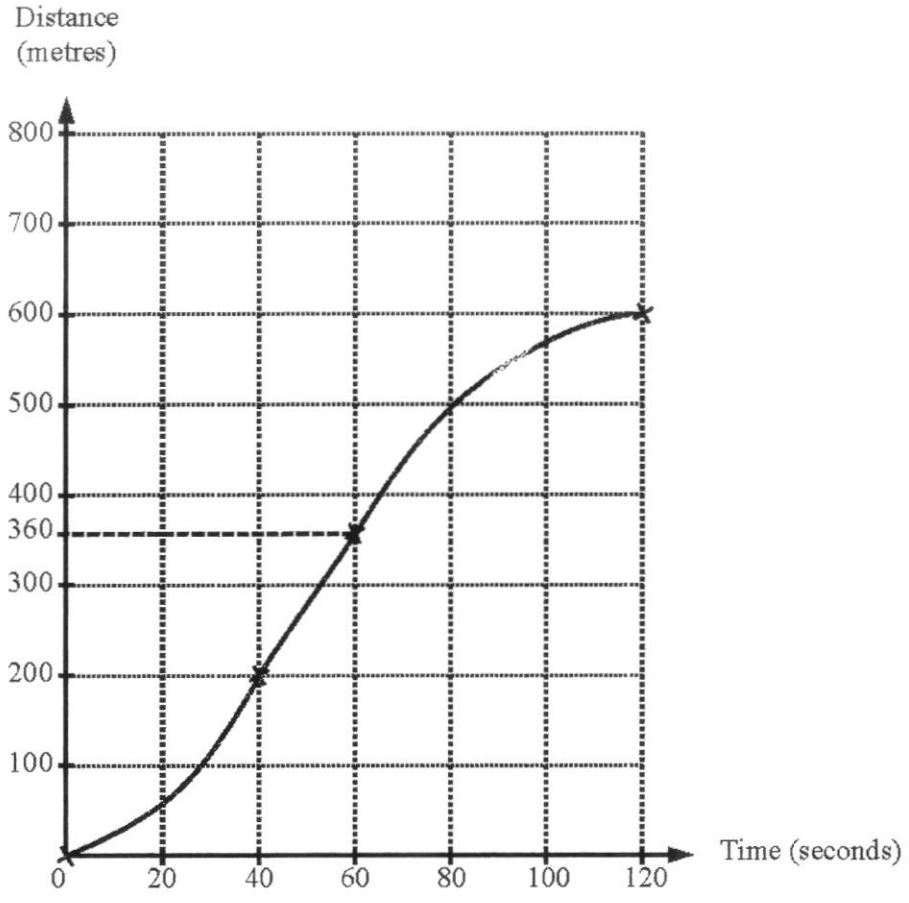
Average speed in last 80s

$$= \frac{400}{80}$$

$$= 5 \text{ m/s}$$

Answer Average speed = ... m/s [2]

(c) On the grid below, sketch the distance-time graph for the journey.



25 (a) The table shows the choice of CCA for Secondary One students in a school.

	Sports	Clubs and Society	Performing Arts	Total
Males	70	26	72	168
Females	38	a	c	d
Total	108	b	141	316

(a) Find the values of a, b, c and d in the table.

$a = 41$
$b = 67$
$c = 69$
$d = 148$

Answer $a =$.....

$b =$ .....

$c =$ .....

$d =$ ..... [2]

(b) A pie chart is to be drawn showing the data for **males**.
Calculate the angle representing the males who choose Sports.

$$\frac{70}{168} \times 360° = 150°$$

Answer ..... [1]

(c) In which group, males or females, did a greater percentage choose Performing Arts.
Explain.

Performing Arts:

Percentage of males

$$= \frac{72}{168} \times 100\%$$

$= 42.857\%$

Percentage of females

$$= \frac{69}{148} \times 100\%$$

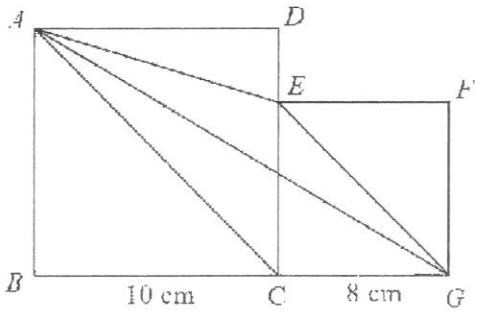
$= 46.6\%$

Since $46.6\% > 42.857\%$

Females have a greater proportion

[2]

26



The diagram shows two squares, $ABCD$ and $CGFE$, of lengths 10 cm and 8 cm respectively.

(a) Find the length of AG.

$$AG = \sqrt{18^2 + 10^2}$$
$$= \sqrt{424}$$
$$= 20.6 \text{ cm}$$

Answer ..... cm [1]

(b) Show that AC is parallel to EG. [2]
Give a reason for each statement you make.

Answer

Angles ACB and angle $EGC = 45°$ since diagonals of squares bisect the interior angles.

Since Angles ACB = angle EGC, corresponding angles, $AC \parallel EG$

Alternative:
Angles ACB = angle CEG (prove by alternate angles)

(c) Find the area of triangle AGE.

$$\angle AGB = \tan^{-1}\frac{10}{18}$$
$$= 29.054°$$
$$EG = \sqrt{8^2 + 8^2}$$
$$= \sqrt{128}$$
$$\angle AGE = 45° - 29.054°$$
$$= 15.947°$$

$$\text{Area} = \frac{1}{2}\left(\sqrt{128}\right)\left(\sqrt{424}\right)\sin 15.947°$$

$$= 32.004$$

$$= 32.0 \text{ cm}^2$$

Alternative Solution

Answer ..... cm^2 [3]

---------- End of Paper ----------

Name	( )	Class	

南华中学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2023

Subject : **Mathematics**
Paper : **4052/02**
Level : **Secondary Four Express**
Date : **21 August 2023**
Duration : **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions. The number of marks is given in brackets [ ] at the end of each question or part question.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	U		TOTAL
									P		
									A		

This paper consists of **22** printed pages and **2** blank pages.

