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NGEE ANN SECONDARY SCHOOL O

PRELIMINARY EXAMINATION

MATHEMATICS **4052/01**

Paper 1 **22 August 2023**

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

For Examiner's Use

Total	**/ 90**

This document consists of **26** printed pages and **0** blank pages.

Answer **all** the questions.

1 Calculate $\dfrac{-(-12)-\sqrt{(-12)^2-4\times17\times(-56)}}{2\times(12)^3}$.

Answer ..... [1]

---

2 Factorise fully $5bm-bn+3mn-15m^2$.

Answer ..... [2]

---

3 Simplify $\left(\dfrac{64a^{18}}{b^0c^{12}}\right)^{-\frac{1}{3}}$.

Answer ..... [2]

---

4 (a) Singapore uses 430 million gallon of water every day.
Write 430 million gallon in litre, give your answer in standard form.
1 gallon = 3.785 litre

Answer ..... litres [1]

(b) It takes 2.5 million litres of water to fill one Olympiad pool.
How many pools can be filled with the daily water usage in Singapore?

Answer ..... pools [1]

5 Write as a single fraction in its simplest form $\frac{2}{x-9}+\frac{3x}{(9-x)^2}$.

Answer ..... [2]

---

6 Tampines Town has an area of $21\ \text{km}^2$. Its area on a map is $525\ \text{cm}^2$.
If the map is drawn to a scale of 1: k, find the value of k.

Answer $k =$ ..... [2]

---

7 A survey was conducted to find the number of cups of bubble tea consumed in a month. The results are shown in the table below.

Cups of bubble tea consumed	0	1	2	3	4	5
Number of people	3	6	y	5	11	7

(a) If the mode is 4, write down the largest possible value of y.

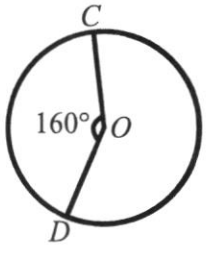
Answer ..... [1]

(b) If the median is 3, write down the greatest possible value of y.

Answer ..... [1]

---

8 In the diagram, it is given that the radius of the circle is 11 cm and $\angle COD = 160°$.



(a) Change 160° to radians, leaving your answer in terms of π.

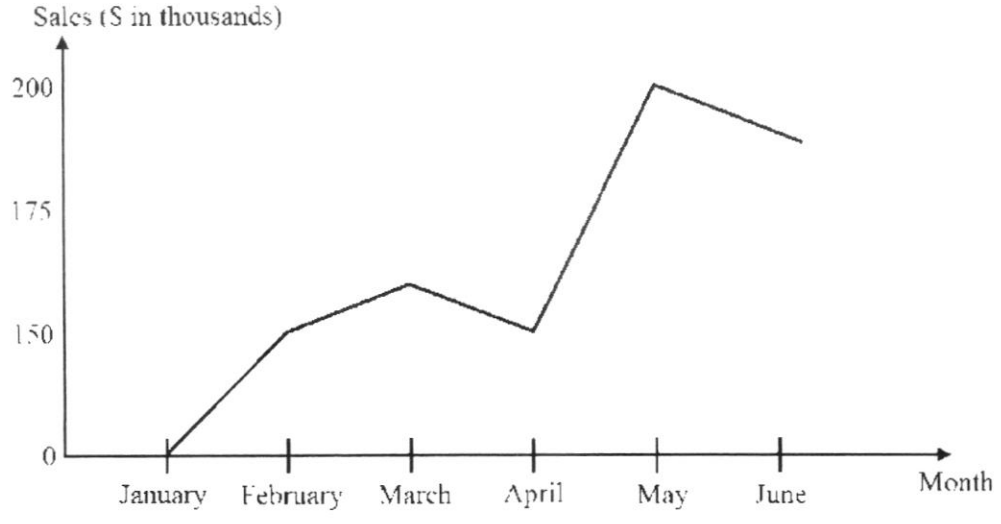
Answer ..... [1]

(b) **Hence**, calculate the length of the minor arc CD, correct to 3 significant figures.

Answer ..... cm [1]

---

9 The chart below shows the monthly sales of an event company from January to June in 2023.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph

Answer .....

.....

.....

.....

.....

..... [2]

10 The table below summarises the times taken by 100 males to complete a cycling race.

Time (t minutes)	$30 \leq t < 40$	$40 \leq t < 50$	$50 \leq t < 60$	$60 \leq t < 70$	$70 \leq t < 80$
Frequency	15	32	30	16	7

(a) Calculate an estimate of the mean time.

Answer ..... minutes [1]

(b) Calculate an estimate of the standard deviation of these times.

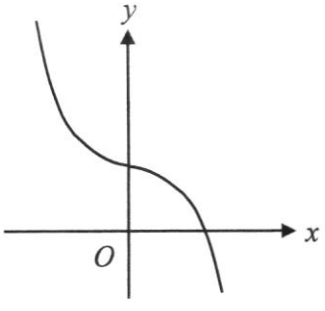
Answer ..... minutes [1]

11

$y = -x^3 + 2$	$y = \frac{5}{x^2}$	$y = -4^{-x}$
$y = 4^{-x}$	$y = x^3 + 2$	$y = \frac{-5}{x^2}$

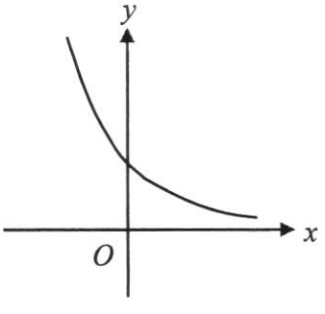
Write down a possible equation for each of the graphs below.
In each case, select one of the equations from the box above.

(a)



Answer ..... [1]

(b)



Answer ..... [1]

12 Viane plans to go to Korea for a holiday.
She goes to the travel fair and is given an early bird discount of 15% for an 8- day tour package that costs $2288. Calculate the amount of money that she needs to pay the tour agency after the addition of 8% Goods and Service Tax (GST) on the discounted price.

Answer $ ..... [2]

---

13 Given that p is inversely proportional to the square root of q, calculate the percentage change in p when q is increased by 400%.

Answer ..... % [3]

---

14 The heights of two geometrically similar cones of same material are in the ratio of 1:30.

(a) Given that the circumference of the base of the larger cone is 450 cm, find the circumference of the base of the smaller cone.

Answer ..... cm [1]

(b) Given that the mass of the larger cone is 12800 g, find the mass of the smaller cone.

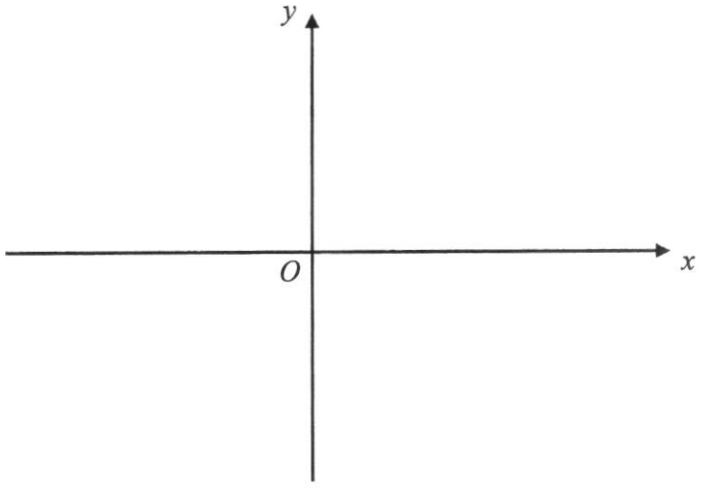
Answer ..... g [2]

15 **(a)** Express $x^2 - 2x - 8$ in the form $(x-a)^2 + b$, where a and b are integers.

Answer ..... [1]

(b) Sketch the graph of $y = x^2 - 2x - 8$.

Answer



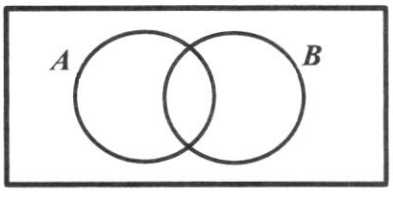
[2]

(c) Write down the coordinates of the minimum point of the curve.

Answer (..... , ...) [1]

16 (a) On the Venn diagram shown below in the answer space, shade the set $A' \cap B'$.

Answer



[1]

(b) ξ = {all real numbers}

A = {all prime numbers}

B = {all rational numbers}

C= {all integers}

D = {all negative numbers}

E = {factors of 12}

(i) List all the elements contained in the set $A \cap E$.

Answer ..... [1]

(ii) Which of the following statement(s) is/are correct?

Statement 1: $C \subset B$

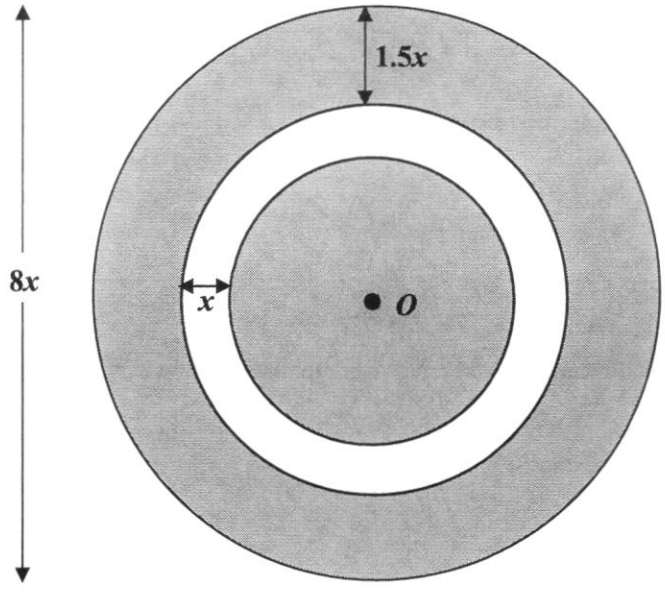
Statement 2: $A \cap C = C$

Statement 3: $A \cap D = \phi$

Statement 4 : $C \cap D = \phi$

Answer Statement(s)..... is/are correct [2]

17 The diagram below shows 3 circles with the centre of circles shown as O.
The diameter of the largest circle is $8x$ cm.
A dart is thrown within the largest circle.



(a) Find the probability that the dart lies inside the shaded region.

Answer ..... [3]

(b) Helen throws two darts and they both lie within the largest circle.
Find the probability that both darts lie inside the shaded region.

Answer ..... [1]

18 Fu Xing needs to pay NT$17000 for his hotel stay in Taiwan.

He is given two options on how he can pay the amount with his credit card.

Option 1	Option 2
• Pays the full amount in New Taiwan Dollars (NT$) • Conversion rate: 1 SGD to NT$22.7 • Incurs 3.5% overseas spending charge	• Pays the full amount in Singapore Dollars (SGD) • Conversion rate: 1 SGD to NT$22.5

Which option should he choose to pay for the amount? Justify your answer with working.

Answer

[4]

19 (a) Solve the inequality $\frac{3}{4}k < \frac{1}{2}(k+1) \leq 5k-1$.

Answer ..... [3]

(b) Solve these simultaneous equations.

$$3x - 2y = 2$$
$$7x + 3y = 43$$

Answer $x =$ ..... , $y =$ ..... [3]

20 (a) Express 6534 as a product of its prime factors.

Answer ..... [1]

(b) Two integers, M and N, can be written as products of prime factors.

$$M = 2 \times p^{r+2} \times q^2 \qquad N = 2 \times p^{r+1} \times q$$

The lowest common multiple (LCM) of M and N is 6534.

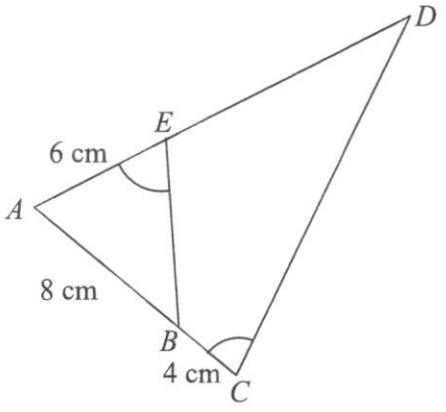
(i) Write down the values of p, q and r.

Answer $p =$ ..... , $q =$ ..... , $r =$ ..... [3]

(ii) Find the highest common factor (HCF) of M and N.

Answer ..... [1]

21 In the diagram below, angle AEB = angle ACD. $AB = 8$ cm, $BC = 4$ cm and $AE = 6$ cm.



(a) Show that triangles AEB and ACD are similar.

Answer .....

.....

.....

..... [2]

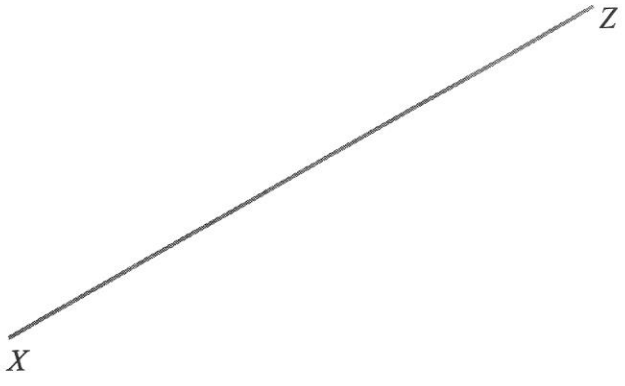
(b) Calculate DE.

Answer ..... cm [2]

(c) Find the ratio of area of quadrilateral $BCDE$: area of triangle ACD.

Answer ..... : ..... [1]

22 (a) Given $XY = 325$ m and $YZ = 200$ m, XZ has been drawn for you.
Complete the scale drawing of a triangular playground XYZ where the given scale is 1: 5000.



[1]

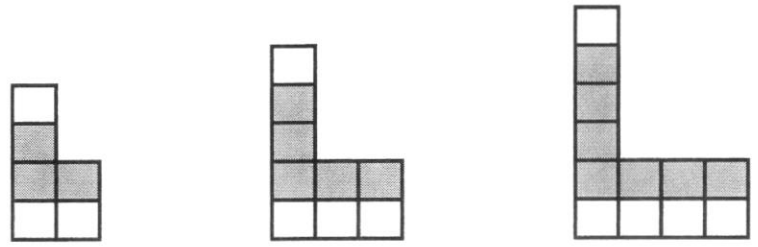
(b) Construct the perpendicular bisector of XZ. [1]

(c) Construct the perpendicular bisector of YX. [1]

(d) Construct the bisector of angle XZY. [1]

(e) A circular fencing is built around the triangular playground XYZ.
Construct a circle touching the three vertices of triangle XYZ. [1]

23 The diagram below consists of patterns made up of shaded and unshaded squares.



PATTERN 1 **PATTERN 2** **PATTERN 3**

(a) State the value of m and of n in the table below.

Pattern Number (P)	Number of shaded squares (Q)	Number of unshaded squares (R)
1	3	3
2	5	4
3	7	5
4	m	n

Answer $m =$ ....., $n =$ ..... [1]

(b) Write down an equation connecting P and Q, leaving your answer in its simplest form.

Answer ..... [1]

(c) Write down an equation connecting P and R, leaving your answer in its simplest form.

