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Geylang Methodist School (Secondary) Preliminary Examination 2024

Candidate
Name

Class

Index
Number

MATHEMATICS

4052/01

Paper 1

4 Express
5 Normal (Academic)

Candidates answer on the Question Paper.

2 hours 15 minutes

Setter: Ms Nainee Ismail

Monday, 5 August 2024

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.

For Examiner's Use

90

This document consists of 20 printed pages including the cover page.

[Turn Over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f} \right)^2}$$

Answer **all** the questions.

- 1 Expand and simplify $(4x - y)(3x + 4y)$.

Answer [2]

- 2 (a) Find the lowest common multiple (LCM) of 108 and 140.

Answer [1]

- (b) Find the highest common factor (HCF) of 108 and 140.

Answer [1]

- 3 Solve $4 - \frac{1}{3}x = 2$.

Answer $x =$ [1]

4 10 23 34 40 25 35 17 44 23

- (a) Find the median of the set of numbers.

Answer [1]

- (b) Find the range of the set of numbers.

Answer [1]

- 5 (a) A hotel made breakfast milkshake for its guests.
A mixture of mango juice, milk and yogurt in the ratio 7 : 5 : 3 are mixed together.
2.5 litres of milk is used in the mixture.

- (i) How much yogurt is used in the milkshake?

Answer litres [1]

- (ii) How much breakfast milkshake is made?

Answer litres [1]

- (b) Another milkshake is made using apple juice, peach juice and milk.
The ratio of apple juice : milk = 2 : 3.
The ratio of milk : peach juice = 5 : 4.

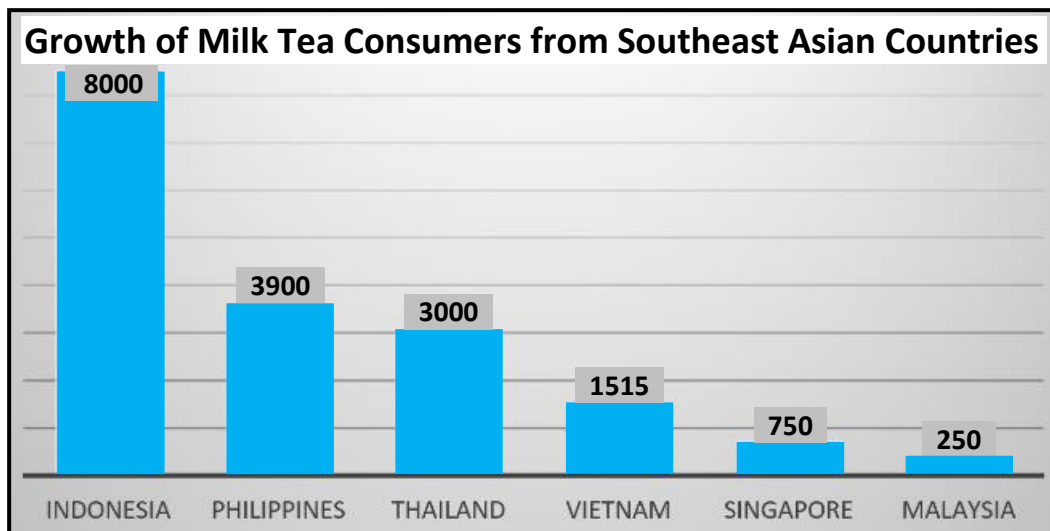
Find the ratio of apple juice : milk : peach juice.

Answer : : [1]

6 Find the equation of the straight line passing through $(7, -7)$ and $(-4, 15)$.

Answer [3]

7 The graph shows the growth in milk tea consumers regionally.



Adapted from source: <https://www.mdpi.com/>

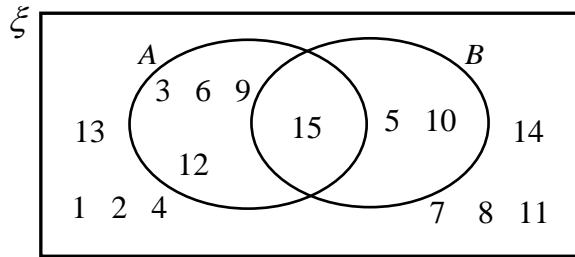
(a) State one misleading feature of the graph.

.....
 [1]

(b) Based on your answer in **part (a)**, explain how this feature affects the reader's interpretation of the graph.

.....
 [1]

8 The Venn diagram shows the elements of $\xi = \{\text{integers } x: 1 \leq x \leq 15\}$ and two sets A and B .



(a) Use one of the symbols below to complete each statement.

$$\emptyset \subseteq \in \cap \not\subseteq$$

(i) $\{ 15 \}$ $A \cap B$ [1]

(ii) 1 $(A \cup B)'$ [1]

(b) Suggest a description to define set A .

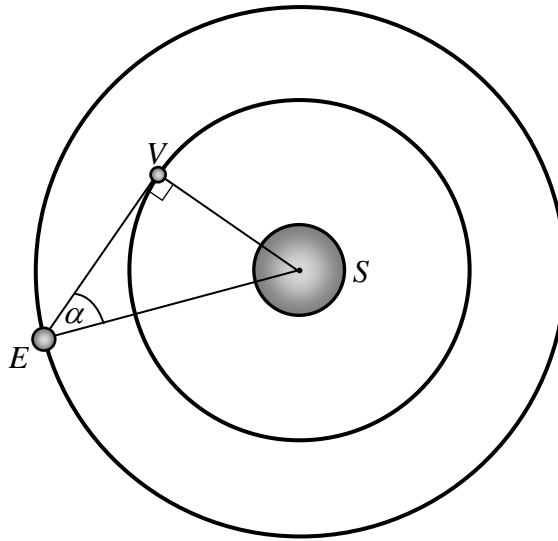
.....

..... [1]

9 The number of social media users globally grew from 4.72 billion in January 2023 to 5.04 billion in January 2024. The number increased by $r\%$ every month during that period. Find the value of r . (1 billion = 1×10^9)

Answer $r =$ % [2]

- 10** The following diagram shows the positions of Earth, E , and Venus, V , and their orbit around the Sun, S . The radius of Venus' orbit is 1.082×10^8 km.
[Follow the degree of accuracy of the values specified in this question and leave your answers in standard form where necessary.]



- (a) If the angle α is 46.054° , calculate the distance between Earth and the Sun.

Answer km [1]

- (b) State the value of α when Earth is furthest from Venus and calculate this distance.

Answer $\alpha =$

..... km [2]

11 Simplify $\frac{x^3 - 4x}{2x^2 - 7x + 6}$.

Answer [3]

- 12 (a) A box is said to contain blue marbles, red marbles and yellow marbles.
 A marble is picked at random from the box.
 The probability that the marble is blue is 0.27.
 The probability that the marble is red is 0.73.

Deduce if there are any yellow marbles in the box, showing the calculations clearly.

.....
 [2]

- (b) Another box contains 10 green marbles, 8 orange marbles and 3 purple marbles.
 The probability of picking a purple marble is $\frac{1}{4}$ when x purple marbles are added in.
 Find the total number of purple marbles in this box.

Answer [2]

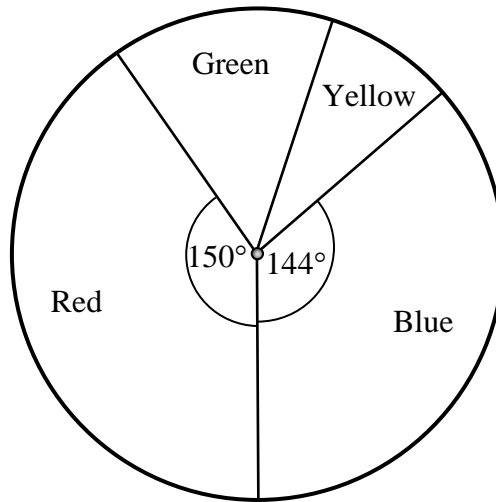
13 Factorise completely $8 - 2x - 24y + 6xy$.

Answer [3]

14 A container of oil is 65% full. 20% of the oil is used for cooking.
15.6 litres of oil is left in the container.
If 14.8 litres of oil is then poured into the container, determine whether there will be a spill.
Show your working.

.....
..... [4]

- 15 A survey is conducted on the number of students in Red, Green, Yellow and Blue House who participated in the Inter-House Games. The results are shown as a pie chart.



There are twice as many students that are from Green House as compared to Yellow House. Explain why this information may not be useful to find the number of students from the Red and Blue House who participated in the Inter-House Games.

.....
 [1]

- 16 The Marina Coastal Expressway (MCE) tunnel is about 3.6 km long. The speed limit in expressway tunnels is 70 km/h. Find the shortest time possible for a car of length 4.8 m to pass through the MCE tunnel **completely**, within the speed regulations. Give your answer in minutes and seconds (nearest whole number).

Answer minutes seconds [3]

17 $N = \frac{a^6}{b^3}$ where a and b are prime numbers.

(a) Explain why N is a perfect cube.

.....
 [1]

(b) Show that M is divisible by 6, given that $\frac{MN}{2} = (3ab)^2$ and M is a natural number.

Answer

[2]

18 The expression $2 + px - x^2$ can be written in the form $q - (x - 3)^2$.

(a) Find the value of p and the value of q .

Answer $p = \dots\dots\dots$

$q = \dots\dots\dots$ [2]

(b) Explain why when $x = 3$, the expression has its maximum value.

.....
 [1]

- 19 (a)** The force, F Newtons, between any two particles in the universe is inversely proportional to the square of the distance, r metres, between them.
It is known that $F = 100$ Newtons for a particular value r .
Find the value of F when this value of r is doubled.

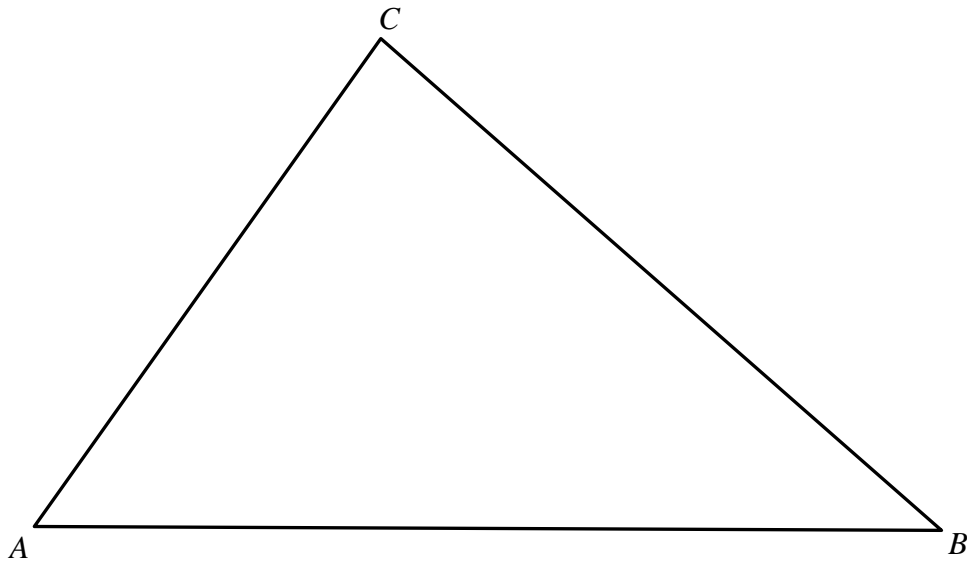
Answer Newtons [2]

- (b)** y is proportional to x^3 .
Explain the effect on the value of x in order for y to be 2700% of its original value.