1 (a) (i) Solve the inequalities $\frac{4x-1}{2} \le \frac{3x}{4} \le \frac{5x+9}{5}$.

$\frac{4x-1}{2} \le \frac{3x}{4} \le \frac{5x+9}{5}$

$\frac{4x-1}{2} \le \frac{3x}{4}$ and $\frac{3x}{4} \le \frac{5x+9}{5}$

$2(4x-1) \le 3x$ $15x \le 20x+36$

$8x-2 \le 3x$ $-5x \le 36$

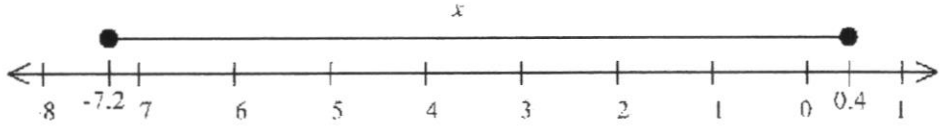
$5x \le 2$

$x \le \frac{2}{5}$ $x \ge -7\frac{1}{5}$

$-7\frac{1}{5} \le x \le \frac{2}{5}$ or $-7.2 \le x \le 0.4$

Answer ..... [2]

(ii) Represent the solutions of the inequalities on the number line below.



[1]

(iii) State the smallest integer which satisfies the inequalities.

-7

Answer ..... [1]

(b) Simplify $\frac{1}{ab^{-2}} \div \left(\frac{2a}{\sqrt[3]{b^5}}\right)^{-3} \times \frac{ab^0}{10}$, leaving your answer in positive index form.

$$\frac{1}{ab^{-2}} \div \left(\frac{2a}{\sqrt[3]{b^5}}\right)^{-3} \times \frac{ab^0}{10}$$
$$= \frac{b^2}{a} \times \frac{8a^3}{b^5} \times \frac{a}{10}$$
$$= \frac{4a^3}{5b^3} \quad \text{or} \quad \frac{4}{5}\left(\frac{a}{b}\right)^3$$

Answer ..... [3]

(c) Express $\frac{x+10}{3x^2-21x+30} + \frac{1}{2-x}$ as a single fraction in its simplest form.

$$\frac{x+10}{3x^2-21x+30} + \frac{1}{2-x}$$
$$= \frac{x+10}{3(x-2)(x-5)} - \frac{1}{x-2}$$
$$= \frac{x+10-3(x-5)}{3(x-2)(x-5)}$$
$$= \frac{25-2x}{3(x-2)(x+5)}$$

Answer ..... [3]

(d) (i) y is inversely proportional to the square of x.

Which of these diagrams represents the graph of y against x?

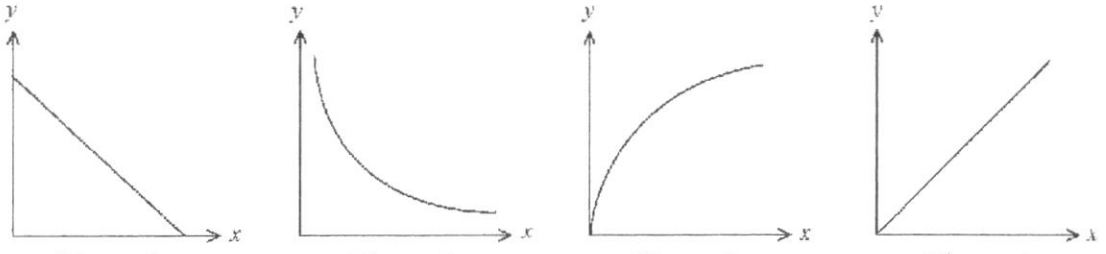


Diagram 1 Diagram 2 Diagram 3 Diagram 4

Answer Diagram ..... **2** ..... [1]

(ii) Find the percentage change in y when x is increased by 150%.

$$y = \frac{k}{x^2}$$

when $x = x_0$, $y = y_0$

$$y_0 = \frac{k}{(x_0)^2}$$

when $x = 2.5x_0$

$$y = \frac{k}{(2.5x_0)^2}$$

$$= \frac{k}{6.25x_0^2}$$

$$= \left(\frac{1}{6.25}\right)y_0 \quad \text{or} \quad \frac{4}{25}y_0$$

$$\% \text{ change} = \frac{\left(\frac{1}{6.25}\right)y_0 - y_0}{y_0} \times 100\%$$

$$= -84\%$$

Answer .....% [3]

2 At a store, 20 red apples cost \$$x$.

The same number of green apples in the store cost \$4 more.

(a) Write an expression, in terms of x, for the number of red apples that can be bought with \$2.40.

$$\frac{48}{x} \text{ or } \frac{20}{x}\times 2.4$$

Answer ..... [1]

(b) Write an expression, in terms of x, for the number of green apples that can be bought with \$2.40.

$$\frac{48}{x+4} \text{ or } \frac{20}{x+4}\times 2.4$$

Answer ..... [1]

(c) \$4.80 can buy a total of 10 apples at the store.

Write down an equation in x to represent this information and show that it reduces to $5x^2-28x-96=0$.