Answer ..... [1]

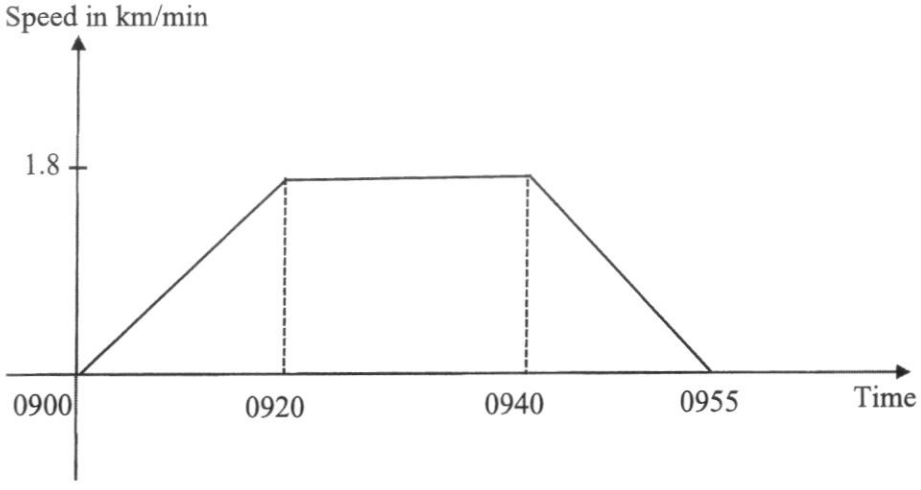
(d) Decide and justify whether it is possible to make a pattern with exactly 50 squares in total.

Answer .....

.....

..... [2]

24 The diagram shows the speed-time graph of a van. The van starts from rest at O and accelerates uniformly until it reaches 1.8 km/min. It then continues at a constant speed before coming to rest at Q.



(a) Find the deceleration during the last 3 minutes in m s^{-2}.

Answer ..... m s^{-2} [2]

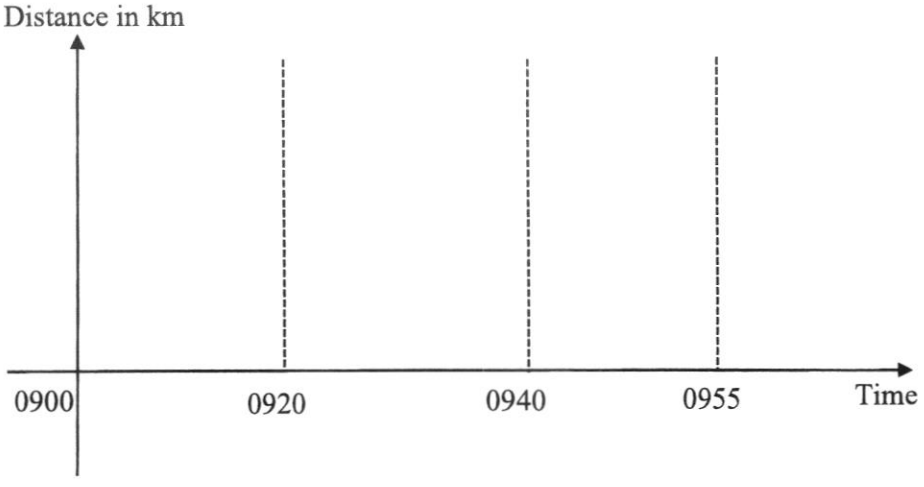
(b) Show that the total distance travelled is 67.5 km.

Answer

[2]

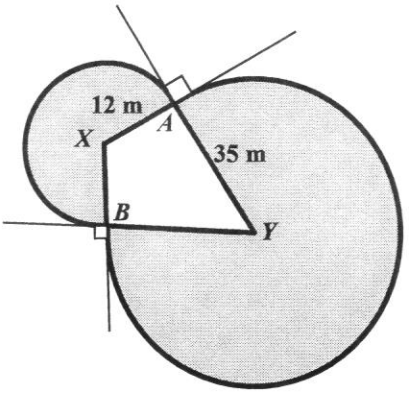
(c) On the axes in the answer space, draw the sketch of the distance-time graph for the whole journey.

Answer



[2]

25 The figure below shows the plan view of a farm. It occupies an area in the shape of two overlapping circles with centres X and Y and radii 12 metres and 35 metres respectively. The circles intersect at the points A and B. XA produced is perpendicular to YA produced. XB produced is perpendicular to YB produced.



(a) Show that angle AXB is 2.48 radians, correct to 3 significant figures.

Answer

[2]

This question continues on the next page.

(b) The shaded area shows the area occupied by the sheep.
Calculate the percentage of the farm area that is occupied by the sheep, using your answer from part **(a)**.

Answer ..... % [5]

26 (a) The interior angles of a quadrilateral $ABCD$ are given below.

$\angle A = 1.25w°$, $\angle B = (80 + w)°$, $\angle C = (2w - 10)°$ and $\angle D = 6(w - 20)°$

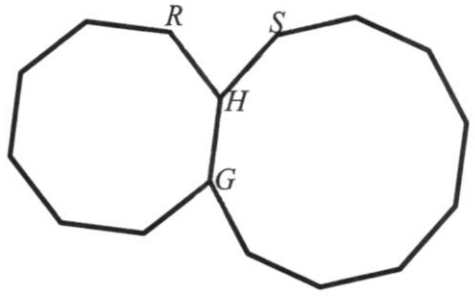
(i) Calculate the value of w.

Answer $w =$ ..... [2]

(ii) State the name for quadrilateral $ABCD$.

Answer ..... [1]

(b) The diagram below shows a regular decagon and a regular octagon with GH as a common side. The points R and S are joined to form a triangle HRS.



Find angle HRS.

Answer ..... [3]

END OF PAPER

Name: ..... Register no: ..... Class: .....

NGEE ANN SECONDARY SCHOOL O

PRELIMINARY EXAMINATION

MATHEMATICS **4052/02**

Paper 2 **24 August 2023**

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

For Examiner's Use

Total	**/ 90**

This document consists of **30** printed pages and **0** blank pages.

Answer **all** the questions

1 **(a)** It is given that $y = 9h^2\left(\frac{x+a}{a-x}\right)$.

(i) Evaluate h when $a = 12$, $x = -6$ and $y = 18.75$.

Answer $h =$ ..... [2]

(ii) Express a in terms of h, x and y.

Answer $a =$ ..... [2]

(b) Solve $\dfrac{15x+2}{x^2-5x+6}+\dfrac{2}{x-3}=3$.

Answer $x =$ ..... or ..... [3]

2 The table below shows the number of loaves of bread sold at 3 different shops on a weekday.

Shop A sells 32 White, 25 Enriched, 15 Fibre and 12 Whole Meal loaves.
Shop B sells 25 White, 18 Enriched, 18 Fibre and 15 Whole Meal loaves.
Shop C sells 28 White, 22 Enriched, 16 Fibre and 14 Whole Meal loaves.

The price per loaf is \$1.20, \$1.40, \$1.70 and \$2.10 for the White, Enriched, Fibre and Whole Meal loaves respectively.

	White	Enriched	Fibre	Whole Meal
Shop A	32	25	15	12
Shop B	25	18	18	15
Shop C	28	22	16	14
Price per loaf	\$1.20	\$1.40	\$1.70	\$2.10

(a) Write down two matrices **X** and **Y** such that matrix multiplication will give the total amount of money received by each shop on that weekday.

Answer **X** = , **Y** = [2]

(b) Hence evaluate the matrix multiplication **XY**.

Answer **XY** = [2]

(c) If discounts of 20%, 10% and 15% are to be given on the purchase of bread from Shop A, B and C respectively, write down a matrix **Z** such that **ZXY** will give the total amount of money received from each of the three shops on that weekday. Hence, find the total amount of money received from each of the three shops, from the selling of bread on that weekday.

Answer Shop A $.....

Shop B $.....

Shop C $..... [2]

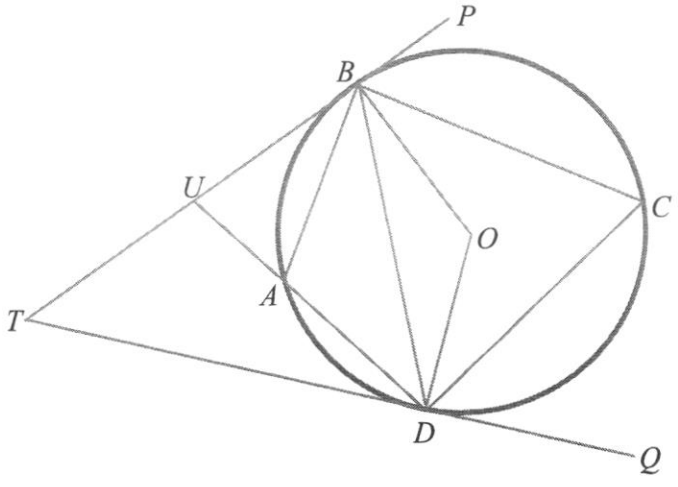
(d) Given that matrix **M** = (1 1 1), explain what the matrix **MZXY** represents.

Answer .....

.....

..... [1]

3 In the diagram, O is the centre of the circle $ABCD$. TBP and TDQ are tangents to the circle. It is given that $\angle BTD = 36^0$ and $AB = AD$.



(a) Find $\angle BCD$.
Give a reason for each step of your working.

Answer ..... [3]

(b) Find $\angle ABO$.
Give a reason for each step of your working.

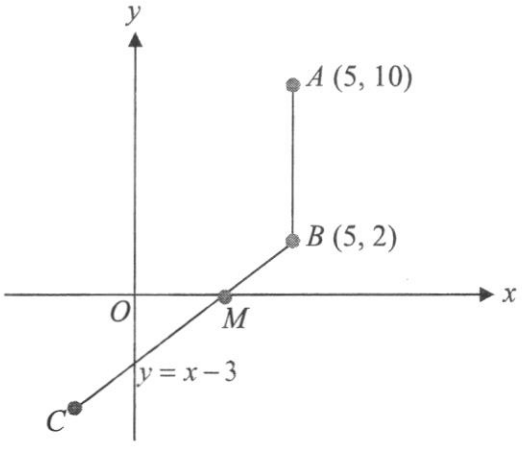
Answer ..... [3]

(c) Show that AB bisects $\angle UBD$.

Answer

[2]

4 The diagram shows a sketch of two straight lines BMC and AB where point A is (5, 10) and point B is (5, 2). It is given that the equation of BC is $y = x - 3$ and that $BC = 4BM$.



(a) Find the coordinates of C.

Answer (....., ...) [2]

(b) D is the point where AM produced meets the y-axis.

Find the length of the line AD.

Answer ..... [4]

(c) $ACBE$ is a parallelogram. Calculate its area.

Answer ..... [2]

5 **(a)** A red plastic bag contains 2 apples and 2 oranges.
The recyclable bag contains 1 apple, 1 orange and 2 bananas.
Matthew picks one fruit from each bag.

(i) Draw a tree diagram to show the probabilities of the possible outcomes.

Answer

[2]

(ii) Find, as a fraction in its simplest form, the probability that 2 apples are selected.

Answer ..... [1]

(iii) Find, as a fraction in its simplest form, the probability that different fruits are selected.

Answer ..... [1]

(iv) Find, as a fraction in its simplest form, the probability that at least one orange is selected.

Answer ..... [1]

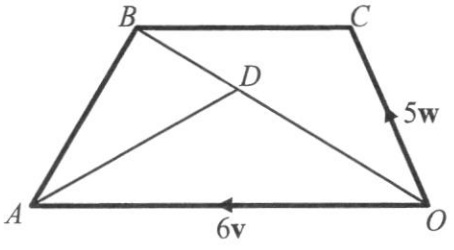
(b) The fruit distribution company employs 15 workers.
One of the 15 workers is selected at random.
The probability that it is a woman who is working part time is $\frac{1}{5}$.
Two of the 15 workers are selected at random.
The probability that they are both men who are working full time is $\frac{1}{5}$.
Complete the table of information about the 15 workers in the company.

Answer

	Part time workers	Full time worker
Men		
Women		4

[4]

6 In the diagram, $\overrightarrow{OA} = 6\mathbf{v}$, $\overrightarrow{OC} = 5\mathbf{w}$, $CB = \frac{2}{3}OA$ and $5OD = 3OB$.



(a) Express each of the following, as simple as possible, in terms of **v** and/or **w**.

(i) $\overrightarrow{OB}$

Answer ..... [1]

(ii) $\overrightarrow{OD}$

Answer ..... [1]

(iii) $\overrightarrow{AD}$

Answer ..... [1]

(iv) $\overrightarrow{DC}$

Answer ..... [1]

(b) Use vectors to show whether A, D and C are collinear.

Answer

[2]

(c) Find the numerical value of

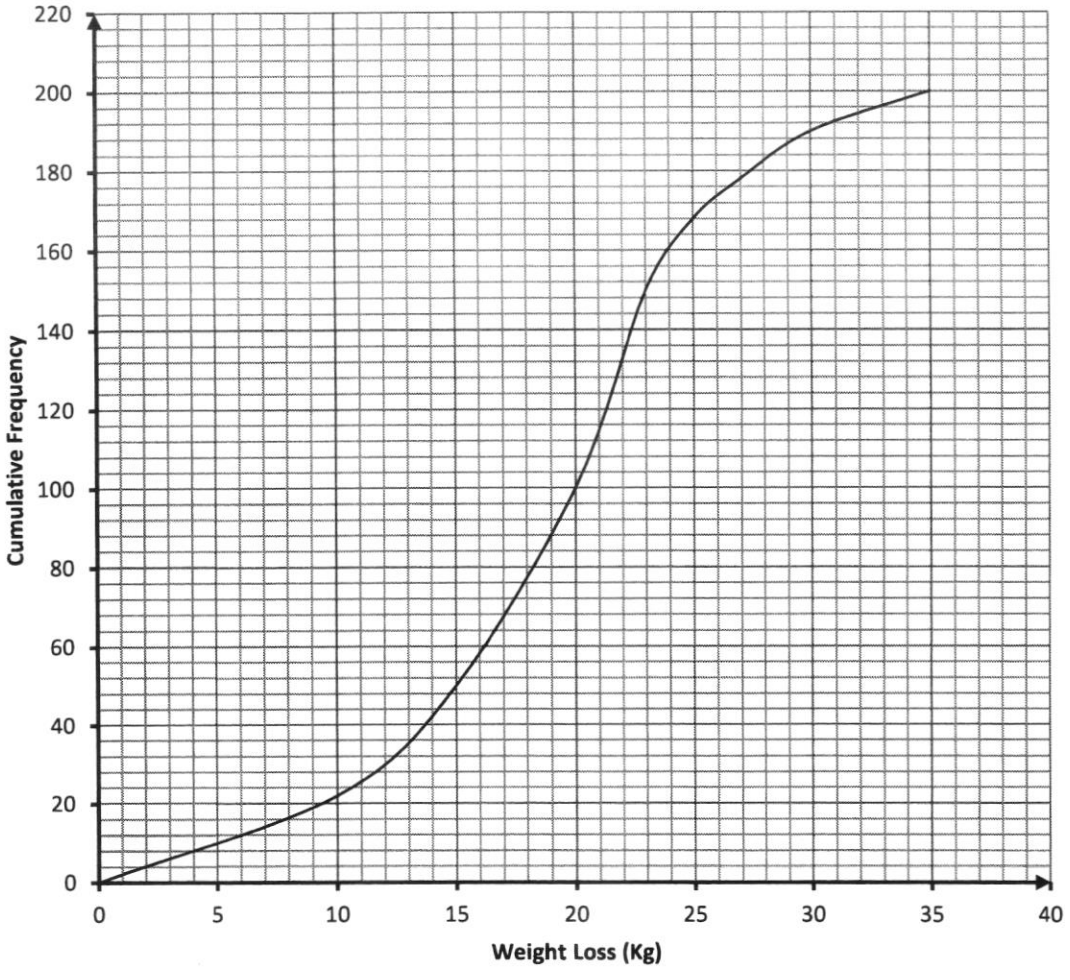
(i) $\dfrac{\text{area of } \Delta DOA}{\text{area of } \Delta DAB}$,

Answer ..... [1]

(ii) $\dfrac{\text{area of } \Delta DBC}{\text{area of } \Delta ABO}$.