.....

..... [1]

- 20** The diagram shows an eco-garden in a school in the shape of triangle ABC .
The diagram is drawn to a scale of 1 : 1000.



- (a) Construct the perpendicular bisector of AB . [1]
- (b) Construct the bisector of angle ABC . [1]
- (c) A flagpole with the school flag is erected at a point P such that it is equidistant from A and B **and** equidistant from AB and BC .
Mark the point P on the diagram and measure the actual length of CP .

Answer $CP = \dots\dots\dots$ m [1]

- (d) A tiled path is to be constructed in the eco-garden, where each tile is equidistant from P . The tiled path also meets the line AB and BC at points X and Y respectively such that $BX = BY$. Describe the shape of the tiled path and find its actual length.

Answer Shape of tiled path: $\dots\dots\dots$

Length of tiled path = $\dots\dots\dots$ m [2]

21 (a) $\frac{2^a \times (\sqrt[3]{64})^c}{16^b} = 1.$

Find an expression for a in terms of b and c .

Answer $a = \dots\dots\dots$ [2]

(b) Simplify $\left(\frac{27y^{12}}{x^6}\right)^{\frac{1}{3}}.$

Answer $\dots\dots\dots$ [2]

22 The table shows the distribution of weight of 40 students in a class.

Weight (x kg)	Frequency
$40 < x \leq 50$	2
$50 < x \leq 60$	13
$60 < x \leq 70$	20
$70 < x \leq 80$	5

(a) Calculate an estimate for

(i) the mean weight of the students,

Answer kg [1]

(ii) the standard deviation of the weight.

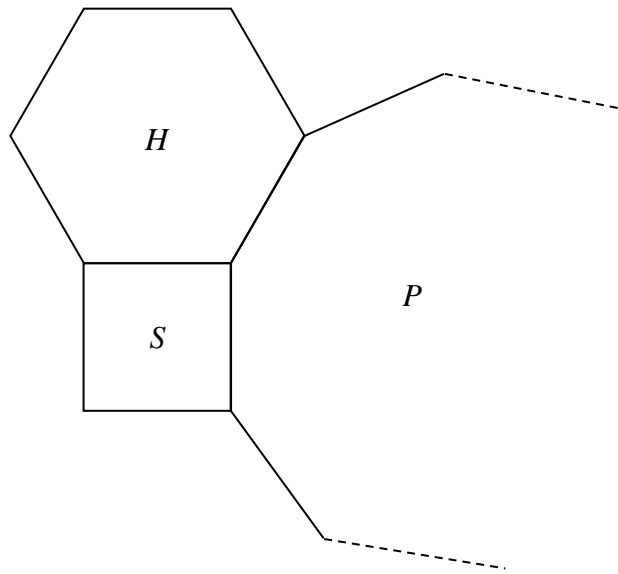
Answer kg [1]

(b) Due to a fault in the weighing machine, the weight recorded was 2 kg lighter than the actual weight. Explain how this may have affected the mean and standard deviation and state its correct value.

.....

..... [2]

- 23 The diagram below shows a square S , a regular hexagon H and a part of a regular n – sided polygon P .



Find the number of sides, n , of the regular polygon P .

Answer $n = \dots\dots\dots$ [4]

24 (a) The first five terms of a sequence are 2, 7, 12, 17 and 22.

- (i) T_n is the n th term of the sequence.
Find an expression, in terms of n for T_n .

Answer $T_n = \dots\dots\dots$ [1]

- (ii) The sum of the first n terms of this sequence is given by $an^2 + bn$.

When $n = 1$, $a + b = 2$.

Show that $4a + 2b = 9$.

Answer

[1]

- (iii) Solve the equations from **part (ii)**.

$$\begin{aligned} a + b &= 2, \\ 4a + 2b &= 9. \end{aligned}$$

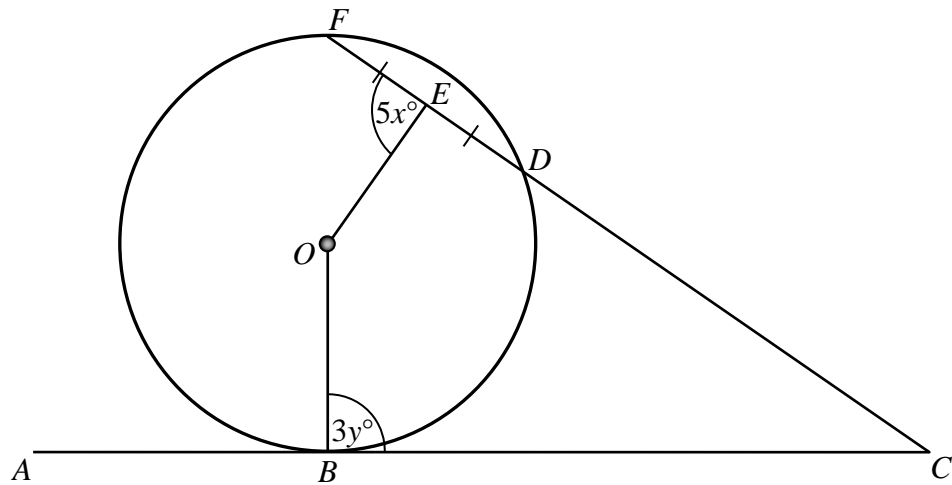
Answer $a = \dots\dots\dots$

$b = \dots\dots\dots$ [2]

- (b) The sum of the first n terms of a different sequence is given by $5n^2 + 3n$.
Find the 11th term of this sequence.

Answer $\dots\dots\dots$ [2]

- 25 In the diagram, B , D and F lie on a circle, centre O .
 AC is a tangent to the circle at B , $DE = FE$, angle $OEF = 5x^\circ$ and angle $OBC = 3y^\circ$.



- (a) A statement was made that $BC = CE$.
 Do you agree? Give reasons for your answer.

.....
 [1]

- (b) Complete these statements.

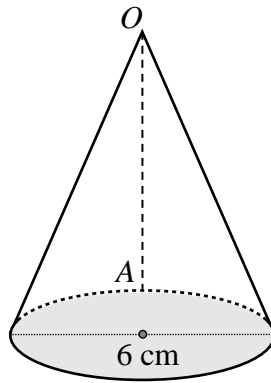
(i) $x =$ because
 [1]

(ii) $y =$ because
 [1]

- (c) Using the answers in **part (b)** and given that angle $BCD = (x + y)^\circ$,
 find the value of reflex angle BOE . Give a reason for each step of your answer.

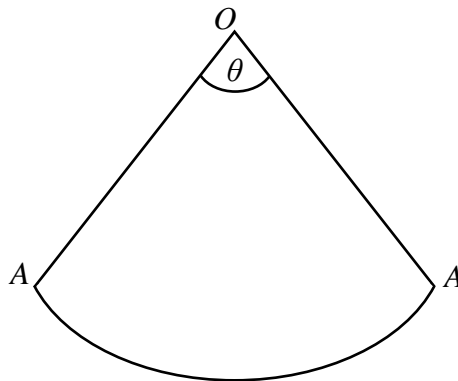
Answer [3]

26



A conical cup, shown above, has a diameter of 6 cm with a volume of $40\pi \text{ cm}^3$.
A cut is made along the dotted line, OA .

The cup is unfolded to form a sector subtended by angle θ , shown below.



- (a) Show that θ is approximately 1.3792 radians (correct to 4 decimal places).

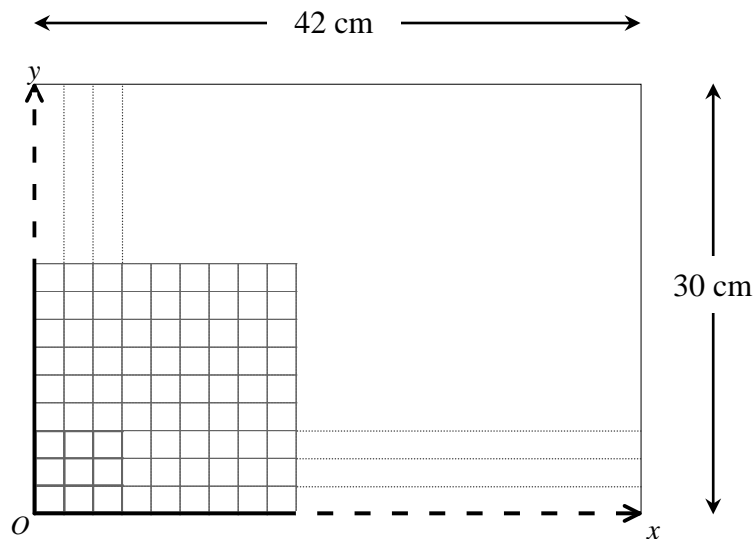
Answer

- (b) Hence, evaluate the area of the sector.

[4]

Answer cm^2 [2]

- 27 A rectangular cardboard, $ABCD$, with dimensions 14 cm by 22 cm is to be drawn exactly at the centre of an A3-sized paper with dimensions 42 cm by 30 cm. For an accurate drawing, a grid with x -axis, y -axis and origin, O , at the bottom left-hand corner of the A3-sized paper, is drawn. A part of an incomplete grid (not drawn to scale) is shown below. A scale of 1 cm to represent 1 unit is used.



- (a) State a possible coordinate for the centre of the A3-sized grid.

Answer (..... ,) [1]

- (b) State the possible coordinates for each of the points of the cardboard, A , B , C and D .

Answer A (..... ,)

B (..... ,)

C (..... ,)

D (..... ,) [2]

- (c) The rectangular cardboard and the A3-sized paper are **not** similar. The A3-sized paper is to be kept as it is as the grid is already drawn as a reference. Suggest how the cardboard can be cut to make it similar to the A3-sized paper, such that there is minimal wastage and cutting.

.....

..... [2]



Geylang Methodist School (Secondary) Preliminary Examination 2024

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Name

WORKED SOLUTIONS & SUGGESTED ANSWER SCHEME

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4052/01

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Mathematical Formulae

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$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

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$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f} \right)^2}$$

Answer **all** the questions.