Answer

$$\frac{48}{x}+\frac{48}{x+4}=10$$
$$48(x+4)+48x=10x(x+4)$$
$$48x+192+48x=10x^2+40x$$
$$10x^2-56x-192=0$$
$$5x^2-28x-96=0$$

[3]

(d) Solve the equation $5x^2 - 28x - 96 = 0$.

$5x^2 - 28x - 96 = 0$

$(5x + 12)(x - 8) = 0$

$x = -2.4$ or 8

$5x^2 - 28x - 96 = 0$

$$x = \frac{-(-28) \pm \sqrt{(-28)^2 - 4(5)(-96)}}{2(5)}$$

$= -2.4$ or 8

Answer $x =$ ..... or ..... [2]

(e) Find the cost of 30 green apples in store A.

rej $x = -2.4$ as price > 0

$\therefore x = 8$

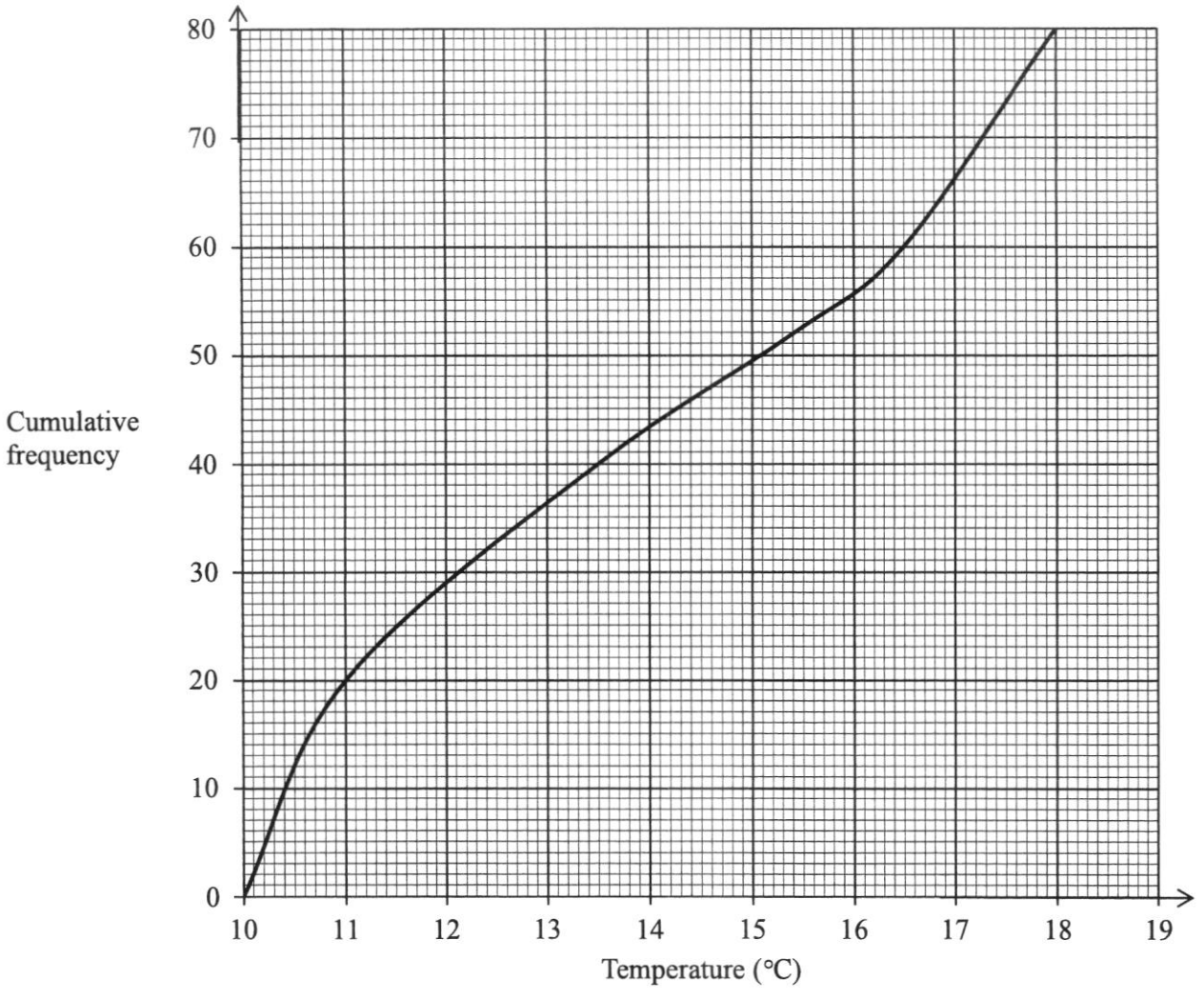
cost of 20 green apples = \$12

cost of 30 green apples $= \$\frac{12}{2} \times 3$

$= \$18$

Answer \$..... [2]

3 The cumulative frequency curve shows the distribution of the daily temperature of city X in the first 80 days of the year 2021.