Answer ..... [2]

7 The cumulative frequency curve below shows the amount of weight loss, measured to the nearest kg, of 200 men in Alpha Centre over 2 years.



The corresponding cumulative frequency table for this distribution is as shown below.

Weight loss (x kg)	≤ 10	≤ 15	≤ 20	≤ 25	≤ 30	≤ 35
Cumulative Frequency	22	p	100	168	190	200

(a) Calculate p.

Answer $p =$ ..... [1]

(b) Use the curve to estimate

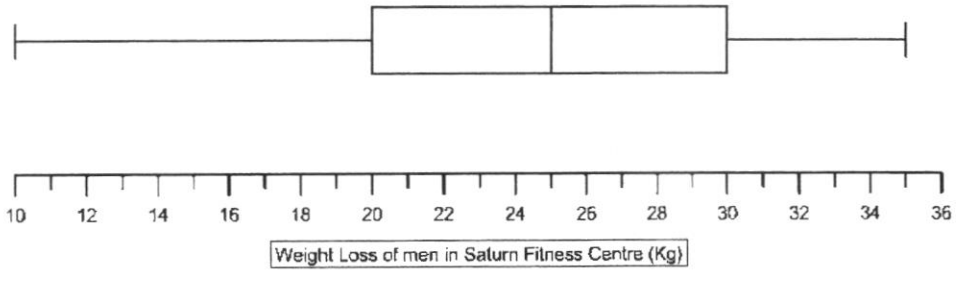
(i) the median weight loss,

Answer ..... kg [1]

(ii) the interquartile range of the weight loss,

Answer ..... kg [2]

(c) A second fitness centre, Saturn Fitness Centre also measured the amount of weight loss by their 240 male members over 2 years.
The box-and-whisker diagram below illustrates their weight loss achieved.



(i) Find the interquartile range of the weight loss.

Answer ..... kg [2]

(ii) Find the number of men who achieved a weight loss of more than 30 kg over 2 years.

Answer ..... [1]

(d) Using the information from parts **(b)** and **(c)**, make two comments comparing the weight loss of the men in Alpha Centre and Saturn Fitness Centre.

Answer .....

.....

.....

.....

.....

..... [2]

8 **(a)** Complete the table of values for $y = \frac{3}{x^2} + x - 2$.

x	0.5	1	1.5	2	2.5	3	4
y	10.5	2		0.75	0.98	1.33	2.19

[1]

(b) On the grid opposite, draw the graph of $y = \frac{3}{x^2} + x - 2$ for $0 \leq x \leq 4$. [3]

(c) Use your graph to find the least value of y.

Answer ..... [1]

(d) By drawing a tangent, find the gradient of the curve at $x = 2.5$.

Answer ..... [2]

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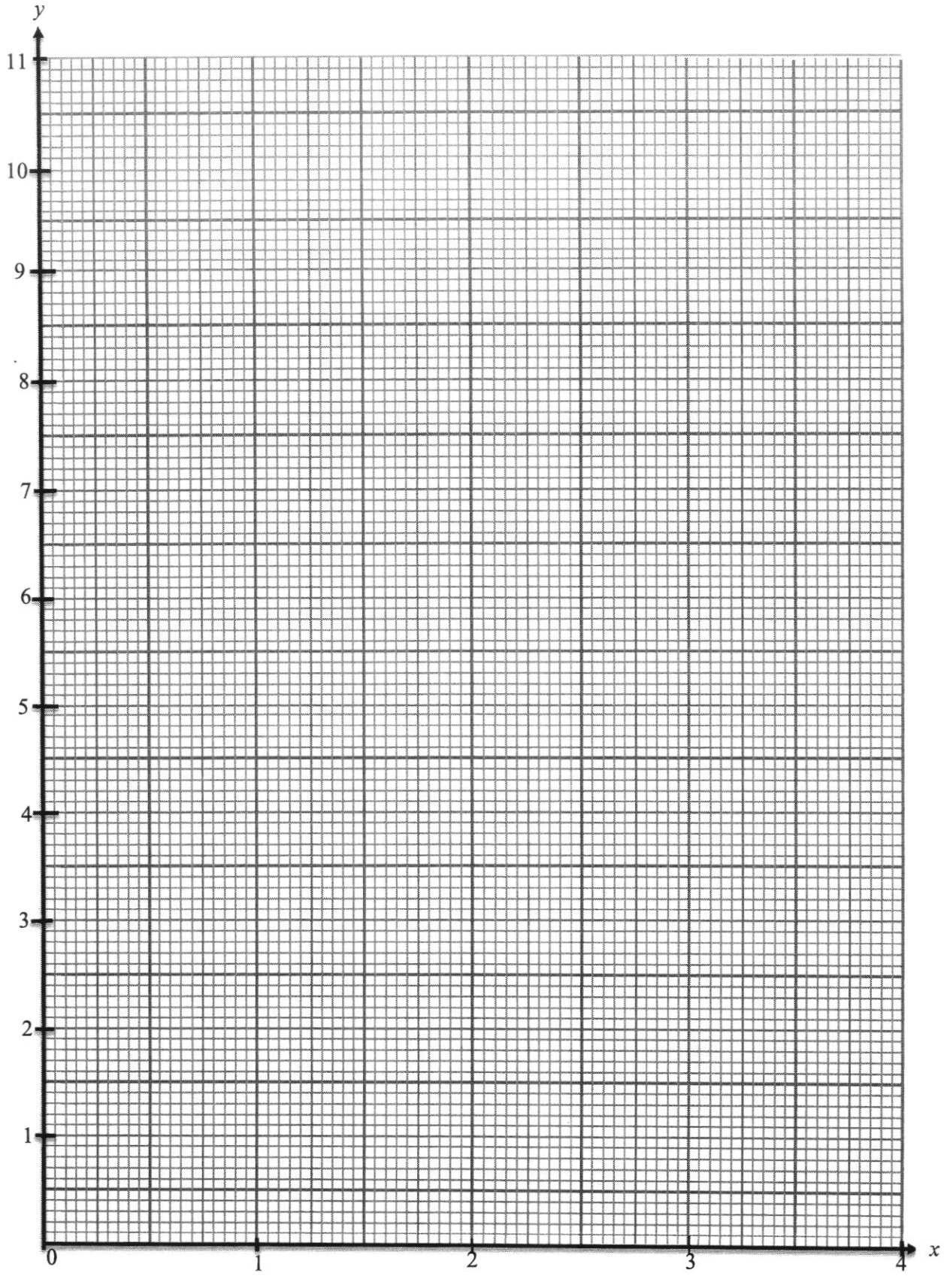

y
11
10
9
8
7
6
5
4
3
2
1
0
1
2
3
4
x

(e) On the same graph, draw a suitable straight line to solve the equation

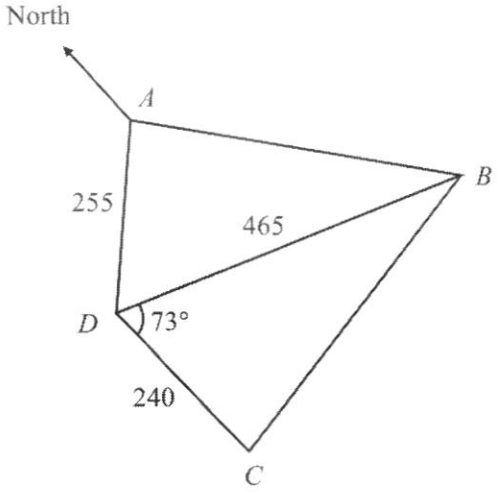
$$\frac{3}{x^2} - x - 4 = 0.$$

Answer $x =$ ..... [2]

(f) Use your graph to find the range of values of x for which $\frac{3}{x^2} + x \leq 4$.

Answer ..... [2]

9 **(a)** $ABCD$ represents a farm on a horizontal field.
$AD = 255$ m, $BD = 465$ m and $CD = 240$ m.
The bearing of B from A is 140° and the bearing of D from A is 224°.
Angle $BDC = 73°$.



(i) Calculate the bearing of D from B.

Answer ..... [3]

(ii) A farmer walks from point D to point C.

Find the shortest distance of the farmer from B during his journey.

Answer ..... m [2]

(iii) The farmer is given a budget of \$57000 to erect electrical barricades around part of the farm, triangle BCD.

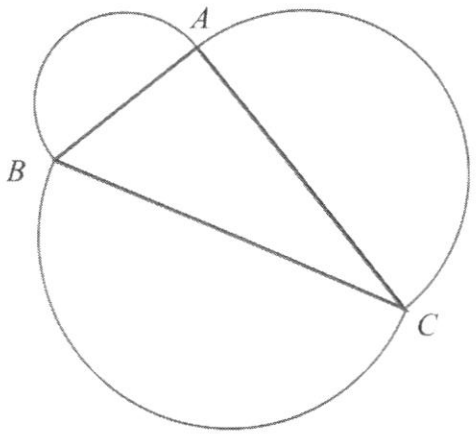
Each 5m barricade is sold at \$245 each.

Determine whether he can complete the job within his budget.
Justify your answer with workings.

Answer

[4]

(b)



Three semicircles are drawn using the sides of a triangle ABC as their diameters.

It is given that the areas of the semicircles are in the ratio 25: 144: 169.

Explain why triangle ABC is right-angled.

Answer

[4]

10 Martin is trying to build a small dam at a river.
In the event of heavy rain, the dam restricts the flow of the water to prevent a flood.

Figure 1 shows a map of the river, drawn to scale.
He hopes to build the dam at the narrowest part of the river.

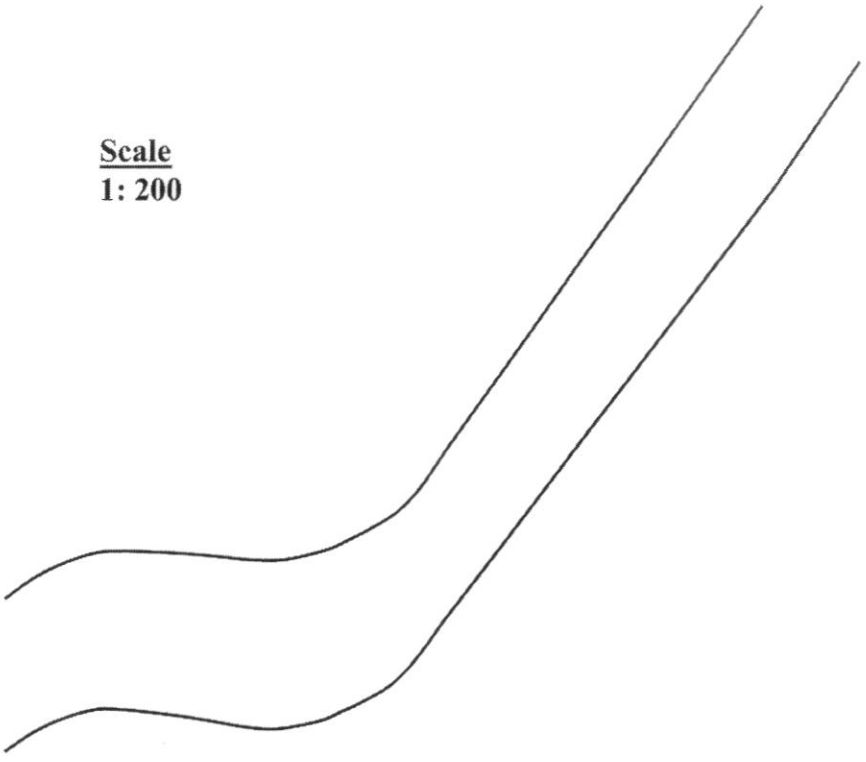


Figure 1 Map of river (drawn to scale)

To better design the dam, Martin measured the depth of the river at 10 different locations during a rainy day. The data obtained is presented in **Figure 2.**

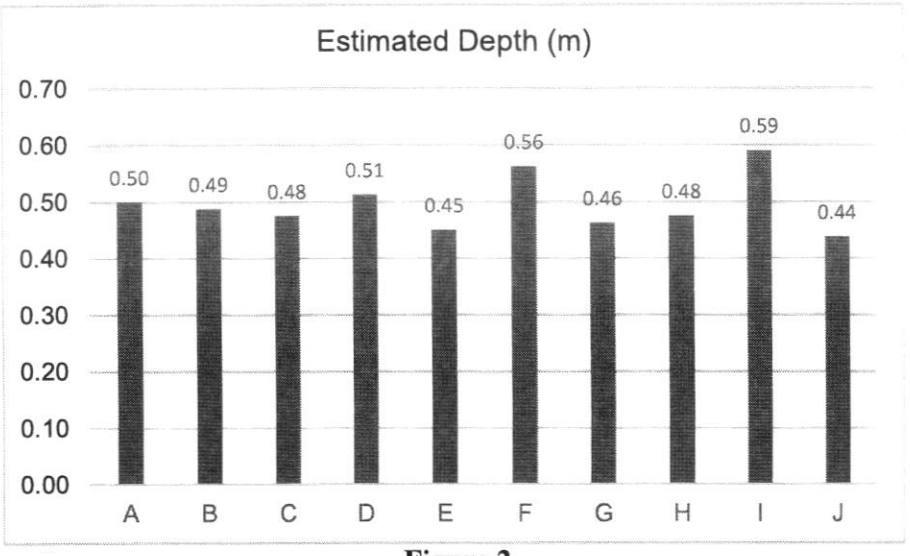


Figure 2

One possible prototype of the dam is shown below in **Figure 3**. The base of the dam is designed to be thicker to account for the greater pressure exerted by the water. The dam has height h m and a length l m and the inclined part of the dam makes an angle of $\theta°$ with the horizontal. The width of the dam, w m, should cover the width of the river completely.