- 1 Expand and simplify $(4x - y)(3x + 4y)$.

$$\begin{aligned}(4x - y)(3x + 4y) \\ &= 12x^2 - 3xy + 16xy - 4y^2 \\ &= 12x^2 + 13xy - 4y^2\end{aligned}$$

\times	$4x$	$-y$
$3x$	$12x^2$	$-3xy$
$4y$	$16xy$	$-4y^2$

Answer $12x^2 + 13xy - 4y^2$ [2]

- 2 (a) Find the lowest common multiple (LCM) of 108 and 140.

By Prime Factorisation,

2	108	140
2	54	70
3	27	35
3	9	35
3	3	35
5	1	7
7	1	1

$$\begin{aligned}\text{LCM} &= 2^2 \times 3^3 \times 5 \times 7 \\ &= 4 \times 27 \times 5 \times 7 \\ &= 3780\end{aligned}$$

Answer 3780 [1]

- (b) Find the highest common factor (HCF) of 108 and 140.

By Prime Factorisation,

2	108	140
2	54	70
	27	35

$$\begin{aligned}\text{HCF} &= 2^2 \\ \text{HCF} &= 4\end{aligned}$$

Answer 4 [1]

- 3 Solve $4 - \frac{1}{3}x = 2$.

$$\begin{aligned}4 - \frac{1}{3}x &= 2 \\ 4 - 2 &= \frac{1}{3}x \\ 2 \times 3 &= x \\ \therefore x &= 6\end{aligned}$$

Answer $x =$ 6 [1]

4 10 23 34 40 25 35 17 44 23

- (a) Find the median of the set of numbers.

Ascending Order:

10 17 23 23 **25** 34 35 40 44

Answer25..... [1]

- (b) Find the range of the set of numbers.

10 17 23 23 25 34 35 40 **44**

Range = Highest – Lowest = 44 – 10 = 34

Answer34..... [1]

- 5 (a) A hotel made breakfast milkshake for its guests.

A mixture of mango juice, milk and yogurt in the ratio 7 : 5 : 3 are mixed together.
2.5 litres of milk is used in the mixture.

- (i) How much yogurt is used in the milkshake?

5 units → 2.5 litres

1 unit → $\frac{2.5}{5} = 0.5$ litres

3 units → $0.5 \times 3 = 1.5$ litres

Answer1.5..... litres [1]

- (ii) How much breakfast milkshake is made?

Total number of units 1 unit → 0.5 litres

= 7 + 5 + 3 15 units → 15 × 0.5

= 15 units → 7.5 litres

Answer7.5..... litres [1]

- (b) Another milkshake is made using apple juice, peach juice and milk.

The ratio of apple juice : milk = 2 : 3.

The ratio of milk : peach juice = 5 : 4.

Find the ratio of apple juice : milk : peach juice.

Apple : Milk : Peach

2(5) : 3(5) : :

: 5(3) : 4(3)

10 : 15 : 12

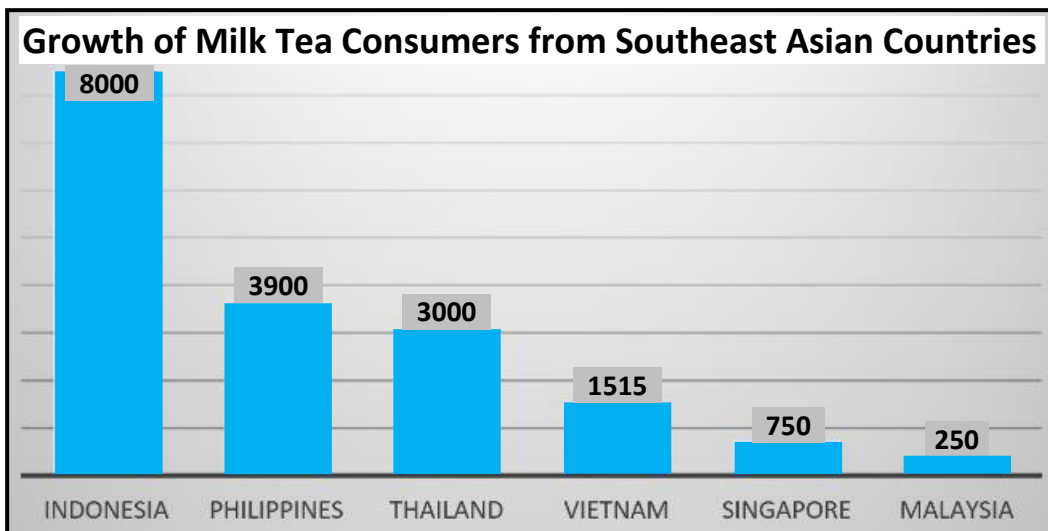
Answer10 : 15 : 12..... [1]

6 Find the equation of the straight line passing through (7, -7) and (-4, 15).

$\begin{aligned} \text{Gradient, } m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{15 - (-7)}{-4 - 7} \\ &= \frac{15 + 7}{-11} \\ &= \frac{22}{-11} \\ \text{Gradient, } m &= -2 \end{aligned}$	$\begin{aligned} \text{y - intercept, } c: \\ y &= mx + c \\ -7 &= (-2)(7) + c \\ -7 &= -14 + c \\ c &= -7 + 14 \\ c &= 7 \end{aligned}$	$\begin{aligned} \text{Equation:} \\ y &= mx + c \\ y &= -2x + 7 \end{aligned}$
--	--	---

Answer $y = -2x + 7$ [3]

7 The graph shows the growth in milk tea consumers regionally.



Adapted from source: <https://www.mdpi.com/>

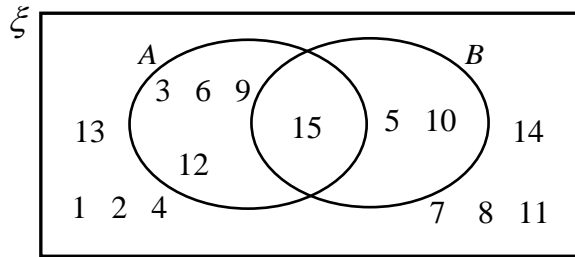
(a) State one misleading feature of the graph.

Evidence: Value does not specify if its actual value or percentage. No y-axis, not indicative of the length of bar and value cannot be verified for accuracy. Title states growth which indicates a basis of comparison which is not shown. [Any comment with substantial and mathematically sound justification can be accepted] [1]

(b) Based on your answer in part (a), explain how this feature affects the reader's interpretation of the graph.

Any **Interpretation** or **Conclusion** with substantial and mathematically sound justification based on the **Evidence** mentioned in part (a) can be accepted. [1]

8 The Venn diagram shows the elements of $\xi = \{\text{integers } x: 1 \leq x \leq 15\}$ and two sets A and B .



(a) Use one of the symbols below to complete each statement.

$$\emptyset \subseteq \in \cap \not\subseteq$$

(i) $\{15\} \dots\dots\dots \subseteq \dots\dots\dots A \cap B$ [1]

(ii) $1 \dots\dots\dots \in \dots\dots\dots (A \cup B)'$ [1]

(b) Suggest a description to define set A .

.....
 $A = \{\text{integer } x: x \text{ is a multiple of } 3 \text{ OR } x \text{ is divisible by } 3\}$
 [1]

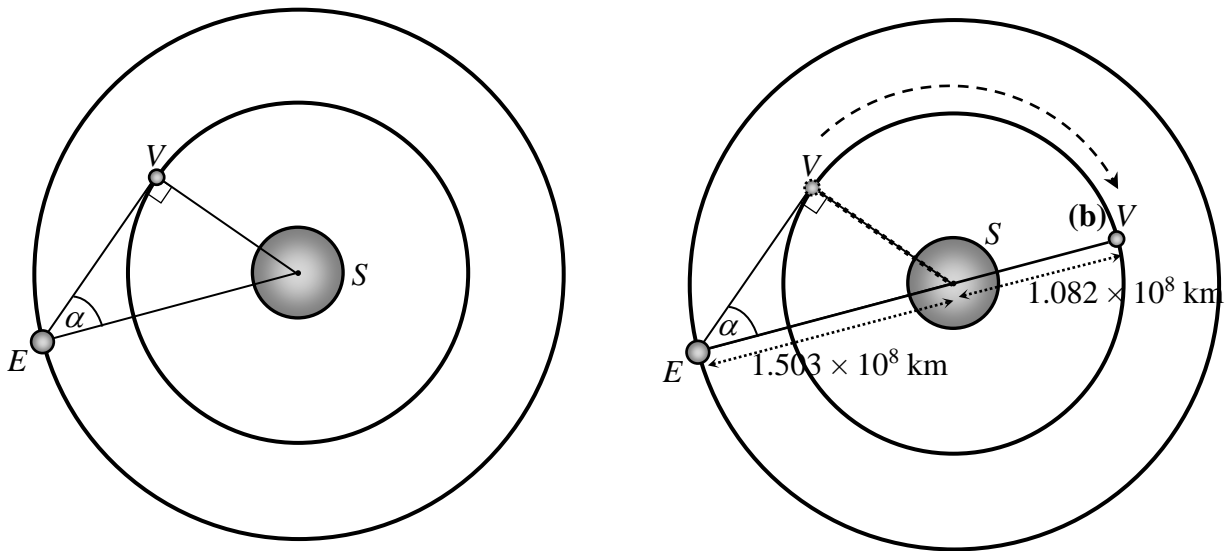
9 The number of social media users globally grew from 4.72 billion in January 2023 to 5.04 billion in January 2024. The number increased by $r\%$ every month during that period. Find the value of r . (1 billion = 1×10^9)

Social media is a system which experiences *exponential* growth.
 [No penalty for not using the info (1 billion = 1×10^9) given:

$$\begin{aligned} \text{Total amount} &= P \left(1 + \frac{r}{100}\right)^n \\ 5.04 \times 10^9 &= 4.72 \times 10^9 \left(1 + \frac{r}{100}\right)^{12} \\ \frac{5.04}{4.72} &= \left(1 + \frac{r}{100}\right)^{12} \\ \sqrt[12]{\frac{5.04}{4.72}} &= 1 + \frac{r}{100} \\ 100 \left(\sqrt[12]{\frac{5.04}{4.72}} - 1\right) &= r \\ r &= 0.5481408453 \\ r &= 0.548 \end{aligned}$$

Answer $r = \dots\dots\dots 0.548 \dots\dots\dots \%$ [2]

- 10 The following diagram shows the positions of Earth, E , and Venus, V , and their orbit around the Sun, S . The radius of Venus' orbit is 1.082×10^8 km.
 [Follow the degree of accuracy of the values specified in this question and leave your answers in standard form where necessary.]