(i) Use the curve to estimate

(a) the median temperature,

Median = 13.5°C

Answer ..... °C [1]

(b) the interquartile range of the temperatures,
Interquartile range = 16.5 – 11 = 5.5°C

Answer ..... °C [2]

(c) the 65th percentile,
65th percentile = 15.4°C

Answer ..... °C [1]

(d) the percentage of days where the temperature was at least 17°C.

$$\text{required } \% = \frac{80-66}{80} \times 100\% = 17.5\%$$

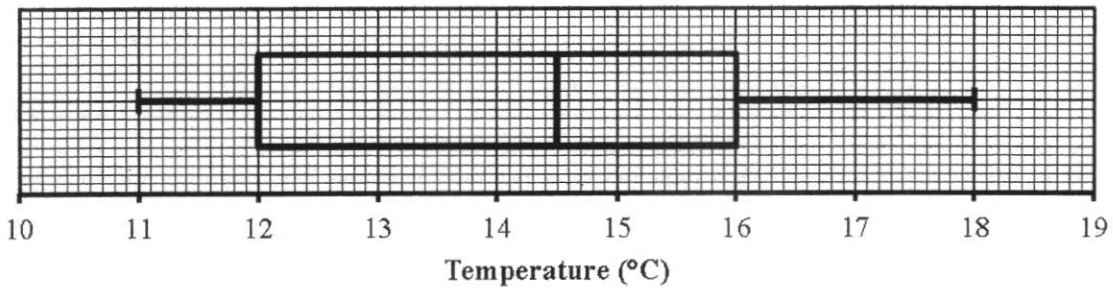
Answer ..... % [1]

(ii) The temperatures of the first 80 days of City X for the year 2022 were also recorded.
The median temperature for the year 2022 is smaller than that of the year 2021.
The interquartile range of the temperatures for the year 2022 is the same as that of the year 2021.
Describe how the cumulative frequency curve for the year 2022 may differ from the curve in the year 2021.

..... The curve of the year 2022 will be to the left of the year 2021. .....

..... [1]

(iii) The temperatures of the first 80 days of City Y in the year 2021 were also recorded.
The box-and-whisker plot shows the distribution of the temperatures.



Make two comments comparing the temperatures of both cities.

The median temperature of City Y is 14.5°C which is higher than that of City X. The temperature in City Y is generally warmer/higher than in City X.

The interquartile range of temperatures of City Y is 4°C which is lower than that of City X. There is a narrower/smaller spread of temperatures in City Y than in city X.

..... [2]

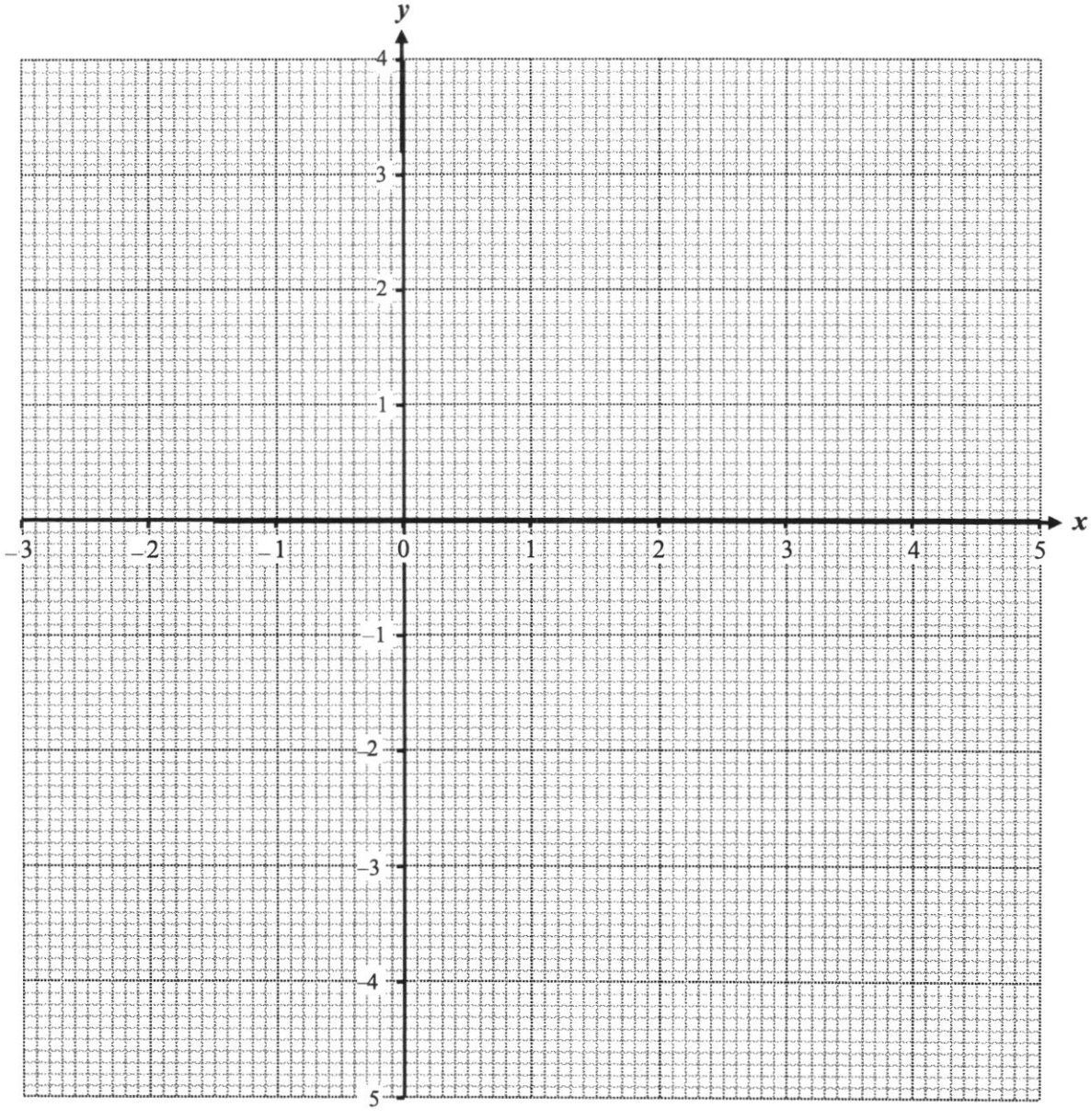
4 **(a)** Complete the table of values for $y = \frac{(x-1)^3}{7} - \frac{4x}{3} + 1$.