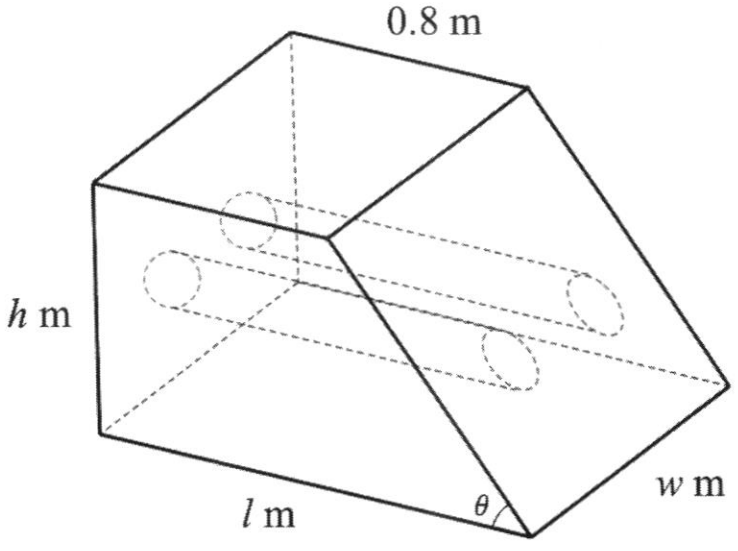


Figure 3

Other design specifications:

- The height of the dam, h m, should be 10% greater than the depth of the water at the location it is built.
- The length of the dam, l m, should be as small as possible and satisfy the inequality

$$\min(2h, w) \leq l \leq 3.$$

[The minimum function **min** gives the smaller value of the two numbers.
For e.g., min(2, 5) = 2]

- The total volume of the dam should exceed 1.5 m^3 for the dam to be operating safely.

(a) **(i)** Find the actual width of the narrowest part of the river.

Answer ..... m [1]

(ii) Calculate a suitable value of θ, leaving your answers corrected to 4 decimal places.

Answer $\theta =$ ..... [3]

(b) Two holes of radius r are drilled through the length of the dam to allow some water to flow through the dam. The hollowed part resembles a truncated cylinder with circular base as shown in **Figure 4.**

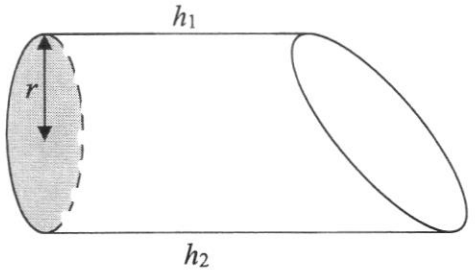


Figure 4

For a truncated cylinder, the volume is given by:

$$V = \frac{1}{2}(h_1 + h_2)\pi r^2$$

Information about the radii of the holes and their corresponding dimensions is shown in the table below.

r (cm)	h_1 (cm)	h_2 (cm)
10	97	113
15	93	116
20	90	120
25	86	124
30	82	128

To ensure a certain rate of water flow, the shaded base area of each hole should be between 700 cm^2 to 1000 cm^2.

This question continues on the next page.

Suggest a suitable radius for the holes to be drilled such that the dam can be operated safely according to all the specifications provided. Justify your decision and show your calculations clearly.

[5]

END OF PAPER

Name: SOLUTIONS ..... Register no: ..... Class: .....

NGEE ANN SECONDARY SCHOOL O

PRELIMINARY EXAMINATION

MATHEMATICS **4052/01**

Paper 1 **22 August 2023**

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use

Total	**/ 90**

This document consists of **26** printed pages and **0** blank pages.

Answer **all** the questions.

1 Calculate $\dfrac{-(-12)-\sqrt{(-12)^2-4\times17\times(-56)}}{2\times(12)^3}$.

Solutions	Marks (Remarks)
$\dfrac{-(-12)-\sqrt{(-12)^2-4\times17\times(-56)}}{2\times(12)^3}=-0.0147178=-0.0147$	B1: Accept answers in 3 sf or more

Answer ..... [1]

---

2 Factorise fully $5bm-bn+3mn-15m^2$.

Solution
$5bm-bn+3mn-15m^2$
$=b(5m-n)+3m(n-5m)$
$=b(5m-n)-3m(5m-n)$
$=(b-3m)(5m-n)$

ALTERNATIVE ANS: $(3m-b)(n-5m)$

Answer ..... [2]

---

3 Simplify $\left(\dfrac{64a^{18}}{b^0c^{12}}\right)^{-\frac{1}{3}}$.

Solutions
$\left(\dfrac{64a^{18}}{b^0c^{12}}\right)^{-\frac{1}{3}}=\left(\dfrac{b^0c^{12}}{64a^{18}}\right)^{\frac{1}{3}}=\dfrac{c^4}{4a^6}$

NOTE: Any negative indices, deduct 1 mark

Answer ..... [2]

---

4 **(a)** Singapore uses 430 million gallon of water every day.
Write 430 million gallon in litre, give your answer in standard form.
1 gallon = 3.785 litre

Solutions
$430\times3.785\times10^{6}=1627.55\times10^{6}=1.62755\times10^{9}$ litres

Answer ..... litres [1]

(b) It takes 2.5 million litres of water to fill one Olympiad pool.
How many pools can be filled with the daily water usage in Singapore?

Solutions
$\frac{1.62755\times10^{9}}{2.5\times10^{6}}=651.02=651$ pools

Answer ..... pools [1]

5 Write as a single fraction in its simplest form $\frac{2}{x-9}+\frac{3x}{(9-x)^2}$.

Solution

$$\frac{2}{x-9}+\frac{3x}{(9-x)^2}$$
$$=\frac{-2}{9-x}+\frac{3x}{(9-x)^2}$$
$$=\frac{-2(9-x)+3x}{(9-x)^2}$$
$$=\frac{5x-18}{(9-x)^2}$$

ALTERNATIVELY,

$$\frac{2}{x-9}+\frac{3x}{(x-9)^2}$$
$$=\frac{2(x-9)+3x}{(x-9)^2}$$
$$=\frac{2x-18+3x}{(x-9)^2}$$
$$=\frac{5x-18}{(x-9)^2}$$

Answer ..... [2]

---

6 Tampines Town has an area of 21 km^2. Its area on a map is 525 cm^2.
If the map is drawn to a scale of 1: k, find the value of k.

Solution

$525cm^2$ represent $21\ km^2$

$1cm^2$ represent $\frac{21}{525}=0.04\ \text{km}^2$

$1cm$ represent $0.2\ \text{km} = 20\,000\ \text{cm}$

$\therefore k = 20000$

Answer $k=$ ..... [2]

---

7 A survey was conducted to find the number of cups of bubble tea consumed in a month. The results are shown in the table below.

Cups of bubble tea consumed	0	1	2	3	4	5
Number of people	3	6	y	5	11	7

(a) If the mode is 4, write down the largest possible value of y.

Solution
10

Answer ..... [1]

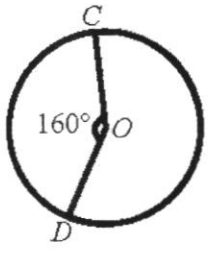
(b) If the median is 3, write down the greatest possible value of y.

Solution
13

Answer ..... [1]

---

8 In the diagram, it is given that the radius of the circle is 11 cm and $\angle COD = 160°$.



(a) Change 160° to radians, leaving your answer in terms of π.

Solution
$160° \times \frac{\pi}{180°} = \frac{8\pi}{9}$

Answer ..... [1]

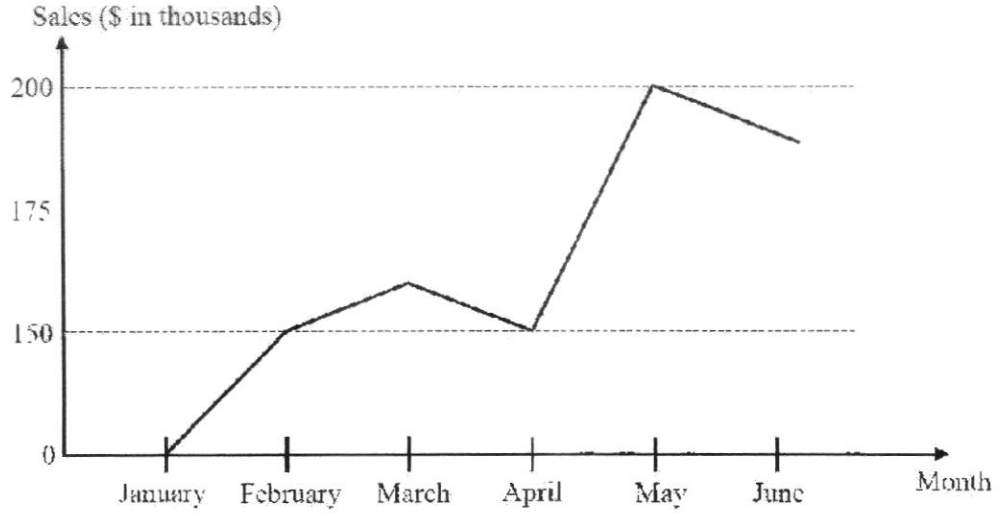
(b) **Hence**, calculate the length of the minor arc CD, correct to 3 significant figures.

Solution
$11 \times \frac{8\pi}{9} = 30.7$ cm

Answer ..... cm [1]

---

9 The chart below shows the monthly sales of an event company from January to June in 2023.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph

Solution
The **scale of the vertical axis** is **not proportional** for the range of zero to 150. The increase of sales from April to May would seem to double of that from January to February. However, the increase during April to May is only $50 000 and the increase during January to February is $150 000.

Answer .....

.....

.....

.....

.....

..... [2]

10 The table below summarises the times taken by 100 males to complete a cycling race.

Time (t minutes)	$30 \le t < 40$	$40 \le t < 50$	$50 \le t < 60$	$60 \le t < 70$	$70 \le t < 80$
Frequency	15	32	30	16	7

(a) Calculate an estimate of the mean time.

Solution
$\text{Mean} = \dfrac{35\times15+45\times32+55\times30+65\times16+75\times7}{100}$
$= 51.8$ minutes (3 significant figures)

Answer ..... minutes [1]

(b) Calculate an estimate of the standard deviation of these times.

Solution
Standard deviation:
$\sqrt{\dfrac{35^2\times15+45^2\times32+55^2\times30+65^2\times16+75^2\times7}{100}-51.8^2}$
$= 11.2$ minutes (3 significant figures)

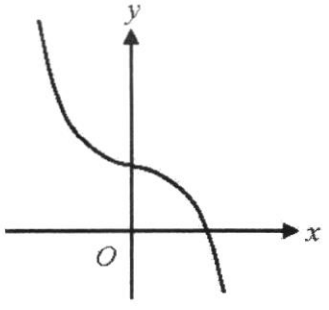
Answer ..... minutes [1]

11

$y = -x^3 + 2$	$y = \frac{5}{x^2}$	$y = -4^{-x}$
$y = 4^{-x}$	$y = x^3 + 2$	$y = \frac{-5}{x^2}$

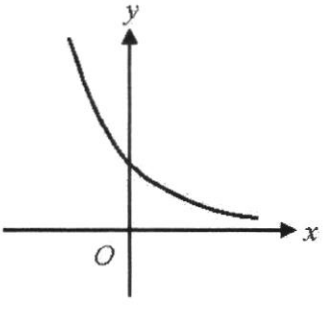
Write down a possible equation for each of the graphs below.
In each case, select one of the equations from the box above.

(a)



Answer $y = -x^3 + 2$ ..... [1]

(b)



Answer $y = 4^{-x}$ ..... [1]

12 Viane plans to go to Korea for a holiday.
She goes to the travel fair and is given an early bird discount of 15% for an 8- day tour package that costs $2288. Calculate the amount of money that she needs to pay the tour agency after the addition of 8% Goods and Service Tax (GST) on the discounted price.

Solution
Discounted price = $0.85 \times 2288 = \$1944.80$ Price after GST = 1944.80×1.08 $= \$2100.38$

Answer $ ..... [2]

---

13 Given that p is inversely proportional to the square root of q, calculate the percentage change in p when q is increased by 400%.

Solution
$p = \frac{k}{\sqrt{q}}$ where k is a constant $p_{new} = \frac{k}{\sqrt{5q}} = \frac{1}{\sqrt{5}}\left(\frac{k}{\sqrt{q}}\right)$ % change : $\frac{\frac{k}{\sqrt{5q}} - \frac{k}{\sqrt{q}}}{\frac{k}{\sqrt{q}}} \times 100\%$ $= -55.27864\ \%$ $= -55.3\ \%$ (3 significant figures)

Answer ..... % [3]

---

14 The heights of two geometrically similar cones of same material are in the ratio of 1:30.

(a) Given that the circumference of the base of the larger cone is 450 cm, find the circumference of the base of the smaller cone.

Solution
$450 \div 30 = 15$

Answer ..... cm [1]

(b) Given that the mass of the larger cone is 12800 g, find the mass of the smaller cone.

Solution
Volume ratio = 1: 27000 128 00 ÷ 27 000 = 0.474 g (3 significant figures)

Answer ..... g [2]

---

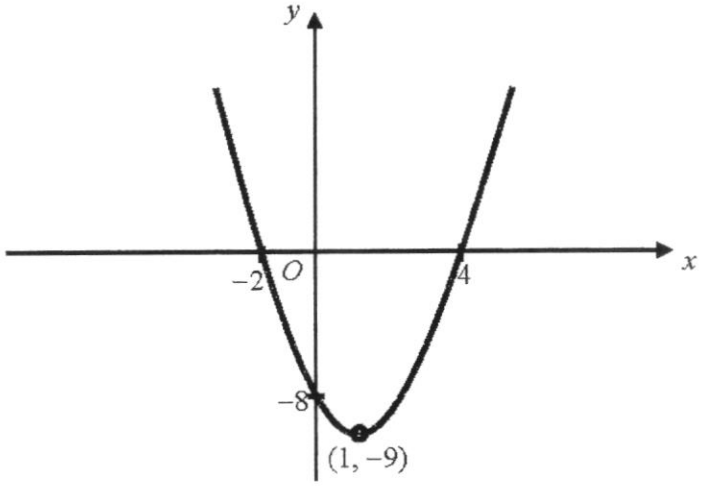
15 **(a)** Express $x^2 - 2x - 8$ in the form $(x-a)^2 + b$, where a and b are integers.

Solution
$x^2 - 2x - 8$
$= (x^2 - 2x) - 8$
$= (x-1)^2 - 8 - 1$
$= (x-1)^2 - 9$

Answer ..... [1]

(b) Sketch the graph of $y = x^2 - 2x - 8$.

Answer



[2]

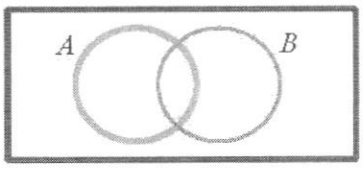
(c) Write down the coordinates of the minimum point of the curve.

Solution
$(1, -9)$

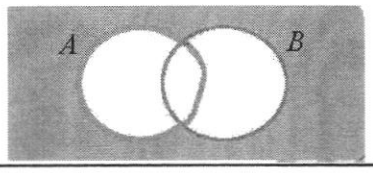
Answer (....., ...) [1]

16 (a) On the Venn diagram shown below in the answer space, shade the set $A' \cap B'$.

Answer



[1]

Solution


(b) ξ = {all real numbers}
A = {all prime numbers}
B = {all rational numbers}
C = {all integers}
D = {all negative numbers}
E = {factors of 12}

(i) List all the elements contained in the set $A \cap E$.