- (a) If the angle α is 46.054° , calculate the distance between Earth and the Sun.
 Distance between Earth and the Sun = ES

$$\sin \alpha^\circ = \frac{VS}{ES}$$

$$\sin 46.054^\circ = \frac{1.082 \times 10^8}{ES}$$

$$ES = \frac{1.082 \times 10^8}{\sin 46.054^\circ}$$

$$ES = 1.502789924 \times 10^8 \text{ km}$$

$$ES = 1.503 \times 10^8 \text{ km}$$

(follow the **standard form** and **degree of accuracy** given in question: **3dp**)

Answer 1.503×10^8 km [1]

- (b) State the value of α when Earth is furthest from Venus and calculate this distance.

Earth furthest from Venus = $ES + VS$
 When $\alpha = 0.000^\circ$ (ES and VS are on the same straight line, collinear)
 (follow degree of accuracy given in question: 3dp)

$$\text{Distance} = ES + VS$$

$$= (1.502789924 \times 10^8) + (1.082 \times 10^8)$$

$$\text{Distance} = 2.584789924 \times 10^8$$

$$ES + VS = 2.585 \times 10^8 \text{ km}$$

(follow **degree of accuracy** given in question: **3dp**)

Answer $\alpha =$ 0.000°
 2.585×10^8 km [2]

11 Simplify $\frac{x^3 - 4x}{2x^2 - 7x + 6}$.

$$\begin{aligned} \frac{x^3 - 4x}{2x^2 - 7x + 6} &= \frac{x(x^2 - 4)}{(x-2)(2x-3)} \\ &= \frac{x(x+2)(x-2)}{(x-2)(2x-3)} \\ &= \frac{x(x+2)}{(2x-3)} \end{aligned}$$

Factorise Numerator fully
Factorise Denominator fully
Simplify fraction correctly

Answer $\frac{x(x+2)}{(2x-3)}$ [3]

- 12 (a) A box is said to contain blue marbles, red marbles and yellow marbles.
A marble is picked at random from the box.
The probability that the marble is blue is 0.27.
The probability that the marble is red is 0.73.

Deduce if there are any yellow marbles in the box, showing the calculations clearly.

Evidence:

$$\begin{aligned} P(\text{yellow}) &= 1 - [P(\text{blue}) + P(\text{red})] \\ &= 1 - [0.27 + 0.73] \\ &= 1 - 1 \end{aligned}$$

$$P(\text{yellow}) = 0$$

Interpretation: $P(\text{yellow}) = 0$ means the chances is NIL and not possible to do so.

Conclusion: There are no yellow marbles in the box. [2]

- (b) Another box contains 10 green marbles, 8 orange marbles and 3 purple marbles.
The probability of picking a purple marble is $\frac{1}{4}$ when x purple marbles are added in.
Find the total number of purple marbles in this box.

New P(purple):

$$\begin{aligned} \frac{3}{10+8+3} \frac{+x}{+x} &= \frac{1}{4} \\ \frac{3+x}{21+x} &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} 12 + 4x &= 21 + x \\ 4x - x &= 21 - 12 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

$$\begin{aligned} \text{Total number of purple marbles} &= 3 + 3 \\ &= 6 \text{ purple marbles} \end{aligned}$$

$$4(3+x) = 1(21+x)$$

Answer 6 [2]

13 Factorise completely $8 - 2x - 24y + 6xy$.

$$\begin{aligned}
 &8 - 2x - 24y + 6xy \\
 &= 2(4 - x - 12y + 3xy) && \text{Factorise common factor 2} \\
 &= 2[1(4 - x) - 3y(4 - x)] && \text{Factorise by grouping} \\
 &= 2(4 - x)(1 - 3y) && \text{Correctly simplified (equivalent)}
 \end{aligned}$$

Answer $2(4 - x)(1 - 3y)$ or $2(x - 4)(3y - 1)$ [3]

14 A container of oil is 65% full. 20% of the oil is used for cooking. 15.6 litres of oil is left in the container. If 14.8 litres of oil is then poured into the container, determine whether there will be a spill. Show your working.

$$\% \text{ of oil (in the container) used for cooking} = (20\% \times 65\%) = 13\%$$

$$\% \text{ of oil (in the container) left} = 65\% - 13\% = 52\%$$

Total capacity of oil still in the container:

$$52\% \rightarrow 15.6 \text{ litres (100\% - 52\% = 48\% to be full)}$$

$$1\% \rightarrow \frac{15.6}{52} \text{ litres}$$

$$\begin{aligned}
 48\% &\rightarrow \frac{15.6}{52} \times 48 \\
 &= 14.4 \text{ litres}
 \end{aligned}$$

$$14.8 \text{ litres} > 14.4 \text{ litres}$$

Amount poured > Capacity (Space) left in container

Hence, there will be a spill.

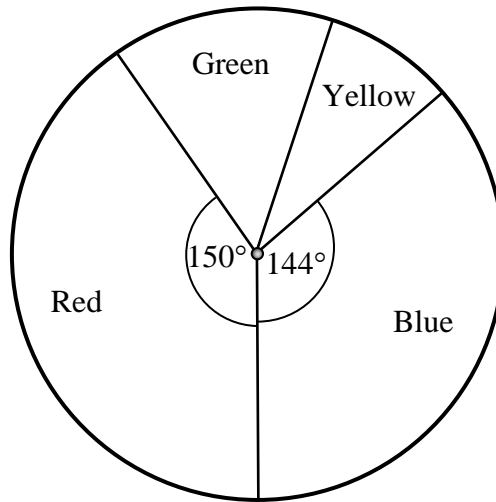
[Accept any reasonable working with method marks allocated accordingly for each step.]

There will be a spill as the total amount of oil poured (14.8 litres) exceeds

the empty capacity (space) in the container (14.4 litres).

[4]

- 15 A survey is conducted on the number of students in Red, Green, Yellow and Blue House who participated in the Inter-House Games. The results are shown as a pie chart.



There are twice as many students that are from Green House as compared to Yellow House. Explain why this information may not be useful to find the number of students from the Red and Blue House who participated in the Inter-House Games.

Evidence: A pie chart displays the breakdown (by angles, percentage, fraction etc) of the survey of the different houses.

Interpretation: However, the actual breakdown of numbers cannot be found.

Conclusion: This can only be found when the total number of students surveyed are given, which is a missing information here.

[1]

- 16 The Marina Coastal Expressway (MCE) tunnel is about 3.6 km long. The speed limit in expressway tunnels is 70 km/h. Find the shortest time possible for a car of length 4.8 m to pass through the MCE tunnel **completely**, within the speed regulations. Give your answer in minutes and seconds (nearest whole number).

$$\begin{aligned}
 \text{Time taken} &= \frac{\text{Total Distance}}{\text{Average Speed}} && \bullet \text{ Adding 4.8 m to Total Distance} \\
 &= \frac{3.6 \text{ km} + 4.8 \text{ m}}{70 \text{ km/h}} && \bullet \text{ Correct Formula for Time and correct value in hours} \\
 &= \frac{3.6 \text{ km} + \frac{4.8}{1000} \text{ km}}{70 \text{ km/h}} && \bullet \text{ Correct Time for minutes and seconds} \\
 &= \frac{3.6048 \text{ km}}{70 \text{ km/h}} \\
 &= 0.05149714286 \text{ (}\times 60 \text{ minutes } \times 60 \text{ seconds)} \\
 &= 185.3897143 \text{ seconds} \\
 \text{Time taken} &= 3 \text{ minutes } 5.3897143 \text{ seconds} \\
 \text{Time taken} &= 3 \text{ minutes } 6 \text{ seconds (round up)} \\
 &\text{(pass through the MCE tunnel **completely**)}
 \end{aligned}$$

Answer³..... minutes⁶..... seconds [3]

17 $N = \frac{a^6}{b^3}$ where a and b are prime numbers.

(a) Explain why N is a perfect cube

$$N = \frac{a^6}{b^3} = \frac{a^{2 \times 3}}{b^{1 \times 3}} = \left(\frac{a^2}{b}\right)^3$$

.....
 $\Rightarrow N = \left(\frac{a^2}{b}\right)^3$; hence N is a perfect cube. Power is a multiple of 3.
 [1]

(b) Show that M is divisible by 6, given that $\frac{MN}{2} = (3ab)^2$ and M is a natural number.

Answer

$$\frac{MN}{2} = (3ab)^2$$

$$MN = 2(3^2)(ab)^2$$

$$M = \frac{18(ab)^2}{N}$$

$$M = 6\left(\frac{3a^2b^2}{N}\right)$$

Hence

$$\Rightarrow M = 6\left(\frac{3a^2b^2}{N}\right)$$

Attempt at

$\rightarrow M$ as subject for formula

$\rightarrow 6$ extracted (multiple of 6 / divisible by 6)

[2]

18 The expression $2 + px - x^2$ can be written in the form $q - (x - 3)^2$.

(a) Find the value of p and the value of q .

$$2 + px - x^2 = q - (x - 3)^2$$

$$2 + px - x^2 = q - (x^2 - 6x + 9)$$

$$2 + px - x^2 = q - x^2 + 6x - 9$$

$$2 + px - x^2 = (q - 9) + 6x - x^2$$

Comparing coefficient of:

$$x : p = 6$$

$$x^0 : 2 = q - 9$$

$$q = 11$$

Answer $p = \dots\dots\dots 6$

$q = \dots\dots\dots 11$ [2]

(b) Explain why when $x = 3$, the expression has its maximum value.