Give your answer correct to 1 decimal place.

x	-3	-2	-1	0	1	2	3	4	5
y	-4.1	-0.2	1.2	0.9	-0.3	-1.5	-1.9	-0.5	3.5

[1]

(b) On the grid, draw the graph of $y = \frac{(x-1)^3}{7} - \frac{4x}{3} + 1$ for $-3 \le x \le 5$.



[3]

(c) The equation $\frac{(x-1)^3}{7}-\frac{4x}{3}=1$ has only one solution.

Explain how this can be seen from your graph.

$\frac{(x-1)^3}{7}-\frac{4x}{3}=1$

$\frac{(x-1)^3}{7}-\frac{4x}{3}+1=2$

By adding in the line $y=2$,
from the graph, there is only one intersection.
$\therefore$ the equation has only one solution

[2]

(d) By drawing a tangent, find the gradient of the curve at $(-2,-0.2)$.

gradient $= 2.52$ (calculated)

Answer ..... [2]

(e) **(i)** On the same axes, draw the line $2y+x=5$ for $-3\le x\le 5$. [1]

(ii) Write down the x-coordinate of the point where this line intersects the curve.

$x=4.3$

Answer $x=$ ..... [1]

(iii) This value of x is the solution of the equation $6(x-1)^3-Ax-B=0$.
Find the value of A and the value of B.

$2\left(\frac{(x-1)^3}{7}-\frac{4x}{3}+1\right)+x=5$

$\frac{2(x-1)^3}{7}-\frac{8x}{3}+2+x-5=0$

$\frac{2(x-1)^3}{7}-\frac{5x}{3}-3=0$

$6(x-1)^3-35x-63=0$

$A=35,\ B=63$

$\frac{(x-1)^3}{7}-\frac{4x}{3}+1=-\frac{x}{2}+\frac{5}{2}$

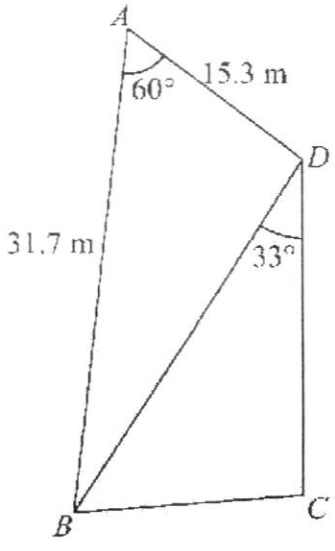
$\frac{(x-1)^3}{7}-\frac{5}{6}x-\frac{3}{2}=0$

$6(x-1)^3-35x-63=0$

$A=35,\ B=63$

.... [2]

5 $ABCD$ represents a plot of land.



D is due north of C.
The bearing of B from C is $257°$.
$AB = 31.7$ m and $AD = 15.3$ m.
Angle $BAD = 60°$ and angle $BDC = 33°$.

(a) Find the length of BD.

$$BD = \sqrt{31.7^2 + 15.3^2 - 2(31.7)(15.3)\cos 60°}$$
$$= 27.459 \ (5sf)$$
$$= 27.5 \text{ m } (3sf)$$

Answer ..... m [2]

(b) Find the length of CD.

$$\angle BCD = 360° - 257° \ (\angle\text{s at a point})$$
$$= 103°$$
$$\angle DBC = 180° - 103° - 33° \ (\angle\text{sum of } \Delta)$$
$$= 44°$$
$$\frac{CD}{\sin 44°} = \frac{27.459}{\sin 103°}$$
$$CD = \frac{27.459}{\sin 103°} \times \sin 44°$$
$$= 19.576 \ (5sf)$$
$$= 19.6 \text{ m } (3sf)$$

Answer ..... m [3]

(c) Find the bearing of A from B.

bearing of D from $B = 33°$ (alternate $\angle$s, // lines)

$$\frac{\sin \angle ABD}{15.3} = \frac{\sin 60°}{27.459}$$

$$\sin \angle ABD = \frac{\sin 60°}{27.459} \times 15.3$$

$$\angle ABD = 28.852°$$

bearing of A from $B = 33° - 28.852°$

$= 004.1°$ $(1dp)$

Answer ..... [2]

(d) Alex walks from point B to point D.

(i) Calculate his distance from D when Alex is due west of C.

let X be the point one BD where Alex is due west of C.

$$\cos 33° = \frac{19.576}{XD}$$

$XD = 23.342$ (5sf)

$= 23.3$ m $(3sf)$

Answer ..... m [2]

(ii) A lamp post of height 11 m is at point C.