Solution
2,3

Answer ..... [1]

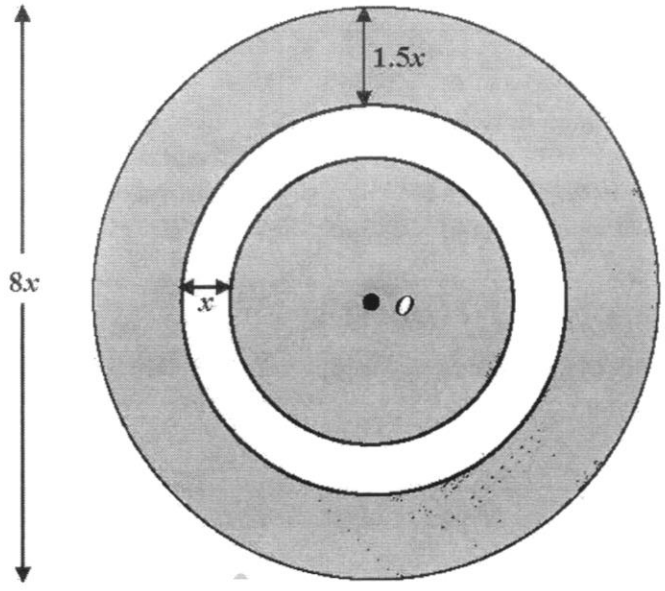
(ii) Which of the following statement(s) is/are correct?

Statement 1: $C \subset B$
Statement 2: $A \cap C = C$
Statement 3: $A \cap D = \phi$
Statement 4 : $C \cap D = \phi$

Solution
Statement 1 Statement 3

Answer Statement(s)..... is/are correct [2]

17 The diagram below shows 3 circles with the centre of circles shown as O.
The diameter of the largest circle is $8x$ cm.
A dart is thrown within the largest circle.



(a) Find the probability that the dart lies inside the shaded region.

Solution
Area of largest circle: $\pi(4x)^2 = 16\pi x^2$ Unshaded area: $\pi(2.5x)^2 - \pi(1.5x)^2$ $= 4\pi x^2$ Probability: $(16 - 4) \div 16$ $= \frac{3}{4}$

Answer ..... [3]

(b) Helen throws two darts and they both lie within the largest circle.
Find the probability that both darts lie inside the shaded region.

Solution
$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

Answer ..... [1]

PartnerInLearning

18 Fu Xing needs to pay NT$17000 for his hotel stay in Taiwan.

He is given two options on how he can pay the amount with his credit card.

Option 1	Option 2
• Pays the full amount in New Taiwan Dollars (NT$) • Conversion rate: 1 SGD to NT$22.7 • Incurs 3.5% overseas spending charge	• Pays the full amount in Singapore Dollars (SGD) • Conversion rate: 1 SGD to NT$22.5

Which option should he choose to pay for the amount? Justify your answer with working.

Answer

Solution
Option 1 $\text{NT\$17000} = \text{SGD}\left(\frac{17000}{22.7}\right)$ $\text{NT\$17000} = \text{SGD } 748.90$ $\text{Amount Payable} = \frac{103.5}{100} \times 748.90 = \text{SGD}775.11$ Option 2 $\text{NT\$17000} = \text{SGD}\left(\frac{17000}{22.5}\right)$ $\text{NT\$17000} = \text{SGD } 755.56$ Fu Xing should pay the full amount using Option 2 as he has to pay less.

[4]

19 (a) Solve the inequality $\frac{3}{4}k < \frac{1}{2}(k+1) \leq 5k-1$.

Solution

$\frac{3}{4}k < \frac{1}{2}(k+1) \leq 5k-1$

$\frac{3}{4}k < \frac{1}{2}(k+1)$

$3k < 2k+2$

$k < 2$

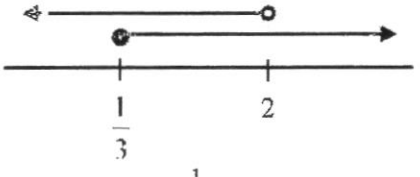
and

$\frac{1}{2}(k+1) \leq 5k-1$

$k+1 \leq 10k-2$

$3 \leq 9k$

$\frac{1}{3} \leq k$



$\therefore \frac{1}{3} \leq k < 2$

Answer ..... [3]

(b) Solve these simultaneous equations.

$3x - 2y = 2$

$7x + 3y = 43$

Solution

$3x - 2y = 2 \quad \Rightarrow \quad 9x - 6y = 6 \quad$ ---(1)

$7x + 3y = 43 \quad \Rightarrow \quad 14x + 6y = 86 \quad$ ---(2)

(1) + (2):

$23x = 92$

$x = 4$

$7(4) + 3y = 43$

$y = 5$

Answer $x =$ ..... , $y =$ ..... [3]

20 (a) Express 6534 as a product of its prime factors.

Solutions
$6534 = 2 \times 11^2 \times 3^3$

Answer ..... [1]

(b) Two integers, M and N, can be written as products of prime factors.

$$M = 2 \times p^{r+2} \times q^2 \qquad N = 2 \times p^{r+1} \times q$$

The lowest common multiple (LCM) of M and N is 6534.

(i) Write down the values of p, q and r.

Solutions
$r = 1, p = 3, q = 11$

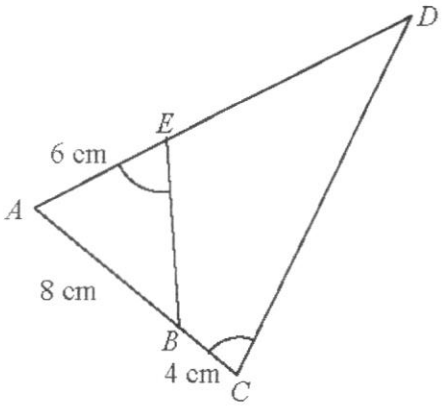
Answer $p =$ ....., $q =$ ....., $r =$ ..... [3]

(ii) Find the highest common factor (HCF) of M and N.

Solutions
HCF = 198

Answer ..... [1]

21 In the diagram below, angle AEB = angle ACD. $AB = 8$ cm, $BC = 4$ cm and $AE = 6$ cm.



(a) Show that triangles AEB and ACD are similar.

Solution
Angle AEB = Angle ACD (given) Angle EAB = Angle CAD (common angle) By AA similarity, triangles AEB and ACD are similar.

Answer .....

.....

..... [2]

(b) Calculate DE.

Solution
$\frac{AD}{AB} = \frac{AC}{AE}$ $\frac{AD}{8} = \frac{12}{6}$ $AD = 2 \times 8 = 16$ $DE = 16 - 6 = 10$ cm

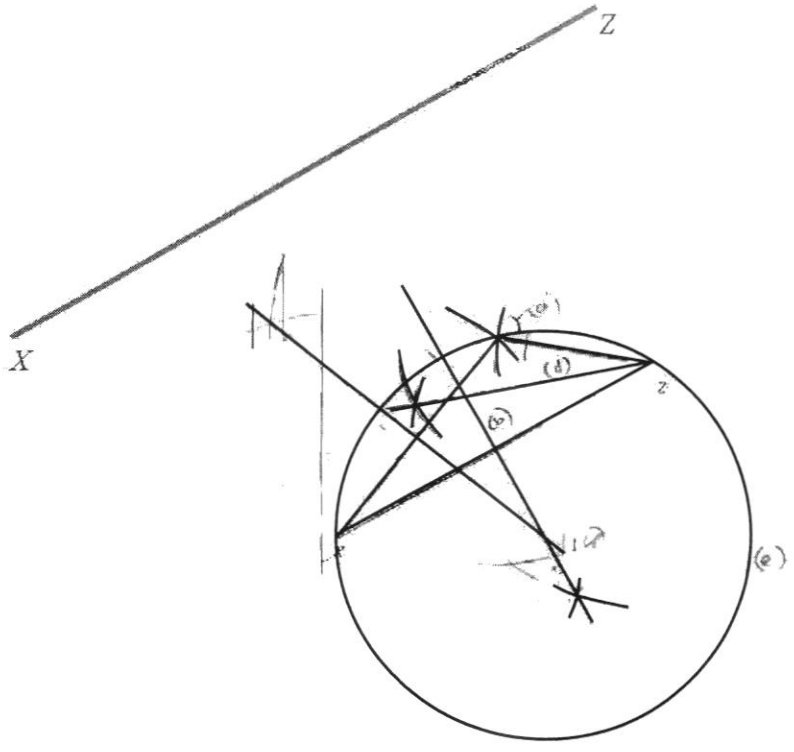
Answer ..... cm [2]

(c) Find the ratio of area of quadrilateral $BCDE$: area of triangle ACD.

Solution
Area ratio of triangles AEB and ACD = 1: 4 Area ratio of quadrilateral $BCDE$ and triangle ACD = 3: 4

Answer ..... : ..... [1]

22 **(a)** Given $XY = 325$ m and $YZ = 200$ m, XZ has been drawn for you.
Complete the scale drawing of a triangular playground XYZ where the given scale is 1: 5000.



[1]

(b) Construct the perpendicular bisector of XZ. [1]

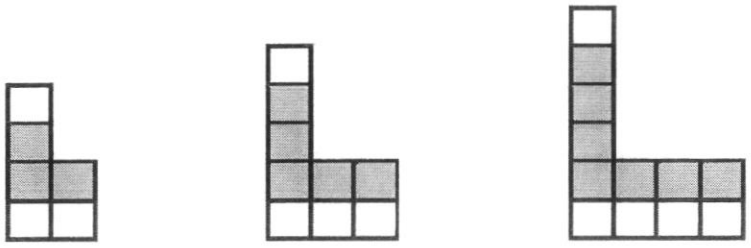
(c) Construct the perpendicular bisector of YX. [1]

(d) Construct the bisector of angle XZY. [1]

(e) A circular fencing is built around the triangular playground XYZ.
Construct a circle touching the three vertices of triangle XYZ. [1]

---

23 The diagram below consists of patterns made up of shaded and unshaded squares.



PATTERN 1 PATTERN 2 PATTERN 3

(a) State the value of m and of n in the table below.

Pattern Number (P)	Number of shaded squares (Q)	Number of unshaded squares (R)
1	3	3
2	5	4
3	7	5
4	m	n

Solution
$m = 9$ $n = 6$

Answer $m =$ ....., $n =$ ..... [1]

(b) Write down an equation connecting P and Q, leaving your answer in its simplest form.

Solution
$Q = 3 + 2(P - 1)$ $Q = 2P + 1$

Answer ..... [1]

(c) Write down an equation connecting P and R, leaving your answer in its simplest form.

Solution
$R = 3 + (P - 1)$ $R = P + 2$

Answer ..... [1]

(d) Decide and justify whether it is possible to make a pattern with exactly 50 squares in total.

Solution
$2P+1+P+2=50$ $3P=47$ $P=15\frac{2}{3}$ Since the value of P is not an integer / a whole number), it is **not possible** to make a pattern with exactly 50 squares in total.

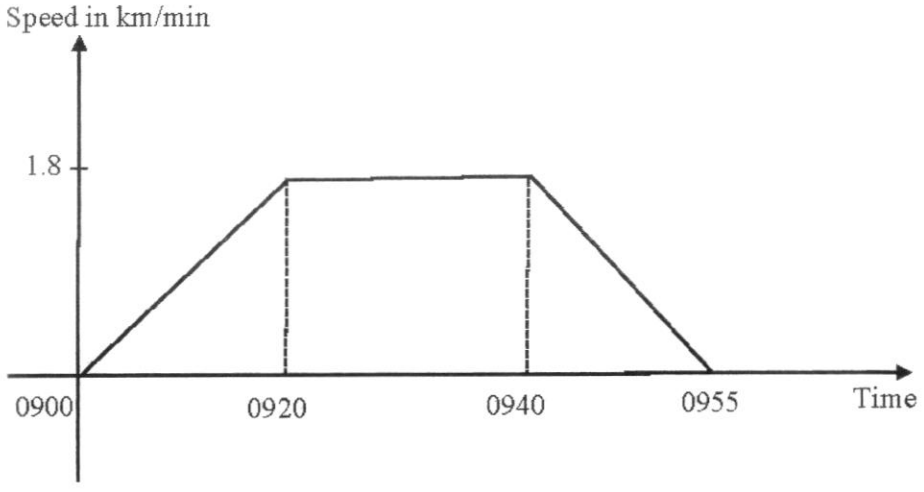
Answer .....

.....

..... [2]

---

24 The diagram shows the speed-time graph of a van. The van starts from rest at O and accelerates uniformly until it reaches 1.8 km/min. It then continues at a constant speed before coming to rest at Q.



(a) Find the deceleration during the last 3 minutes in m s^{-2}.

Solution
$\text{Deceleration} = \dfrac{1.8\times1000}{60\times60\times15}$ $= \dfrac{1}{30} m/s^2$ or 0.0333m/s^2

Answer ..... m s^{-2} [2]

(b) Show that the total distance travelled is 67.5 km.

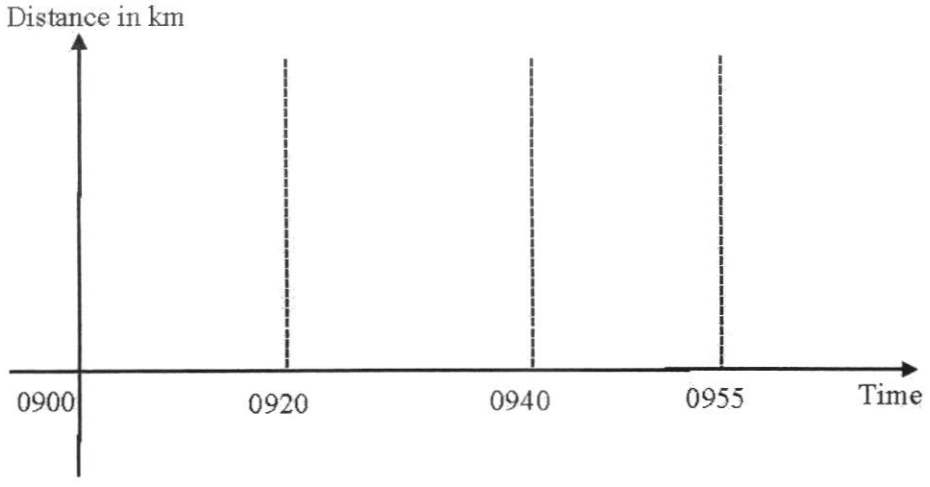
Answer

Solution
Distance travelled
$= \dfrac{1}{2}\times1.8\times20+1.8\times20+\dfrac{1}{2}\times1.8\times15$
$= 18+36+13.5$ km
$=67.5$ km

[2]

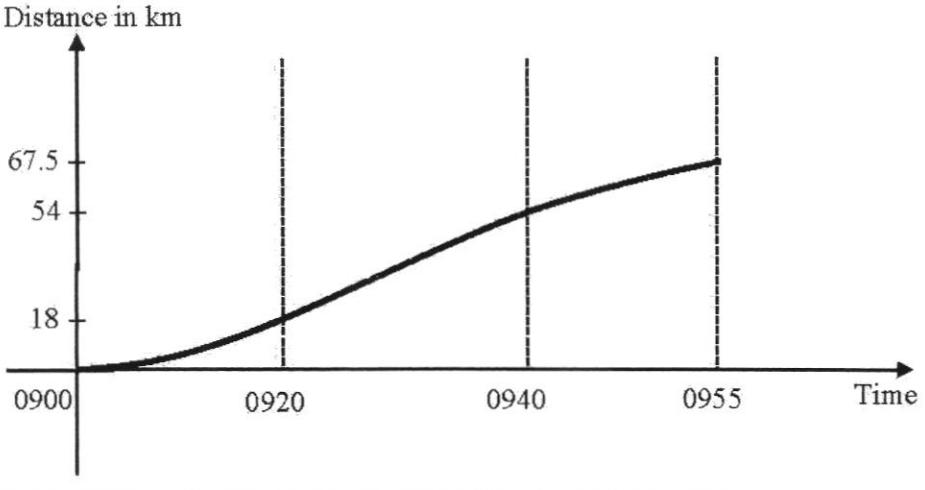
(c) On the axes in the answer space, draw the sketch of the distance-time graph for the whole journey.