Evidence: $(x - 3)^2$ in $q - (x - 3)^2$ is of minimum value when $(x - 3)^2 = 0$, which leads to $(x - 3) = 0$ and $x = 3$. [$x^2 \geq 0$]
Interpretation: Coefficient of x^2 is negative hence $-(x - 3)^2 = 0$ is maximum when $x = 3$.
Conclusion: Hence when $x = 3$, $2 + px - x^2$ has its **maximum** value. [1]

- 19 (a) The force, F Newtons, between any two particles in the universe is inversely proportional to the square of the distance, r metres, between them.
It is known that $F = 100$ Newtons for a particular value r .
Find the value of F when this value of r is doubled.

$$\begin{array}{ll}
 F \propto \frac{1}{r^2} & F_2 = \frac{k}{(r_2)^2} \quad [r_2 = 2r] \\
 F = \frac{k}{r^2} & F_2 = \frac{100r^2}{(2r)^2} \\
 100 = \frac{k}{r^2} & F_2 = \frac{100r^2}{4r^2} \\
 k = 100r^2 & F_2 = 25
 \end{array}$$

Answer²⁵..... Newtons [2]

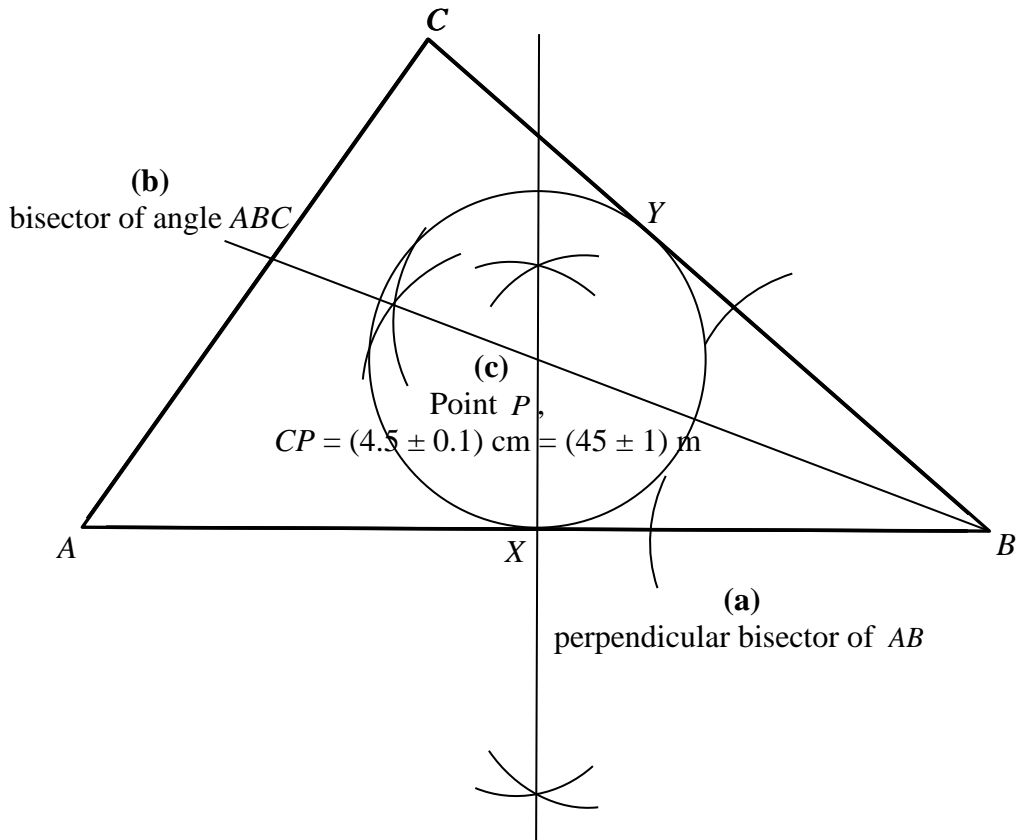
- (b) y is proportional to x^3 .
Explain the effect on the value of x in order for y to be 2700% of its original value.

$$\begin{array}{ll}
 y_2 = 2700\% \text{ of } y & (\Rightarrow 27y) \\
 x_2 = \sqrt[3]{2700\%} \text{ of } y & (\Rightarrow \sqrt[3]{27} x) \\
 x_2 = \sqrt[3]{27} x & \\
 x_2 = 3x &
 \end{array}$$

.....
 $x_2 = 3x$ **OR** x is tripled of its original value **OR** x is 300% of its original value.

..... [1]

- 20 The diagram shows an eco-garden in a school in the shape of triangle ABC .
The diagram is drawn to a scale of 1 : 1000.



(a) Construct the perpendicular bisector of AB . [1]

(b) Construct the bisector of angle ABC . [1]

(c) A flagpole with the school flag is erected at a point P such that it is equidistant from A and B **and** equidistant from AB and BC .
Mark the point P on the diagram and measure the actual length of CP .

1 : 1000
 1 cm : 1000 cm
 (1 m = 100 cm)
 1 cm : (1000 ÷ 10) m
 1 cm : 10 m
 $CP = (4.5 \pm 0.1) \text{ cm} = (45 \pm 1) \text{ m}$
 Answer $CP = \dots\dots\dots (45 \pm 1) \text{ m}$ [1]

(d) A tiled path is to be constructed in the eco-garden, where each tile is equidistant from P . The tiled path also meets the line AB and BC at points X and Y respectively such that $BX = BY$. Describe the shape of the tiled path and find its actual length.

Tiled path is a circle with radius, $r = (2.2 \pm 0.1) \text{ cm} = (22 \pm 1) \text{ m}$
 Length = $2\pi r$
 $= 2\pi(22) [r = 22 \pm 1]$
 Length = $44\pi r = 138.230 \dots \text{ m}$
 Length = 138 m (3sf)
 $131.9 \text{ m} \leq \text{Length} \leq 144.5 \text{ m}$
 $BX = BY$
 (Tangents from external point, B , are equal)

Answer Shape of tiled path: $\dots\dots\dots$ Circle

Length of tiled path = $\dots\dots\dots 138 \dots\dots\dots \text{ m}$ [2]

21 (a) $\frac{2^a \times (\sqrt[3]{64})^c}{16^b} = 1.$

Find an expression for a in terms of b and c .

$$\frac{2^a \times (\sqrt[3]{64})^c}{16^b} = 1 \quad \Rightarrow a + 2c = 4b$$

$$\therefore a = 4b - 2c$$

$$2^a \times 4^c = 16^b$$

$$(2^a) \times (2^2)^c = (2^4)^b \quad \begin{array}{l} \text{Simplify using Indices Law} \\ \text{Simplify to base 2} \end{array}$$

$$2^a \times 2^{2c} = 2^{4b}$$

$$2^{a+2c} = 2^{4b}$$

Answer $a = \dots\dots\dots 4b - 2c \dots\dots\dots$ [2]

(b) Simplify $\left(\frac{27y^{12}}{x^6}\right)^{-\frac{1}{3}}$.

$$\left(\frac{27y^{12}}{x^6}\right)^{-\frac{1}{3}} = \left(\frac{x^6}{27y^{12}}\right)^{\frac{1}{3}} = \left(\frac{x^6}{3^3 y^{12}}\right)^{\frac{1}{3}}$$

$$= \left(\frac{x^{6 \times \frac{1}{3}}}{3^{3 \times \frac{1}{3}} y^{12 \times \frac{1}{3}}}\right) = \frac{x^2}{3y^4}$$

Answer $\dots\dots\dots \frac{x^2}{3y^4} \dots\dots\dots$ [2]

- 22 The table shows the distribution of weight of 40 students in a class.

Weight (x kg)	Frequency
$40 < x \leq 50$	2
$50 < x \leq 60$	13
$60 < x \leq 70$	20
$70 < x \leq 80$	5

- (a) Calculate an estimate for

- (i) the mean weight of the students,

$$\begin{aligned} \text{Mean} &= \frac{\sum fx}{\sum f} \\ &= \frac{(2 \times 45) + (13 \times 55) + (20 \times 65) + (5 \times 75)}{40} = \frac{2480}{40} \end{aligned}$$

$$\text{Mean} = 62 \text{ kg}$$

Answer 62 kg [1]

- (ii) the standard deviation of the weight.

$$\begin{aligned} \text{Standard deviation} &= \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} \\ &= \sqrt{\frac{(2 \times 45^2) + (13 \times 55^2) + (20 \times 65^2) + (5 \times 75^2)}{40} - 62^2} \\ &= 7.483314774 \end{aligned}$$

$$\text{Standard Deviation} = 7.48 \text{ kg}$$

Answer 7.48 kg [1]

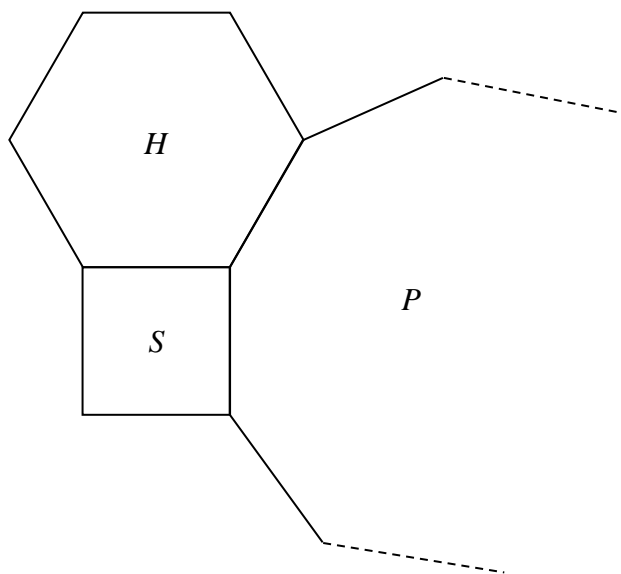
- (b) Due to a fault in the weighing machine, the weight recorded was 2 kg lighter than the actual weight. Explain how this may have affected the mean and standard deviation and state its correct value.

Mean calculated is lower and correct value will increase by 2 kg ($62 + 2 = 64$ kg).

Standard deviation will remain the same at 7.48 kg.

..... [2]

- 23 The diagram below shows a square S , a regular hexagon H and a part of a regular n – sided polygon P .



Find the number of sides, n , of the regular polygon P .

Each interior angle of regular hexagon, H

$$\begin{aligned} &= \frac{(6-2) \times 180^\circ}{6} \\ &= \frac{4 \times 180^\circ}{6} \\ &= \frac{720^\circ}{6} \\ &= 120^\circ \end{aligned}$$

Each interior angle of square, $S = 90^\circ$

Each interior angle of regular polygon, P

$$\begin{aligned} &= 360^\circ - 120^\circ - 90^\circ \\ &= 150^\circ \end{aligned}$$

[angles at a point]

Each exterior angle of regular polygon, P

$$\begin{aligned} &= 180^\circ - 150^\circ \\ &= 30^\circ \end{aligned}$$

Number of sides, n , of regular polygon P

$$\begin{aligned} n &= \frac{360^\circ}{30^\circ} \\ n &= 12 \text{ sides} \end{aligned}$$

[Accept working with sound properties of angles used]

Answer $n = \dots\dots\dots 12 \dots\dots\dots$ [4]

24 (a) The first five terms of a sequence are 2, 7, 12, 17 and 22.

(i) T_n is the n th term of the sequence.

Find an expression, in terms of n for T_n .

$$T_n = a + (n - 1) d \quad [a = 2, d = 7 - 2 = 5]$$

$$T_n = 2 + (n - 1) (5)$$

$$= 2 + 5n - 5$$

$$T_n = 5n - 3$$

$$\text{Answer } T_n = \dots\dots\dots \frac{5n - 3}{\dots\dots\dots} \text{ (or equivalent)} \dots\dots\dots [1]$$

(ii) The sum of the first n terms of this sequence is given by $an^2 + bn$.

When $n = 1$, $a + b = 2$.

Show that $4a + 2b = 9$.