Calculate the greatest possible angle of elevation of the top of the lamp post from Alex during his walk.

let the shortest distance from C to BD be CY.

let the greatest angle of elevation be θ

$$\sin 33° = \frac{CY}{19.576}$$

$CY = 10.662$ $(5sf)$

$$\tan \theta = \frac{11}{10.662}$$

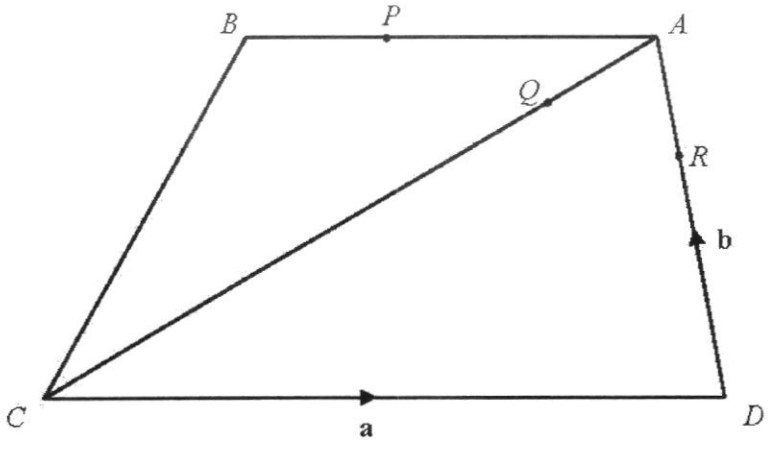
$\theta = 45.9°$ $(1dp)$

Answer ..... [2]

6 $ABCD$ is a trapezium where BA is parallel to CD and $3AB = 2CD$.

$AP = 3BP$, $2DA = 3DR$, and $QA = \frac{1}{4}CQ$.

$\overrightarrow{CD} = \mathbf{a}$ and $\overrightarrow{DA} = \mathbf{b}$.



(a) **(i)** Express $\overrightarrow{QA}$ in terms of **a** and **b**, as simply as possible.

$$\begin{aligned}\overrightarrow{QA} &= \frac{1}{5}\overrightarrow{CA} \\ &= \frac{1}{5}\left(\overrightarrow{CD} + \overrightarrow{DA}\right) \\ &= \frac{1}{5}(\mathbf{a} + \mathbf{b})\end{aligned}$$

Answer $\overrightarrow{QA} =$ ..... [1]

(ii) Express $\overrightarrow{QR}$ in terms of **a** and **b**, as simply as possible.

$$\begin{aligned}\overrightarrow{QR} &= \overrightarrow{QA} + \overrightarrow{AR} \\ &= \frac{1}{5}(\mathbf{a} + \mathbf{b}) - \frac{1}{3}\mathbf{b} \\ &= \frac{1}{5}\mathbf{a} - \frac{2}{15}\mathbf{b} \\ &= \frac{1}{15}(3\mathbf{a} - 2\mathbf{b})\end{aligned}$$

Answer $\overrightarrow{QR} =$ ..... [1]

(iii) Express $\dot{PR}$ in terms of **a** and **b**, as simply as possible.

$$\begin{aligned}\dot{PR} &= \dot{PA} + \dot{AR} \\ &= \frac{3}{4}\overrightarrow{BA} - \frac{1}{3}\mathbf{b} \\ &= \frac{3}{4}\left(\frac{2}{3}\mathbf{a}\right) - \frac{1}{3}\mathbf{b} \\ &= \frac{1}{2}\mathbf{a} - \frac{1}{3}\mathbf{b} \\ &= \frac{1}{6}(3\mathbf{a} - 2\mathbf{b})\end{aligned}$$

Answer $\dot{PR} =$ ..... [2]

(b) Explain why P, Q and R are collinear.

$$\dot{PR} = \frac{1}{6}(3\mathbf{a} - 2\mathbf{b})$$

$$\overrightarrow{QR} = \frac{1}{15}(3\mathbf{a} - 2\mathbf{b}) = \frac{2}{5}\left(\frac{1}{6}\right)(3\mathbf{a} - 2\mathbf{b}) = \frac{2}{5}\overrightarrow{PR}$$

QR is parallel to PR. Since R is a common point, P, Q and R are collinear.

[2]

(c) Given that the area of triangle $ABC = 13$ cm^2, find the area of trapezium $ABCD$.

$$\begin{aligned}\frac{\text{area of trapezium } ABCD}{\text{area of triangle } ABC} &= \frac{\frac{1}{2} \times h \times (BA + CD)}{\frac{1}{2} \times h \times BA} \\ &= \frac{BA + CD}{BA} \\ &= \frac{5}{2}\end{aligned}$$

$$\begin{aligned}\text{area of trapezium } ABCD &= \frac{5}{2} \times 13 \\ &= 32.5 \text{ cm}^2\end{aligned}$$

Answer ..... cm^2 [2]

(d) The point S is the midpoint of CD. Determine if CA is parallel to SR.

$\overrightarrow{CA} = \mathbf{a} + \mathbf{b}$

$\dot{SR} = \dot{SD} + \dot{DR}$

$= \frac{1}{2}\mathbf{a} + \frac{2}{3}\mathbf{b}$

Since $\dot{CA} \neq k\dot{SR}\ \forall k$, CA is not parallel to SR

.....

..... [2]

7 A café offers three set meals, A, B and C, during lunch and dinner on weekdays.

The matrix **M** shows the number of orders for set meals on a particular day.

$$\mathbf{M} = \begin{matrix} \\ \text{lunch} \\ \text{dinner} \end{matrix} \begin{matrix} \begin{matrix} \text{Set } A & \text{Set } B & \text{Set } C \end{matrix} \\ \begin{pmatrix} 19 & 23 & 32 \\ 25 & 12 & 29 \end{pmatrix} \end{matrix}$$

(a) The café charges \$20 for each set lunch, and \$30 for each set dinner.

Represent the prices in a 1×2 matrix **P**.