Answer



[2]

Solutions

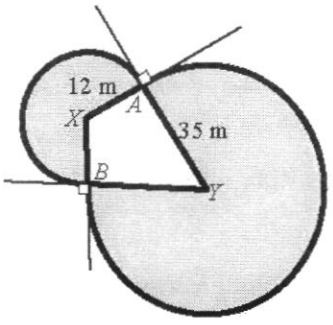


Distance (0900 to 0920) = 18 km

Distance (0920 to 0940) = 20 x 1.8 = 36 km

Distance (0940 to 0955) = $\frac{1}{2} \times 1.8 \times 15 = 13.5$ km

25 The figure below shows the plan view of a farm. It occupies an area in the shape of two overlapping circles with centres X and Y and radii 12 metres and 35 metres respectively. The circles intersect at the points A and B. XA produced is perpendicular to YA produced. XB produced is perpendicular to YB produced.



(a) Show that angle AXB is 2.48 radians, correct to 3 significant figures.

Answer

Solution
$\tan \angle AXY = \frac{35}{12}$
$\angle AXY = 1.2405$ (to 5 s.f.)
$\angle AXB = 2 \times 1.2405$
$= 2.4810$ *
$= 2.48$ (to 3 s.f.)

[2]

This question continues on the next page.

(b) The shaded area shows the area occupied by the sheep.
Calculate the percentage of the farm area that is occupied by the sheep, using your answer from part **(a)**.

Solution

Reflex $\angle AXB = 2\pi - 2.48 = 3.8032$ rad (to 5 s.f.)

$$\angle AYB = 2\pi - \frac{\pi}{2} - \frac{\pi}{2} - 2.48 = \pi - 2.48$$
$$= 0.6616 \text{ rad (to 5 s.f.)}$$

Reflex $\angle AYB = 2\pi - (\pi - 2.48)$
$= \pi + 2.48 = 5.6216$ rad (to 5 s.f.)

Shaded area of small circle

$$= \frac{1}{2}(12)^2(2\pi - 2.48) = 273.83 \text{ m}^2 \text{ (to 5 s.f.)}$$

Shaded area of big circle

$$= \frac{1}{2}(35)^2(\pi + 2.48) = 3443.2 \text{ m}^2 \text{ (to 5 s.f.)}$$

Area of kite $AXBY = 2 \times \left(\frac{1}{2} \times 12 \times 35\right) = 420 \text{ m}^2$

Percentage required

$$= \frac{273.83 + 3443.2}{273.83 + 3443.2 + 420}$$
$= 89.8\%$ (to 3 s.f.)

Answer ..... % [5]

26 (a) The interior angles of a quadrilateral $ABCD$ are given below.

$\angle A = 1.25w°$, $\angle B = (80+w)°$, $\angle C = (2w-10)°$ and $\angle D = 6(w-20)°$

(i) Calculate the value of w.

Solution
$1.25w+80+w+2w-10+6w-120=360$
$10.25w=410$
$w=40$

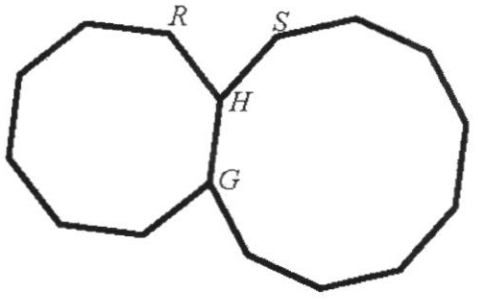
Answer $w =$ ..... [2]

(ii) State the name for quadrilateral $ABCD$.

Solution
$[\angle A=50°, \angle B=120°, \angle C=70°$ and $\angle D=120°]$
Kite

Answer ..... [1]

(b) The diagram below shows a regular decagon and a regular octagon with GH as a common side. The points R and S are joined to form a triangle HRS.



Find angle HRS.

Solution
$\angle RHS =$ Exterior $\angle$ of decagon + Exterior $\angle$ of octagon
$= \frac{360}{10} + \frac{360}{8}$
$= 81°$
$\angle HRS = \frac{180°-81°}{2}$ (base $\angle$s of isosceles Δ)
$= 49.5°$

Answer ..... [3]

END OF PAPER

Name: SOLUTIONS ..... Register no: ..... Class: .....

NGEE ANN SECONDARY SCHOOL

O

PRELIMINARY EXAMINATION

MATHEMATICS **4052/02**

Paper 2 **24 August 2023**

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

For Examiner's Use

Total	**/ 90**

This document consists of **30** printed pages and **0** blank pages.

Answer **all** the questions

1 (a) It is given that $y = 9h^2\left(\frac{x+a}{a-x}\right)$.

(i) Evaluate h when $a = 12$, $x = -6$ and $y = 18.75$.

Solution
$18.75 = 9h^2\left[\frac{-6+12}{12-(-6)}\right]$
$18.75 = 3h^2$
$h^2 = 6.25$
$h = \pm 2.5$
Accept $h = \pm\frac{5}{2}$

Answer $h =$ ..... [2]

(ii) Express a in terms of h, x and y.

Solution
$y = 9h^2\left(\frac{x+a}{a-x}\right)$
$y(a-x) = 9h^2(x+a)$
$ay - xy = 9h^2x + 9h^2a$
$ay - 9h^2a = 9h^2x + xy$
$a = \frac{9h^2x + xy}{y - 9h^2}$

Answer $a =$ ..... [2]

(b) Solve $\dfrac{15x+2}{x^2-5x+6}+\dfrac{2}{x-3}=3$.

Solution
$\dfrac{15x+2}{x^2-5x+6}+\dfrac{2}{x-3}=3$
$\dfrac{15x+2}{(x-2)(x-3)}+\dfrac{2}{x-3}=3$
$\dfrac{15x+2+2(x-2)}{(x-2)(x-3)}=3$
$17x-2=3x^2-15x+18$
$3x^2-32x+20=0$
$(3x-2)(x-10)=0$
$x=\dfrac{2}{3}$ or $x=10$

Answer $x=$ ..... or ..... [3]

2 The table below shows the number of loaves of bread sold at 3 different shops on a weekday.

Shop A sells 32 White, 25 Enriched, 15 Fibre and 12 Whole Meal loaves.
Shop B sells 25 White, 18 Enriched, 18 Fibre and 15 Whole Meal loaves.
Shop C sells 28 White, 22 Enriched, 16 Fibre and 14 Whole Meal loaves.

The price per loaf is \$1.20, \$1.40, \$1.70 and \$2.10 for the White, Enriched, Fibre and Whole Meal loaves respectively.

	White	Enriched	Fibre	Whole Meal
Shop A	32	25	15	12
Shop B	25	18	18	15
Shop C	28	22	16	14
Price per loaf	\$1.20	\$1.40	\$1.70	\$2.10

(a) Write down two matrices **X** and **Y** such that matrix multiplication will give the total amount of money received by each shop on that weekday.

Solution

$$\mathbf{X} = \begin{pmatrix} 32 & 25 & 15 & 12 \\ 25 & 18 & 18 & 15 \\ 28 & 22 & 16 & 14 \end{pmatrix}, \quad \mathbf{Y} = \begin{pmatrix} 1.20 \\ 1.40 \\ 1.70 \\ 2.10 \end{pmatrix}$$

Answer $\mathbf{X} =$, $\mathbf{Y} =$ [2]

(b) Hence evaluate the matrix multiplication **XY**.

Solution

$$\mathbf{XY} = \begin{pmatrix} 32 & 25 & 15 & 12 \\ 25 & 18 & 18 & 15 \\ 28 & 22 & 16 & 14 \end{pmatrix} \begin{pmatrix} 1.20 \\ 1.40 \\ 1.70 \\ 2.10 \end{pmatrix}$$

$$= \begin{pmatrix} 124.10 \\ 117.30 \\ 121.00 \end{pmatrix}$$

Answer $\mathbf{XY} =$ [2]

(c) If discounts of 20%, 10% and 15% are to be given on the purchase of bread from Shop A, B and C respectively, write down a matrix **Z** such that **ZXY** will give the total amount of money received from each of the three shops on that weekday. Hence, find the total amount of money received from <u>each of the three shops</u>, from the selling of bread on that weekday.

Solution

$$\mathbf{Z} = \begin{pmatrix} 0.8 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.85 \end{pmatrix}$$

$$\begin{pmatrix} 0.8 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.85 \end{pmatrix} \begin{pmatrix} 124.10 \\ 117.30 \\ 121.00 \end{pmatrix} = \begin{pmatrix} 99.28 \\ 105.57 \\ 102.85 \end{pmatrix}$$

The total amount received is \$99.28, \$105.57 and \$102.85 from Shop A, B and C respectively.

Answer Shop A \$.....

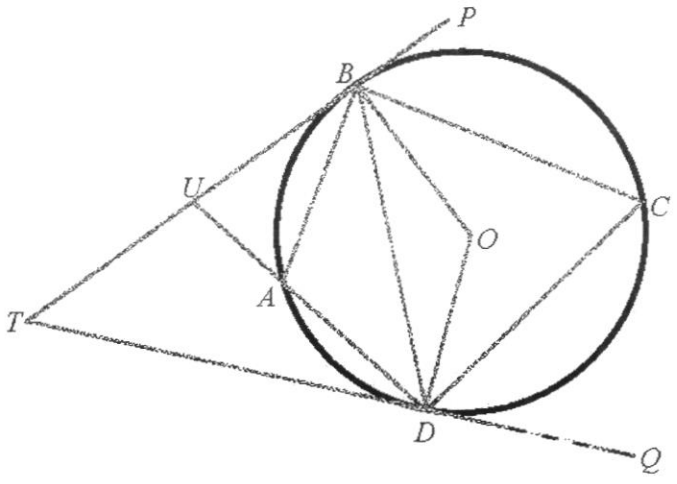
Shop B \$.....

Shop C \$..... [2]

(d) Given that matrix **M** = (1 1 1), explain what the matrix **MZXY** represents.

Answer It represents the **<u>total amount received</u>** by **<u>all three shops</u>** ..... **<u>on the day of the discount</u>**. .... [1]

3 In the diagram, O is the centre of the circle $ABCD$. TBP and TDQ are tangents to the circle. It is given that $\angle BTD = 36^0$ and $AB = AD$.



(a) Find $\angle BCD$.
Give a reason for each step of your working.

Solution
$\angle TBO = \angle TDO = 90^0$ (radius perpendicular tangent)
$\therefore \angle BOD = 360^0 - 90^0 - 90^0 - 36^0 = 144^0$ ($\angle$s sum of quadrilateral)
$\angle BCD = \frac{144^0}{2} = 72^0$ (angle at centre = twice angle at circumference)

Answer ..... [3]

(b) Find $\angle ABO$.
Give a reason for each step of your working.

Solution
$\angle BAD = 180^0 - 72^0 = 108^0$ (angles in opposite segments are supplementary)
$\angle ABD = \frac{180^0 - 108^0}{2} = 36^0$ (base angles of isosceles triangle)
$\angle ODB = \frac{180^0 - 144^0}{2} = 18^0$ (base angles of isosceles triangle)
$\angle ABO = 18^0 + 36^0 = 54^0$

Answer ..... [3]

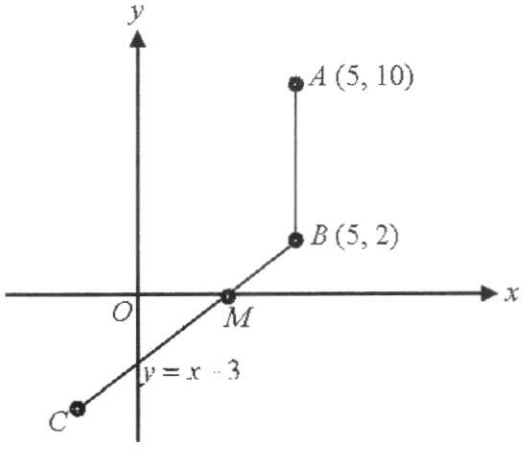
(c) Show that AB bisects $\angle UBD$.

Answer

Solution
$\angle UBD = \frac{180^0 - 36^0}{2} = 72^0$ (tangents from external point)
$\angle ABD = 36^0 = \frac{1}{2}\angle UBD$
$\therefore AB$ bisects $\angle UBD$

[2]

4 The diagram shows a sketch of two straight lines BMC and AB where point A is (5, 10) and point B is (5, 2). It is given that the equation of BC is $y = x - 3$ and that $BC = 4BM$.



(a) Find the coordinates of C.

Solution
When $y = 0$, $x - 3 = 0$ $x = 3$ M is (3, 0) Given $BC = 4BM$, $MC = 3BM$ Coordinates of C $= (3 - 6, 0 - 6)$ $= (-3, -6)$

Answer (....., ...) [2]

(b) D is the point where AM produced meets the y-axis.

Find the length of the line AD.

Solution
Gradient of $AM = \frac{10}{2} = 5$ $y = 5x + c$ $10 = 5(5) + c$ $c = -15$ Equation of AM is $y = 5x - 15$. Point D is $(0, -15)$. Length of AD $= \sqrt{(5-0)^2 + (10+15)^2}$ $= 25.5$ units (to 3 s.f.)

Answer ..... [4]

(c) $ACBE$ is a parallelogram. Calculate its area.

Solution
Area of $ACBD$ $= 2\times$ Area of ΔACB $= 2 \times \frac{1}{2} \times 8 \times 8$ $= 64$ units^2

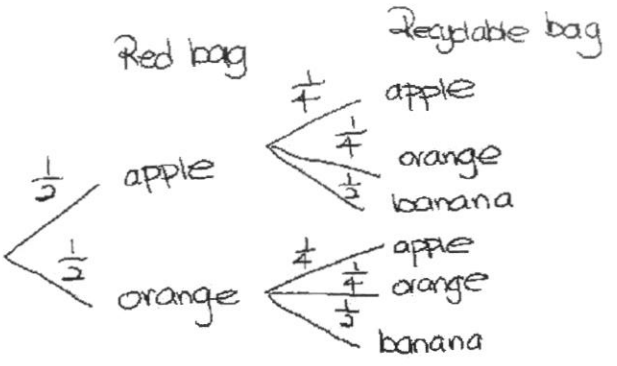
Answer ..... [2]

5 (a) A red plastic bag contains 2 apples and 2 oranges.
The recyclable bag contains 1 apple, 1 orange and 2 bananas.
Matthew picks one fruit from each bag.