Answer

Given: Sum $S_n = an^2 + bn$

When $n = 2$, $S_2 = a(2)^2 + b(2)$

$$2 + 7 = 4a + 2b$$

$$4a + 2b = 9 \quad \text{(Shown)}$$

[1]

(iii) Solve the equations from part (ii).

$$a + b = 2 \quad (1)$$

$$4a + 2b = 9 \quad (2)$$

$$(1): \quad a = 2 - b$$

$$(2): \quad 4(2 - b) + 2b = 9$$

$$8 - 4b + 2b = 9$$

$$-2b = 9 - 8$$

$$-2b = 1$$

$$b = -\frac{1}{2}$$

$$a + b = 2, \quad (1): \quad a = 2 - b$$

$$4a + 2b = 9. \quad a = 2 - \left(-\frac{1}{2}\right)$$

$$a = 2 + \frac{1}{2}$$

$$a = 2\frac{1}{2}$$

$$\text{Answer } a = \dots\dots\dots \frac{2\frac{1}{2}}{\dots\dots\dots}$$

$$b = \dots\dots\dots \frac{-\frac{1}{2}}{\dots\dots\dots} \dots\dots\dots [2]$$

(b) The sum of the first n terms of a different sequence is given by $5n^2 + 3n$.
Find the 11th term of this sequence

Given: Sum $S_n = 5n^2 + 3n$

$$T_{11} = S_{11} - S_{10}$$

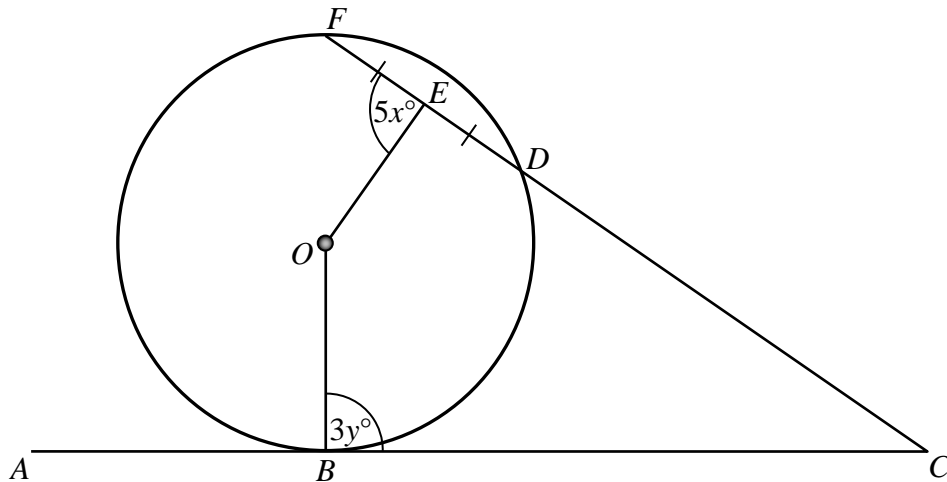
$$= [5(11)^2 + 3(11)] - [5(10)^2 + 3(10)]$$

$$= 638 - 530$$

$$T_{11} = 108$$

$$\text{Answer } \dots\dots\dots \frac{108}{\dots\dots\dots} \dots\dots\dots [2]$$

- 25 In the diagram, B , D and F lie on a circle, centre O .
 AC is a tangent to the circle at B , $DE = FE$, angle $OEF = 5x^\circ$ and angle $OBC = 3y^\circ$.



- (a) A statement was made that $BC = CE$.
 Do you agree? Give reasons for your answer.

Evidence: AC is a tangent at B . In this case, CF is NOT a tangent at E .

Interpretation: For $BC = CE$, both AC and FC has to be tangents which are drawn from external point C (tangents from external point). E is not on the circumference.

Conclusion: Disagree that $BC = CE$ based on the evidence above and the interpretation. [1]

- (b) Complete these statements.

Angle $OEF = 5x^\circ = 90^\circ$ (OE is a perpendicular bisector of chord DF)

$$5x = 90 \Rightarrow x = \frac{90}{5} = 18$$

- (i) $x = \dots\dots\dots 18 \dots\dots\dots$ because OE is a perpendicular bisector of chord DF
 (no marks awarded for correct value of x but incorrect reasons.) [1]

Angle $OBC = 3y^\circ = 90^\circ$ (AC is a tangent perpendicular to radius OB)

$$3y = 90 \Rightarrow y = \frac{90}{3} = 30$$

- (ii) $y = \dots\dots\dots 30 \dots\dots\dots$ because AC is a tangent perpendicular to radius OB
 (no marks awarded for correct value of y but incorrect reasons.) [1]

- (c) Using the answers in part (b) and given that angle $BCD = (x + y)^\circ$,
 find the value of reflex angle BOE . Give a reason for each step of your answer.

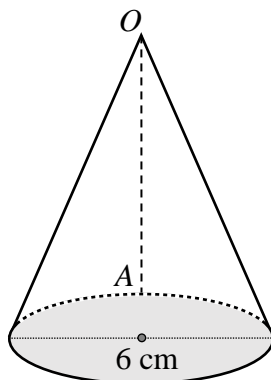
Angle $BCD = (x + y)^\circ = (18 + 30)^\circ = 48^\circ$ [From part (b)]

Obtuse angle $BOE = [360 - 90 - 90 - 48]^\circ = 132^\circ$
 [sum of angles in a quadrilateral, \perp bisector of chord, tangent perpendicular to radius]

Reflex angle $BOE = (360 - 132)^\circ = 228^\circ$
 [angles at a point]

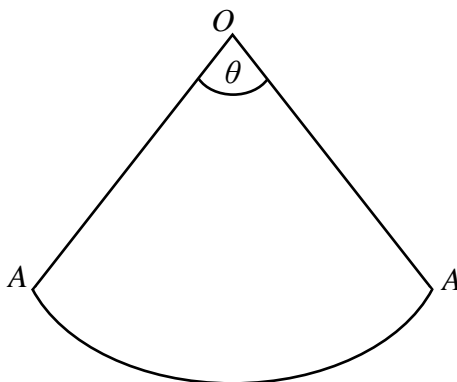
Answer $\dots\dots\dots 228^\circ \dots\dots\dots$ [3]

26



A conical cup, shown above, has a diameter of 6 cm with a volume of $40\pi \text{ cm}^3$. A cut is made along the dotted line, OA .

The cup is unfolded to form a sector subtended by angle θ , shown below.



- (a) Show that θ is approximately 1.3792 radians (correct to 4 decimal places).

Answer

$$\text{Radius of base of cup, } r = \frac{6}{2} = 3 \text{ cm}$$

Perpendicular height of cone, h

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$40\pi = \frac{1}{3}\pi(3)^2 h$$

$$40 = 3h$$

$$h = \frac{40}{3} = 13\frac{1}{3} \text{ cm}$$

$$OA^2 = h^2 + r^2$$

$$OA^2 = \left(\frac{40}{3}\right)^2 + 3^2 = \frac{1681}{9}$$

$$OA = \sqrt{\frac{1681}{9}} = \frac{41}{3} = 13\frac{2}{3} \text{ cm}$$

Circumference of base of circle (radius = r_1)
= Arc length of sector (radius = r_2)

$$2\pi r_1 = r_2 \theta$$

$$2\pi(3) = (OA)\theta$$

$$6\pi = \left(\frac{41}{3}\right)\theta$$

$$\theta = 6\pi \div \frac{41}{3}$$

$$\theta = 1.379235799$$

$$\theta = 1.3792 \text{ (4dp)} \quad (\text{Shown})$$

Alternative:

Curved surface area of cone = Area of sector

$$\pi(r_1)^2 = \frac{1}{2}(r_2)^2\theta \quad [4]$$

- (b) Hence, evaluate the area of the sector.

$$\text{Area of sector} = \frac{1}{2}r^2\theta \quad \left[r = r_2 = \frac{41}{3}\right]$$

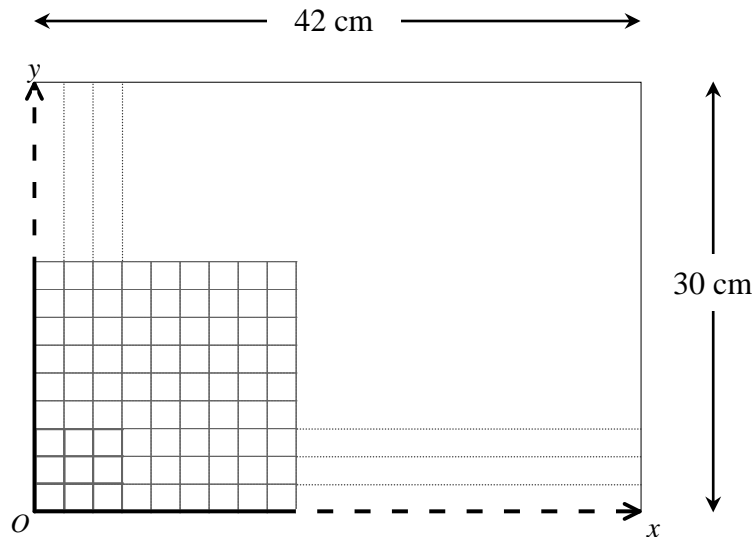
$$= \frac{1}{2}\left(\frac{41}{3}\right)^2(1.3792\dots) \quad (\text{ecf})$$

$$= 128.8052988$$

$$\text{Area of sector} = 129 \text{ cm}^2 \text{ (3sf)}$$

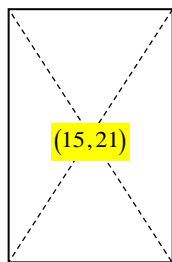
Answer 129 cm² [2]

- 27 A rectangular cardboard, $ABCD$, with dimensions 14 cm by 22 cm is to be drawn exactly at the centre of an A3-sized paper with dimensions 42 cm by 30 cm. For an accurate drawing, a grid with x -axis, y -axis and origin, O , at the bottom left-hand corner of the A3-sized paper, is drawn. A part of an incomplete grid (not drawn to scale) is shown below. A scale of 1 cm to represent 1 unit is used.

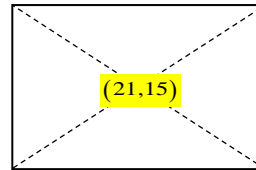


- (a) State a possible coordinate for the centre of the A3-sized grid.

Two possible answers:



Orientation: **Portrait**
(15, 21)

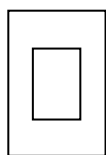


Orientation: **Landscape**
(21, 15)

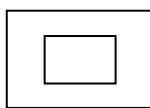
Answer (..... ,) [1]

- (b) State the possible coordinates for each of the points of the cardboard, A , B , C and D .