$\mathbf{P} = \begin{pmatrix} 20 & 30 \end{pmatrix}$

Answer $\mathbf{P} =$ [1]

(b) Evaluate the matrix $\mathbf{N} = 5\mathbf{M}$.

$$\mathbf{N} = 5\begin{pmatrix} 19 & 23 & 32 \\ 25 & 12 & 29 \end{pmatrix} = \begin{pmatrix} 95 & 115 & 160 \\ 125 & 60 & 145 \end{pmatrix}$$

Answer $\mathbf{N} =$ [1]

(c) Evaluate the matrix $\mathbf{T} = \mathbf{PN}$.

$$\mathbf{T} = \begin{pmatrix} 20 & 30 \end{pmatrix}\begin{pmatrix} 95 & 115 & 160 \\ 125 & 60 & 145 \end{pmatrix} = \begin{pmatrix} 5650 & 4100 & 7550 \end{pmatrix}$$

Answer $\mathbf{T} =$ [2]

(d) State what each of the elements of T represents.

The estimated total amount collected from the sales of each type of set meal in a particular week.

..... [1]

(e) Using matrix multiplication only, evaluate the total number of set meals sold on this particular day.

$$\begin{pmatrix} 19 & 23 & 32 \\ 25 & 12 & 29 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 74 \\ 66 \end{pmatrix} \quad \textit{or} \quad \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 19 & 23 & 32 \\ 25 & 12 & 29 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 44 & 35 & 61 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 74 \\ 66 \end{pmatrix} = \begin{pmatrix} 140 \end{pmatrix} \qquad = \begin{pmatrix} 140 \end{pmatrix}$$

Answer ..... [2]

8 Bag A contains 4 red discs, 5 blue discs and 8 yellow discs.

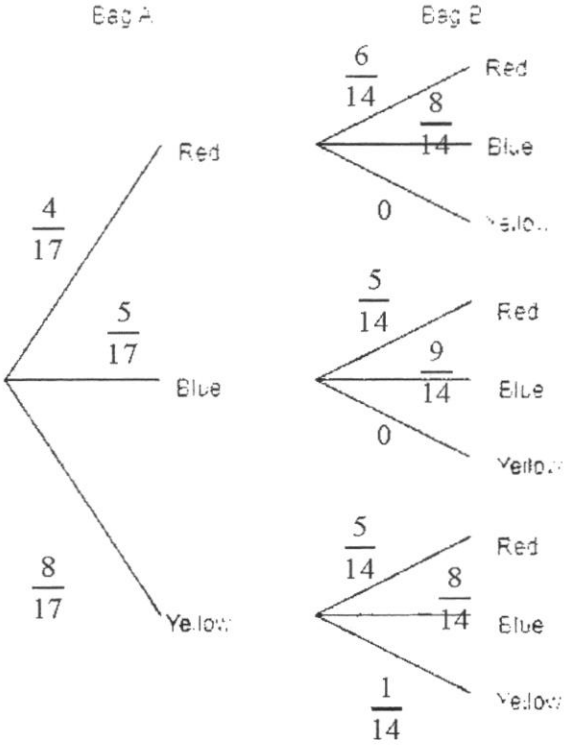
Bag B contains 5 red discs and 8 blue discs.

A disc is chosen at random from Bag A and placed into Bag B.

A disc is then chosen at random from Bag B.

(a) Draw a tree diagram to show the possible outcomes and their probabilities.

Answer



[2]

(b) **(i)** Find, as a fraction in its simplest form, the probability that both discs chosen are the same colour.

$$\begin{aligned}\mathrm{P}(\text{same colour}) &= \frac{4}{17}\times\frac{6}{14}+\frac{5}{17}\times\frac{9}{14}+\frac{8}{17}\times\frac{1}{14}\\ &= \frac{11}{34}\end{aligned}$$

Answer ..... [2]

(ii) Find, as a fraction in its simplest form, the probability that one disc is red, and the other disc is blue.

$$\begin{aligned}\mathrm{P}(\text{one red, one blue}) &= \frac{4}{17}\times\frac{8}{14}+\frac{5}{17}\times\frac{5}{14}\\ &= \frac{57}{238}\end{aligned}$$

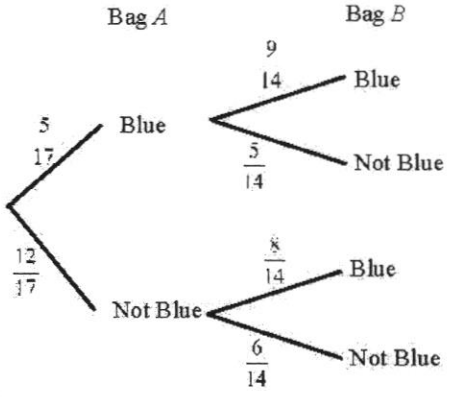
Answer ..... [2]

(iii) Find, as a fraction in its simplest form, the probability that the disc chosen from bag A is not blue.

$$\mathrm{P}(\text{bag } A \text{ not blue}) = \frac{4}{17}+\frac{8}{17}=\frac{12}{17}$$

Answer ..... [1]

(iv) Find, as a fraction in its simplest form, the probability that at least one disc chosen is blue.



$$\begin{aligned}\mathrm{P}(\text{at least one blue}) &= 1-\mathrm{P}(\text{none blue})\\ &= 1-\left(\frac{12}{17}\times\frac{6}{14}\right)\\ &= \frac{83}{119}\end{aligned}$$

or

$$\begin{aligned}\mathrm{P}(\text{at least one blue}) &= \left(\frac{4}{17}\times\frac{8}{14}\right)+\frac{5}{17}+\left(\frac{8}{17}\times\frac{8}{14}\right)\\ &= \frac{83}{119}\end{aligned}$$

Answer ..... [2]

9 Yan is intending to travel from Singapore to Kuala Lumpur with his family.

He is considering between the following three different modes of transportation.