(i) Draw a tree diagram to show the probabilities of the possible outcomes.

Answer

Solution



[2]

(ii) Find, as a fraction in its simplest form, the probability that 2 apples are selected.

Solution

$$\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$$

Answer ..... [1]

(iii) Find, as a fraction in its simplest form, the probability that different fruits are selected.

Solution

$$\left(\frac{1}{2}\times\frac{3}{4}\right)+\left(\frac{1}{2}\times\frac{3}{4}\right)=\frac{3}{4}$$

Answer ..... [1]

(iv) Find, as a fraction in its simplest form, the probability that at least one orange is selected.

Solution
$1-\left(\frac{1}{2}\times\frac{3}{4}\right)=\frac{5}{8}$

Answer ..... [1]

(b) The fruit distribution company employs 15 workers.
One of the 15 workers is selected at random.
The probability that it is a woman who is working part time is $\frac{1}{5}$.
Two of the 15 workers are selected at random.
The probability that they are both men who are working full time is $\frac{1}{5}$.
Complete the table of information about the 15 workers in the company.

Answer

	Part time workers	Full time worker
Men		
Women		4

Solution

	Part time workers	Full time worker
Men	1	7
Women	3	4

Let the number of part time woman worker be y.

$\frac{y}{15}=\frac{1}{5} \quad \Rightarrow \quad y=3$

Let the number of full-time man workers be x.

$\frac{x}{15}\times\frac{x-1}{14}=\frac{1}{5}$

$x(x-1)=42$

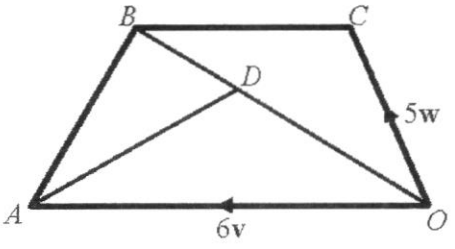
$x^2-x-42=0$

$(x-7)(x+6)=0$

$x=7$ or $x=-6$ (rejected)

[4]

6 In the diagram, $\overrightarrow{OA} = 6\mathbf{v}$, $\overrightarrow{OC} = 5\mathbf{w}$, $CB = \frac{2}{3}OA$ and $5OD = 3OB$.



Deduct 1 mark overall for any missing under thilde.

(a) Express each of the following, as simple as possible, in terms of v and/or w.

(i) $\overrightarrow{OB}$

Solution
$\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB}$
$= 5\mathbf{w} + \frac{2}{3}\overrightarrow{OA}$
$= 5\mathbf{w} + \frac{2}{3}(6\mathbf{v})$
$= 5\mathbf{w} + 4\mathbf{v}$

Answer ..... [1]

(ii) $\overrightarrow{OD}$

Solution
$\overrightarrow{OD} = \frac{3}{5}\overrightarrow{OB}$
$= \frac{3}{5}(5\mathbf{w} + 4\mathbf{v})$
$= 3\mathbf{w} + \frac{12}{5}\mathbf{v}$

Answer ..... [1]

(iii) $\overrightarrow{AD}$

Solution
$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$ $= 3\mathbf{w} + \frac{12}{5}\mathbf{v} - 6\mathbf{v}$ $= 3\mathbf{w} - \frac{18}{5}\mathbf{v}$

Answer ..... [1]

(iv) $\dot{DC}$

Solution
$\dot{DC} = \dot{OC} - \dot{OD}$ $= 5\mathbf{w} - \left(3\mathbf{w} + \frac{12}{5}\mathbf{v}\right)$ $= 2\mathbf{w} - \frac{12}{5}\mathbf{v}$

Answer ..... [1]

(b) Use vectors to show whether A, D and C are collinear.

Answer

Solution
$\overrightarrow{AD} = 3\mathbf{w} - \frac{18}{5}\mathbf{v} = \frac{3}{2}\left(2\mathbf{w} - \frac{12}{5}\mathbf{v}\right)$ $\overrightarrow{DC} = 2\mathbf{w} - \frac{12}{5}\mathbf{v}$ Since $\overrightarrow{AD} = \frac{3}{2}\overrightarrow{DC}$, A, D and C are collinear.

[2]

(c) Find the numerical value of

(i) $\dfrac{\text{area of } \Delta DOA}{\text{area of } \Delta DAB}$,

Solution
$\dfrac{\text{area of } \Delta DOA}{\text{area of } \Delta DAB} = \dfrac{3}{2}$

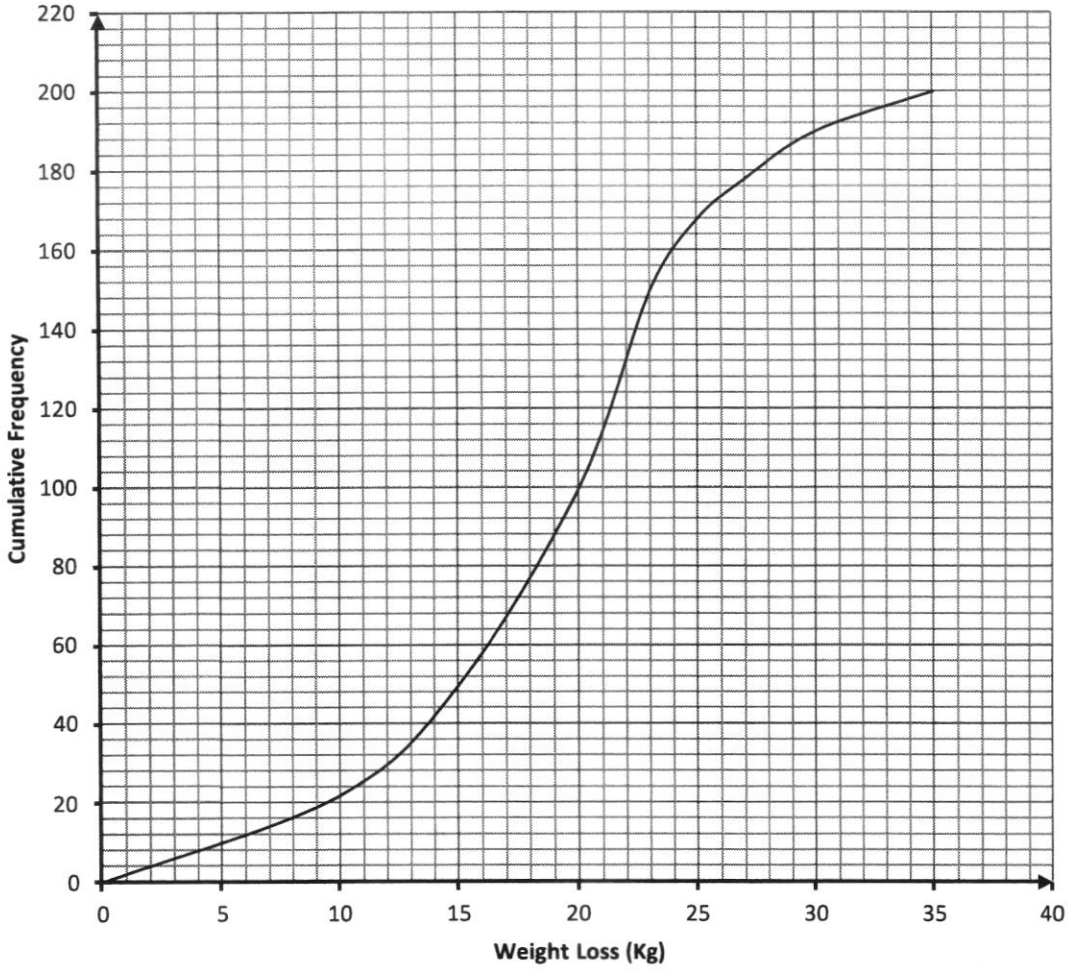
Answer ..... [1]

(ii) $\dfrac{\text{area of } \Delta DBC}{\text{area of } \Delta ABO}$.

Solution
Since ΔDBC and ΔDOA are similar,
$\dfrac{\text{area of } \Delta DBC}{\text{area of } \Delta DOA} = \left(\dfrac{CB}{OA}\right)^2$
$= \left(\dfrac{2}{3}\right)^2$
$= \dfrac{4}{9}$
$\dfrac{\text{area of } \Delta DOA}{\text{area of } \Delta AOB} = \dfrac{3}{5}$
$\dfrac{\text{area of } \Delta DBC}{\text{area of } \Delta ABO} = \dfrac{\text{area of } \Delta DBC}{\text{area of } \Delta DOA} \times \dfrac{\text{area of } \Delta DOA}{\text{area of } \Delta AOB}$
$= \dfrac{4}{9} \times \dfrac{3}{5}$
$= \dfrac{4}{15}$

Answer ..... [2]

7 The cumulative frequency curve below shows the amount of weight loss, measured to the nearest kg, of 200 men in Alpha Centre over 2 years.



The corresponding cumulative frequency table for this distribution is as shown below.

Weight loss (x kg)	≤ 10	≤ 15	≤ 20	≤ 25	≤ 30	≤ 35
Cumulative Frequency	22	p	100	168	190	200

(a) Calculate p.

Solution
$p = 50$

Answer $p =$ ..... [1]

(b) Use the curve to estimate

(i) the median weight loss,

Solution
20 kg

Answer ..... kg [1]

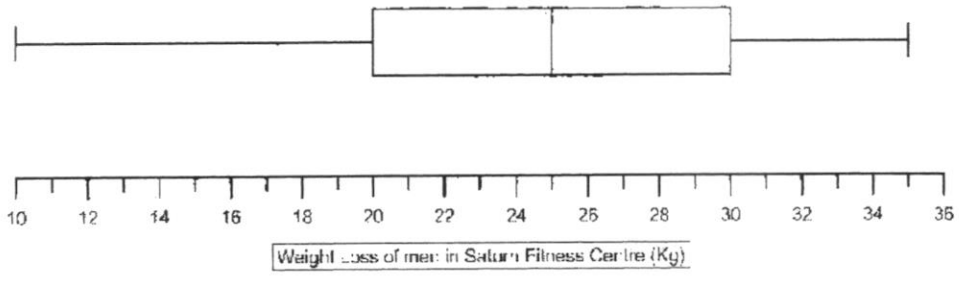
(ii) the interquartile range of the weight loss,

Solution
23 – 15 =8 kg

Answer ..... kg [2]

(c) A second fitness centre, Saturn Fitness Centre also measured the amount of weight loss by their 240 male members over 2 years.

The box-and-whisker diagram below illustrates their weight loss achieved.



(i) Find the interquartile range of the weight loss.

Solution
30 – 20 =10 kg

Answer ..... kg [2]

(ii) Find the number of men who achieved a weight loss of more than 30 kg over 2 years.

Solution
$240 \div 4 = 60$

Answer ..... [1]

(d) Using the information from parts **(b)** and **(c)**, make two comments comparing the weight loss of the men in Alpha Centre and Saturn Fitness Centre.

Solution
Men in Saturn Fitness Centre lost more weight as the median weight loss for men in Saturn Fitness Centre (25 kg) is greater than the median weight loss for men in Alpha Centre (20 kg). The weight loss for men in Alpha Centre is more consistent/has a lesser spread as the interquartile range of the weight loss for Alpha Centre (8 kg) is lesser than the interquartile range of the weight loss for Saturn Fitness Centre (10 kg).

Answer .....

.....

.....

..... [2]

8 **(a)** Complete the table of values for $y = \frac{3}{x^2} + x - 2$.

x	0.5	1	1.5	2	2.5	3	4
y	10.5	2	*0.83*	0.75	0.98	1.33	2.19

[1]

(b) On the grid opposite, draw the graph of $y = \frac{3}{x^2} + x - 2$ for $0 \leq x \leq 4$. [3]

(c) Use your graph to find the least value of y.

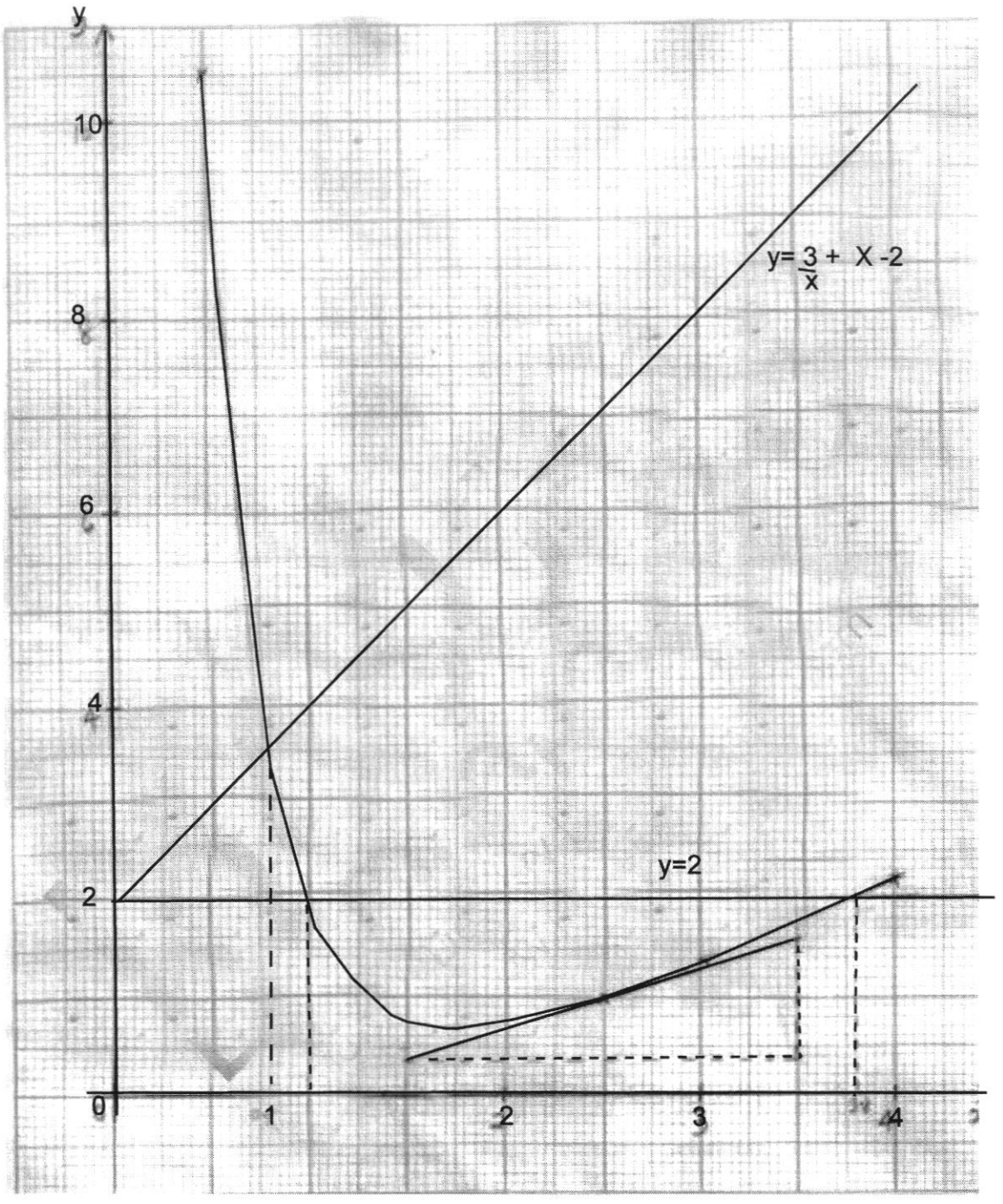
Solution	
Least value of $y = 0.7$	Accept $0.65 \leq y \leq 0.7$

Answer ..... [1]

(d) By drawing a tangent, find the gradient of the curve at $x = 2.5$.