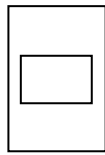
Four possible answers: Coordinates must be in order $ABCD$, $BCDA$, $CDAB$, $DABC$ etc



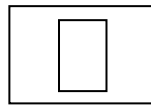
(8,10)
(8,32)
(22,32)
(22,10)



(10,8)
(10,22)
(32,22)
(32,8)



(4,14)
(4,28)
(26,28)
(26,14)



(14,4)
(14,26)
(28,26)
(28,4)

Answer A (..... ,)

B (..... ,)

C (..... ,)

D (..... ,) [2]

- (c) The rectangular cardboard and the A3-sized paper are **not** similar.

The A3-sized paper is to be kept as it is as the grid is already drawn as a reference. Suggest how the cardboard can be cut to make it similar to the A3-sized paper, such that there is minimal wastage and cutting.

Best Option: Cut the **longer side** of cardboard by **2.4 cm**. Ratio of length = $\frac{14}{30} = \frac{22 - 2.4}{42} \left(= \frac{19.6}{42} \right) = \frac{7}{15}$

Next Option: Cut the **longer side** of cardboard by **12 cm**. Ratio of length = $\frac{14}{42} = \frac{22 - 12}{30} \left(= \frac{10}{30} \right) = \frac{1}{3}$ [2]

(Minimal wastage and cutting required so BEST OPTION is the correct answer.)



Geylang Methodist School (Secondary) Preliminary Examination 2024

Candidate Name			
Class		Index Number	

MATHEMATICS

4052/02

Paper 2

4 Express
5 Normal (Academic)

Candidates answer on the Question Paper.

2 hours 15 minutes

Setter: Ms Nainee Ismail and Mr Kenneth Tan

Thursday, 15 August 2024

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.

For Examiner's Use

90

This document consists of **20** printed pages including the cover page.

[Turn Over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

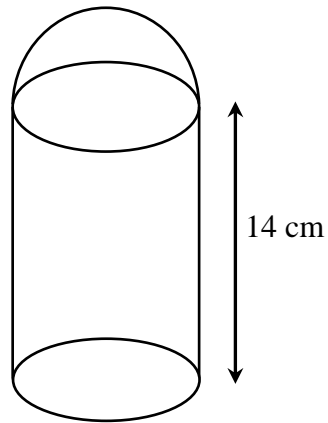
$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f} \right)^2}$$

- 1 A model of a thermos flask consists of a solid hemisphere attached to a solid cylinder as shown. The height of the cylinder is 14 cm.



The curved surface area of the cylinder is twice the curved surface area of the hemisphere.

- (a) Calculate the radius of the base of the cylinder.

Answer cm [2]

- (b) Calculate the height of the model of the thermos flask.

Answer cm [1]

- (c) Find the ratio volume of hemisphere : volume of cylinder.

Answer : [2]

-
- 2 (a) (i) Solve the inequalities $-9 < 5 - 4x \leq 6$.

Answer [2]

- (ii) Write down all the integers that satisfy $-9 < 5 - 4x \leq 6$.

Answer [1]

(b) $y = \frac{5 - x^2}{x^2 + w}$

- (i) Evaluate the value of y when $x = 2.41$ and $w = 1.908$.
Write your answer correct to three decimal places.

Answer $y =$ [2]

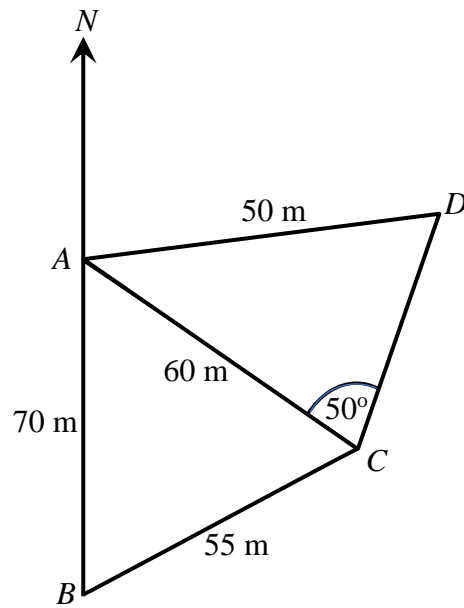
- (ii) Rearrange the formula to make x the subject.

Answer $x =$ [4]

2 (c) Solve $\frac{6}{x-1} - \frac{4-x}{3-x} = 1$.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [4]

3



A , B , C and D are four points on horizontal ground.

$AB = 70$ m, $BC = 55$ m, $CA = 60$ m, $AD = 50$ m, angle $ACD = 50^\circ$ and angle ADC is acute.

(a) Calculate angle BAC .

Answer Angle $BAC = \dots\dots\dots$ [3]

(b) Calculate angle DAC .

Answer Angle $DAC = \dots\dots\dots$ [3]

3 (c) Calculate the bearing of A from D .

Answer [2]

(d) A man walks northwards from point A . A vertical pole is placed at point D .
The greatest angle of elevation of the pole when viewed along this path is 7° .
Calculate the height of the pole.

Answer m [2]

- 4 Mr Tan planned to travel to Kuala Lumpur for a family trip.
Before the trip, he exchanged Singapore Dollars (S\$) for Malaysian Ringgit (RM).
The exchange rate is S\$ 1 = RM x .
- (a) Write down an expression, in terms of x , for the Singapore Dollars he exchanged if he received RM 2000.

Answer S\$ [1]

- (b) After the trip, Mr Tan found out that he still had RM 300.
He exchanged the Malaysian Ringgit back to Singapore Dollars.
The exchange rate is now S\$ 1 = RM $(x + 0.05)$.
Write down an expression, in terms of x , for the Singapore Dollars he exchanged for RM 300.

Answer S\$ [1]

- (c) Mr Tan used up S\$ 485 for the trip.
Form an equation in x and show that it reduces to $1940x^2 - 6703x - 400 = 0$.

Answer

[3]

- 4 (d) Solve the equation $1940x^2 - 6703x - 400 = 0$,
giving your solutions correct to two decimal places.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [3]

- (e) From your solution to **part (d)**, state the exchange rate in the form is S\$ 1 = RM x ,
to two decimal places, and hence, find the total amount of Singapore Dollars (S\$)
he initially converted to Malaysian Ringgit (RM).

Answer S\$ 1 = RM $\dots\dots\dots$

S\$ $\dots\dots\dots$ [2]

- 5 A shop sells two flavours of ice-cream, Raisins (R) and Chunkies (C) in three different sizes, small (S), medium (M) and large (L). The sales on two successive days are given in the table below.

Size	Saturday			Sunday		
	small	medium	large	small	medium	large
Number of cups sold (Raisins)	12	17	8	14	12	10
Number of cups sold (Chunkies)	18	15	11	13	21	16

The information for Saturday’s sales can be represented by the matrix

$$\mathbf{M} = \begin{matrix} & \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \\ \begin{pmatrix} 12 & 17 & 8 \\ 18 & 15 & 11 \end{pmatrix} & \begin{matrix} \text{R} \\ \text{C} \end{matrix} \end{matrix} .$$

- (a) Represent the information for Sunday’s sales in a 2×3 matrix \mathbf{N} .

Answer
$$\mathbf{N} = \begin{matrix} & \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \\ \left(\begin{matrix} \\ \\ \end{matrix} \right) & \begin{matrix} \text{R} \\ \text{C} \end{matrix} \end{matrix} \quad [1]$$

- (b) The cost of each cup of ice-cream is \$2.50 for small (S), \$3.20 for medium (M) and \$4.50 for large (L).

Represent the cost of ice cream in a 3×1 matrix \mathbf{C} .

Answer
$$\mathbf{C} = \begin{matrix} \$ \\ \left(\begin{matrix} \\ \\ \end{matrix} \right) & \begin{matrix} \text{S} \\ \text{M} \\ \text{L} \end{matrix} \end{matrix} \quad [1]$$

- (c) Evaluate the matrix $\mathbf{P} = \mathbf{M} + \mathbf{N}$.

Answer
$$\mathbf{P} = \quad [1]$$

- (d) State what the elements of \mathbf{P} represent.

.....
 [1]

- 5 (e) Evaluate the matrix $\mathbf{Q} = \frac{1}{2}\mathbf{PC}$.

Answer $\mathbf{Q} =$ [2]

- (f) State what the elements of \mathbf{Q} represent.

.....
 [1]

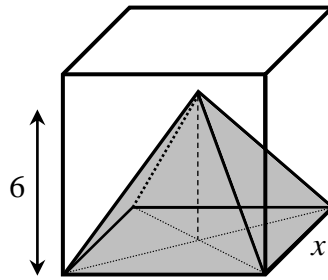
- (g) Evaluate the matrix $\mathbf{R} = (1 \ 1)\mathbf{PC}$.

Answer $\mathbf{R} =$ [2]

- (h) State what the elements of \mathbf{R} represent.

.....
 [1]

- 6 The following diagram shows a solid which is made up of a cube whose edge is x cm with a right square pyramid removed from the base of the cube. The pyramid has the same square base as the cube and its vertical height is 6 cm.



- (a) Show that the volume of the remaining solid, y , is $x^3 - 2x^2$.

Answer

[1]

- (b) Complete the table of values for $y = x^3 - 2x^2$.
Values are given to one decimal place where appropriate.

x	0	1	1.5	2	3	4	5	6	6.5
y	0		-1.1		9	32	75	144	190.1

[1]

- (c) On the grid opposite, draw the graph of $y = x^3 - 2x^2$ for $0 \leq x \leq 6.5$. [3]
- (d) Use your graph to estimate the volume of the remaining solid when the pyramid has a base area of 24 cm^2 .

Answer Volume = cm^3 [2]

- (e) (i) Use your graph, for $0 < x < 2$, to give a possible deduction on the volume of the remaining solid, y .

.....