Option 1: Direct flight from Singapore to Kuala Lumpur

Option 2: Direct drive from Singapore to Kuala Lumpur

Option 3: Trains following this route from Singapore to Kuala Lumpur

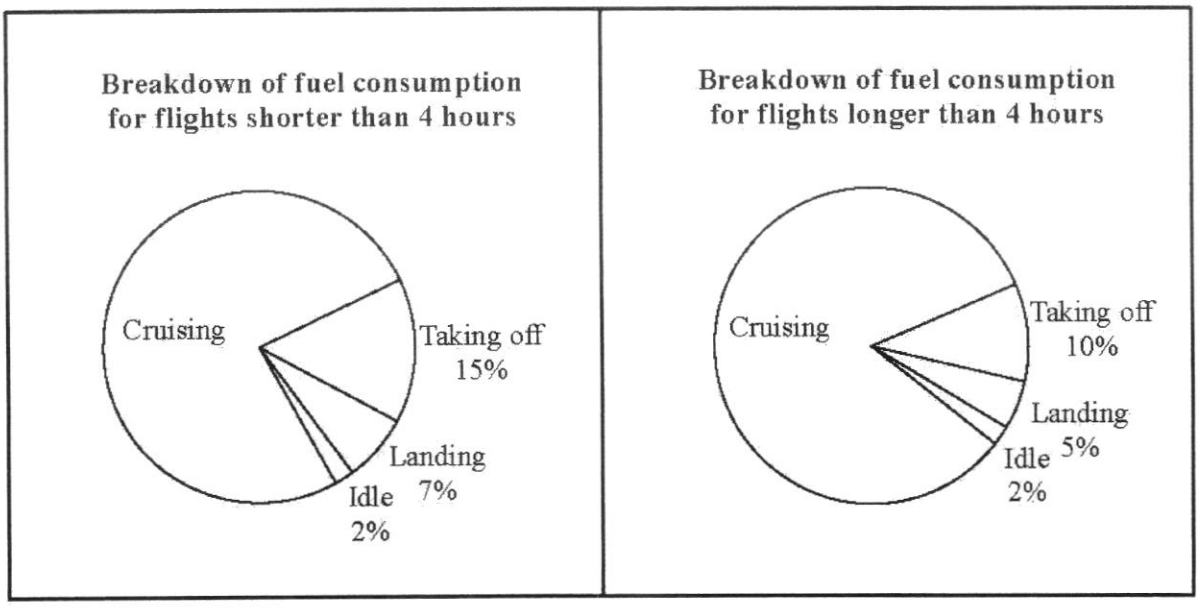
Yan finds the information below.

Mode of transportation	**Plane**	**Train (from Woodlands to JB Sentral)**	**Train (from JB Sentral to Gemas)**	**Train (from Gemas to KL Sentral)**	**Car**
Passenger capacity	300	320	150	125	5
Travel duration	60 – 65 minutes	5 minutes	4 hours 45 minutes	2 hours 35 minutes	6 – 7 hours
Distance covered (km)	310	5	205	130	370
Amount of fuel		6 gallons	200 gallons	130 gallons	45 litres

Other useful information:

- Fuel efficiency is a measure of how much energy is produced by an engine in relation to the amount of fuel that it uses. In other words, it shows how far your vehicle can travel with a certain amount of fuel. It is normally measured in litres per kilometre.
- 1 gallon = 3.785 litres

(a) The pie charts below show the breakdown of the total fuel consumption for a flight.



The amount of fuel needed for cruising during any flight is 5760 *l* / 1000 km.
In case of emergencies, it is mandatory for all flights to carry an additional 20% of the total fuel needed to complete the flight.

Find the amount of fuel, in litres, required for the plane to fly from Singapore to Kuala Lumpur.

$$\begin{aligned}\text{total \% for non-cruising phase} &= 15\% + 7\% + 2\% \\ &= 24\% \\ \text{amount of fuel for cruising} &= \frac{5760}{1000} \times 310 \\ &= 1785.6l \\ \text{total fuel needed for flight} &= 1785.6 \div 76\% \times 120\% \\ &= 2819.37l \ (6sf) \\ &= 2820l \ (3\text{sf})\end{aligned}$$

Answer ..... *l* [3]

(b) An environmentalist, Yan is looking to travel using the mode that is the most fuel efficient per passenger.

Suggest a suitable mode of transport for Yan and his family to travel to Kuala Lumpur. Justify the decision you made and show your calculations clearly.

Answer

$$\text{fuel efficiency per passenger for car} = \frac{45}{370 \times 5}$$
$$= 0.024324 \ l/\text{km} \ (5sf)$$

$$\text{fuel efficiency per passenger for plane} = \frac{2819.37}{310 \times 300}$$
$$= 0.030316 \ l/\text{km} \ (5sf)$$

$$\text{fuel efficiency per passenger for train from Singapore to JB} = \frac{6 \times 3.785}{5 \times 320}$$
$$= 0.014194$$

$$\text{fuel efficiency per passenger for train from JB to Gemas} = \frac{200 \times 3.785}{205 \times 150}$$
$$= 0.024618$$

$$\text{fuel efficiency per passenger for train from Gemas to KL} = \frac{130 \times 3.785}{130 \times 125}$$
$$= 0.03028$$

$$\text{average fuel efficiency for train} = \frac{0.014194 + 0.024618 + 0.03028}{3}$$
$$= 0.023031 \ l/\text{km} \ (5sf)$$

Yan should choose the train as it is the most fuel efficient per passenger.

[6]

(c) State one assumption you have made in the above calculations.

Answer The vehicles are at full capacity. [1]

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