Solution	
Gradient at $x = 2.5$ is , $= \frac{1.6 - 0.35}{3.5 - 1.5}$ $= 0.625$	Accept 0.610 – 0.640

Answer ..... [2]


y
10
8
6
4
2
0
1
2
3
4
y= 3/x + X -2
y=2

(e) On the same graph, draw a suitable straight line to solve the equation

$$\frac{3}{x^2} - x - 4 = 0.$$

Solution
$\frac{3}{x^2} - x - 4 = 0$
$\frac{3}{x^2} - x + 2x - 2 - 2 = 2x$
$\frac{3}{x^2} + x - 2 = 2x + 2$
Draw the line $y = 2x + 2$.

x	0	2	4
y	2	6	10

From the graph, $x = 0.8$.

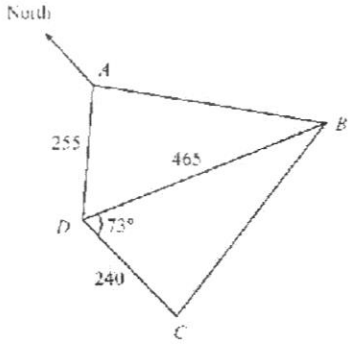
Answer $x =$ ..... [2]

(f) Use your graph to find the range of values of x for which $\frac{3}{x^2} + x \leq 4$.

Solution
$\frac{3}{x^2} + x \leq 4$
$\frac{3}{x^2} + x - 2 \leq 2$
$y \leq 2$
From the graph, the solution is $1 \leq x \leq 3.8$.

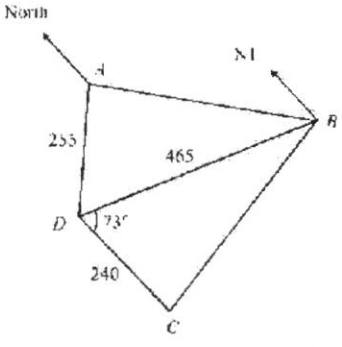
Answer ..... [2]

9 (a) $ABCD$ represents a farm on a horizontal field.
$AD = 255$ m, $BD = 465$ m and $CD = 240$ m.
The bearing of B from A is $140°$ and the bearing of D from A is $224°$.
Angle $BDC = 73°$.



(i) Calculate the bearing of D from B.

Solution



Let N1 denote the north of B.
$\angle ABN1 = 180° - 140° = 40°$
$\angle BAD = 224° - 140° = 84°$

By sine rule,

$$\frac{\sin \angle ABD}{255} = \frac{\sin 84°}{465}$$

$\angle ABD = 33.051°$ (3 d.p.)
Bearing $= 360° - 40° - 33.051° = 286.949°$
Bearing $= 286.9°$ (1 d.p.)

Answer ..... [3]

(ii) A farmer walks from point D to point C.

Find the shortest distance of the farmer from B during his journey.

Solution
Shortest distance be d.
$\sin 73° = \frac{d}{465}$
$d = 445$ m (3 s.f.)

Answer ..... m [2]

(iii) The farmer is given a budget of \$57000 to erect electrical barricades around part of the farm, triangle BCD.

Each 5m barricade is sold at \$245 each.

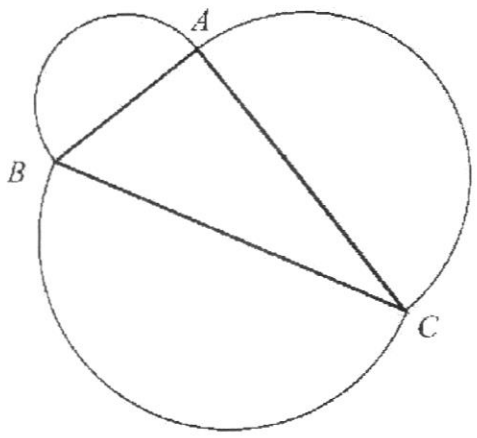
Determine whether he can complete the job within his budget.
Justify your answer with workings.

Answer

Solution
$BC^2 = 465^2 + 240^2 - 2(465)(240)\cos 73°$
$BC = 456.692$ (6 s.f.)
Total no of barricades needed $= \frac{456.692}{5} + \frac{465}{5} + \frac{240}{5}$
$= 232.3384$ $= 233$ (rounded up)
Total cost $= 233 \times \$245 = \$57085 > \$57000$
Since the total cost exceeds the budget, he is **not** able to complete the job within his budget.

[4]

(b)



Three semicircles are drawn using the sides of a triangle ABC as their diameters.

It is given that the areas of the semicircles are in the ratio 25: 144: 169.

Explain why triangle ABC is right-angled.

Answer

Solution
Since the ratios of their areas is 25: 144: 169, the ratios of their radii will be 5:12: 13. Their diameters will be in the form 5x, 12x and 13x respectively. That is, $AB = 5x$, $AC = 12x$, and $BC = 13x$. $AB^2 + AC^2 = (5x)^2 + (12x)^2 = 169x^2$ $BC^2 = (13x)^2 = 169x^2$ Since $\boldsymbol{AB^2 + AC^2 = BC^2}$, **by converse of the Pythagoras' theorem**, ABC is a right-angled triangle. *Award max 2 marks if student use AB = 5cm, AC = 12 cm, and BC = 13 cm.

[4]

10 Martin is trying to build a small dam at a river.
In the event of heavy rain, the dam restricts the flow of the water to prevent a flood.

Figure 1 shows a map of the river, drawn to scale.
He hopes to build the dam at the narrowest part of the river.

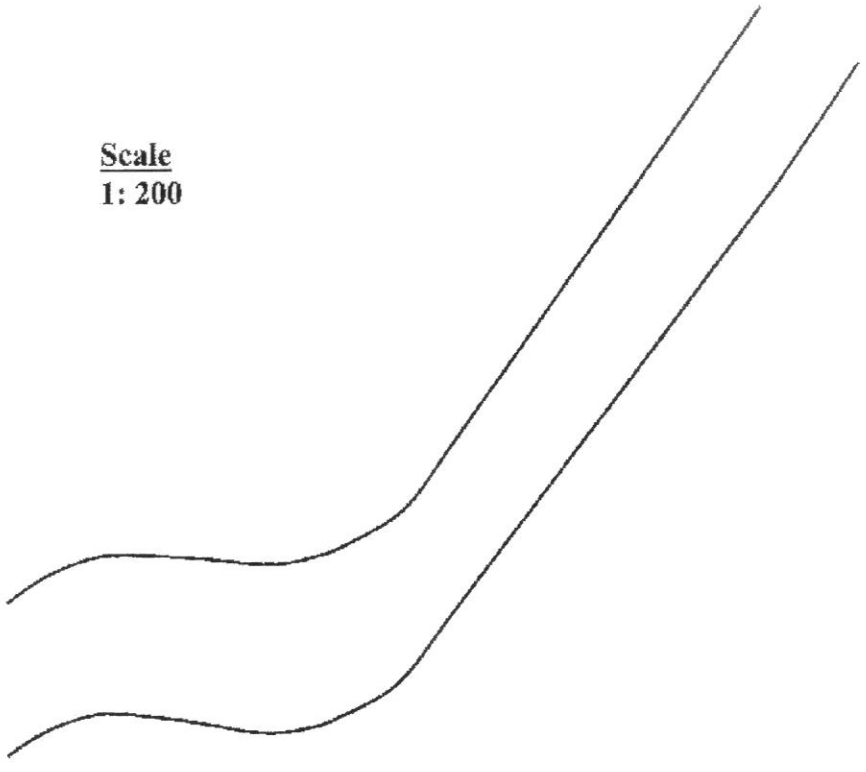


Figure 1 Map of river (drawn to scale)

To better design the dam, Martin measured the depth of the river at 10 different locations during a rainy day. The data obtained is presented in **Figure 2.**

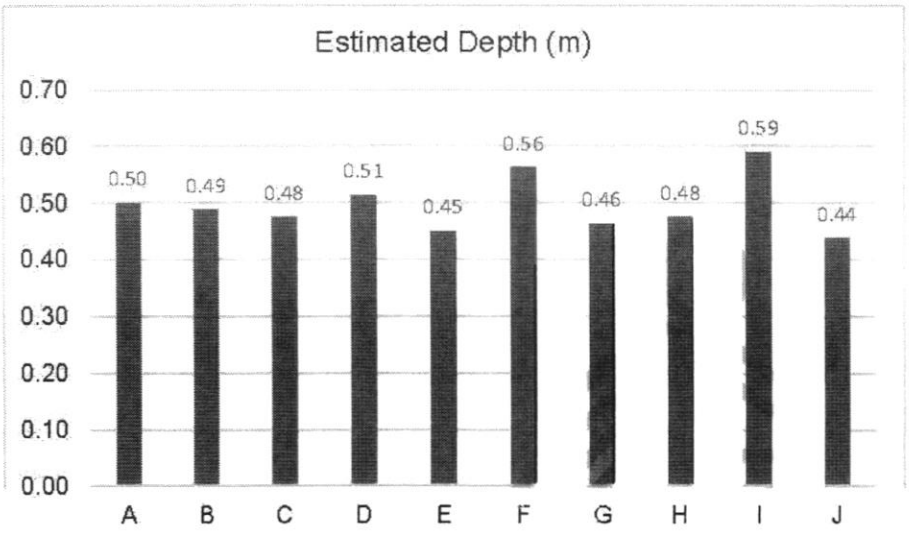


Figure 2

One possible prototype of the dam is shown below in **Figure 3**. The base of the dam is designed to be thicker to account for the greater pressure exerted by the water. The dam has height h m and a length l m and the inclined part of the dam makes an angle of $\theta°$ with the horizontal. The width of the dam, w m, should cover the width of the river completely.

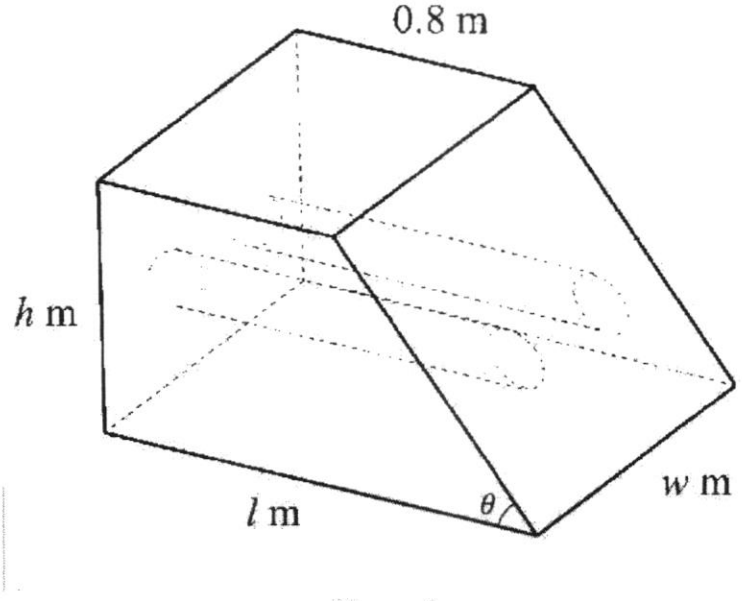


Figure 3

Other design specifications:

- The height of the dam, h m, should be 10% greater than the depth of the water at the location it is built.
- The length of the dam, l m, should be as small as possible and satisfy the inequality

$$\min(2h, w) \leq l \leq 3.$$

[The minimum function **min** gives the smaller value of the two numbers.
For e.g., min(2, 5) = 2]

- The total volume of the dam should exceed 1.5 m^3 for the dam to be operating safely.

(a) (i) Find the actual width of the narrowest part of the river.

Solution
Measure $w = 1.4\text{cm} \pm 0.05$ cm $w = 1.4 \times \frac{200}{100}$ $w = 2.8$ m Accept 2.7m, 2.8m or 2.9 m.

Answer ..... m [1]

(ii) Calculate a suitable value of θ, leaving your answers corrected to 4 decimal places.

Solution
Decide on depth = ~ 0.59 m (note that this is the only acceptable answer to ensure that the water doesn't overflow even with 10% greater) $h = \frac{110}{100} \times 0.59$ $= 0.649$ m $\min(0.649 \times 2, 2.8) = 1.298$ A suitable length for l is 1.298 m. $\theta = \tan^{-1}\left(\frac{0.649}{1.298-0.8}\right)$ $= 52.49973°$ (6 d.p.) $= 52.4997°$ (4 d.p.)

Answer $\theta =$ ..... [3]

(b) Two holes of radius r are drilled through the length of the dam to allow some water to flow through the dam. The hollowed part resembles a truncated cylinder with circular base as shown in **Figure 4**.

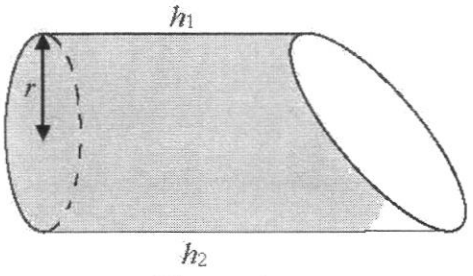


Figure 4

For a truncated cylinder, the volume is given by:

$$V = \frac{1}{2}(h_1 + h_2)\pi r^2$$

Information about the radii of the holes and their corresponding dimensions is shown in the table below.

r (cm)	h_1 (cm)	h_2 (cm)
10	97	113
15	93	116
20	90	120
25	86	124
30	82	128

To ensure a certain rate of water flow, the shaded base area of each hole should be between 700 cm^2 to 1000 cm^2.

This question continues on the next page.

Suggest a suitable radius for the holes to be drilled such that the dam can be operated safely according to all the specifications provided. Justify your decision and show your calculations clearly.

Answer

Solution
Volume of the prism (before drilling) $V = \frac{1}{2}(0.8 + l) \times h \times w$ $V = \frac{1}{2}(0.8 + 1.298) \times 0.649 \times 2.8$ $= 1.9062\ m^3$ (5 s.f.) <u>Volume of truncated cylinder</u> Choose $r = 15$ cm For $r = 15$, $A = \pi(15)^2 = 706.86\ \text{cm}^2$ For $r = 20$, $A = \pi(20)^2 = 1256.6\ \text{cm}^2$ (reject) $V = \frac{1}{2}(h_1 + h_2) \times \pi \times r^2$ $V = \frac{1}{2}(0.93 + 1.16) \times \pi \times 0.15^2$ $V = 0.073867\ \text{m}^3$ $Total\ volume = 1.9062 - 2 \times 0.073867$ $= 1.76\ m^3$(3 s.f.) Since $\mathbf{1.76\ m^3 > 1.5\ m^3}$, the prototype can be operated safely when $r = 15$ cm.

[5]

END OF PAPER