..... [1]

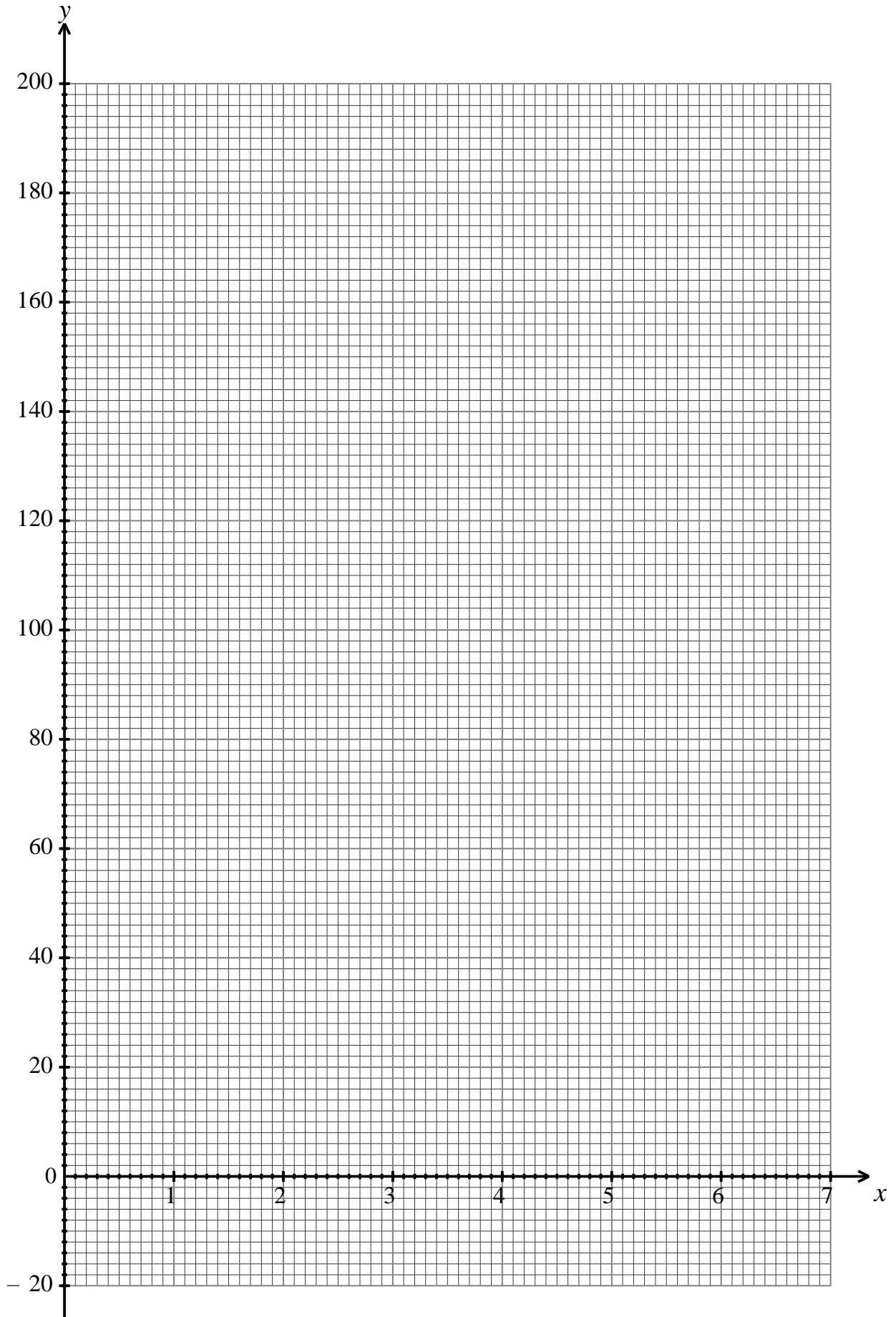
- (ii) Suggest a suitable range of x such that it is possible to remove a full right square pyramid from the cube.

Answer [1]

- (f) By drawing the line $y = ax + b$, solve the equation $-x^3 + 2x^2 + 15x + 20 = 0$.

Answer $x =$ [2]

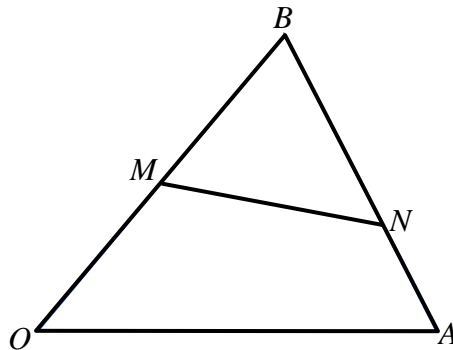
6 (c)



- 7 (a) The point P is $(5, -8)$ and the point Q is $(-4, 7)$.
 The point L is such that $\overrightarrow{QP} = \frac{1}{2}\overrightarrow{PL}$ and O is the origin.
 Find $|\overrightarrow{OL}|$.

Answer [4]

- (b) In the diagram $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
 M is a point on OB where $OB = 2MB$ and the point N lies on AB
 such that $3BN = 2BA$.



- (i) Express \overrightarrow{BN} , as simply as possible, in terms of \mathbf{a} and \mathbf{b} .

Answer $\overrightarrow{BN} = \dots\dots\dots$ [1]

- 7 (b) (ii) Express \overrightarrow{MN} , as simply as possible, in terms of \mathbf{a} and \mathbf{b} .

Answer $\overrightarrow{MN} = \dots\dots\dots$ [2]

- (iii) The point P lies on OA produced such that $OA : OP = 1 : 2$.
Determine whether the points M, N and P are collinear.
Justify your answer.

.....

..... [2]

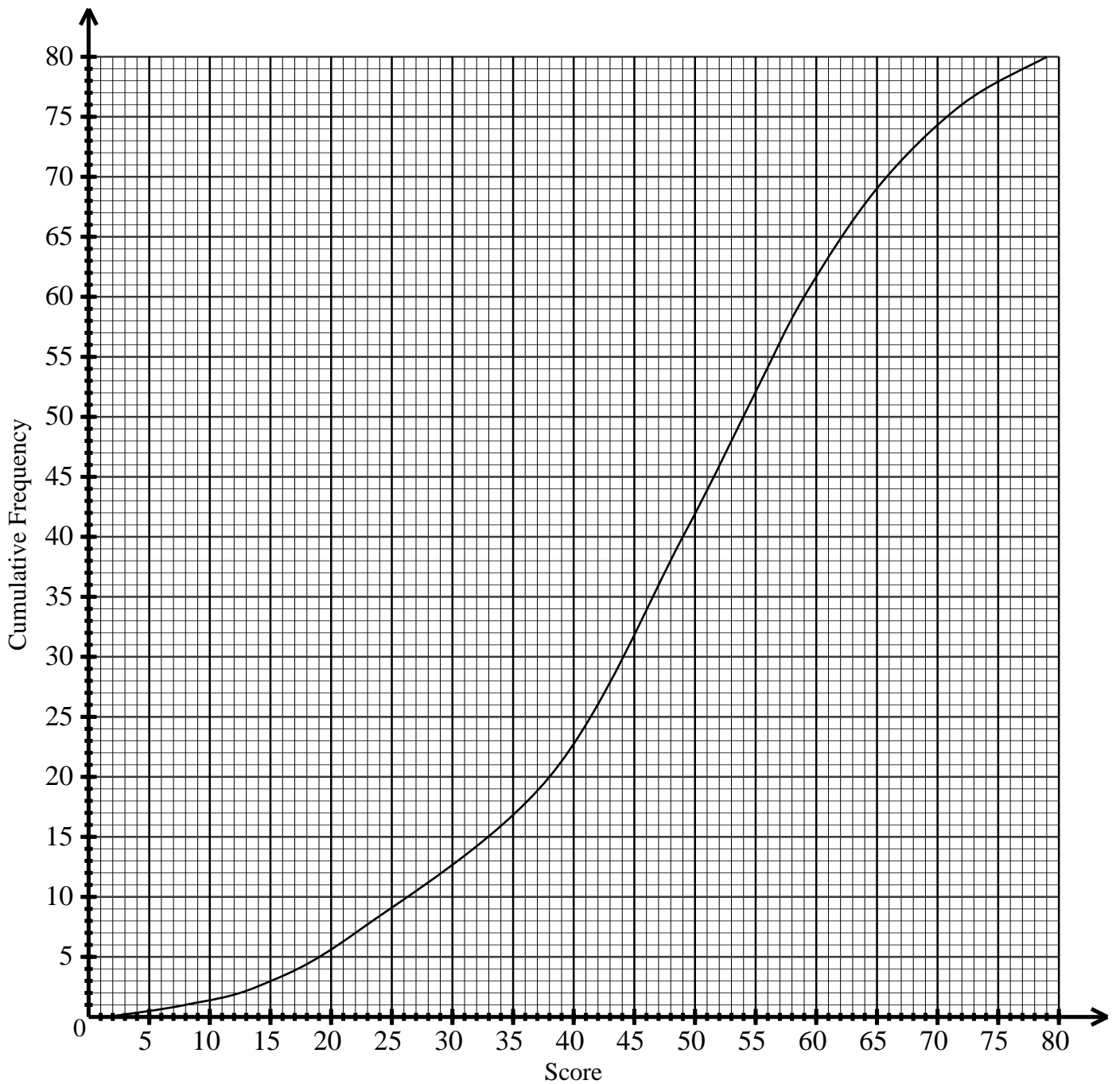
- (iv) The point Q is such that $k\overrightarrow{BQ} = \mathbf{a}$ and ONQ is a straight line.
Write down the value of k .

Answer $k = \dots\dots\dots$ [2]

- (v) Find the ratio area of triangle BMN : area of triangle OBA .

Answer : [1]

- 8 The cumulative frequency curve shows the distribution of the test scores of a group of 80 students.



- (a) Use the curve to estimate
- (i) the 65th percentile,
 - (ii) the interquartile range.

Answer [1]

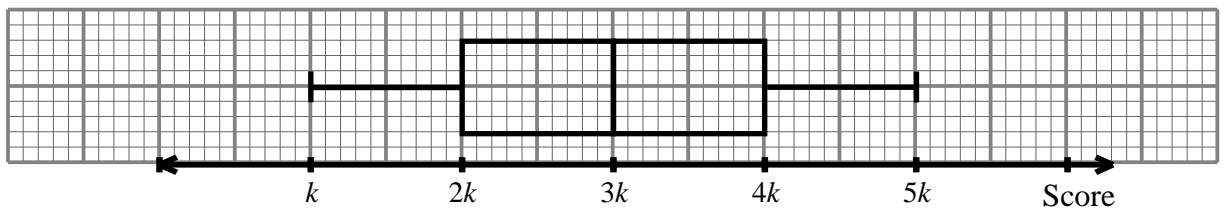
Answer [2]

8 (b) (i) Complete the table by using the estimations from the curve.

Lowest Score	
Lower Quartile	
Median	
Upper Quartile	
Highest Score	

[2]

(ii) The distribution of the scores of the 80 students is represented in a box-and-whisker plot below, where k is a constant.
Do you agree with the representation?
Provide justifications by using your answer from **part (b)(i)**.



Answer

.....

..... [2]

(c) Students who scored 33 marks and below are required to attend remedial.
A student is selected at random from the group.
Find the probability that the student selected is not required to attend remedial.

Answer [2]

- 9 Ann was planning to buy a car and chanced upon the following while doing her research.



Adapted from: <https://blog.seedly.sg/factors-cost-car-price-singapore/>

Her annual salary is \$108 000. When calculating her budget, she planned to set aside 27% of her monthly salary for the expenses incurred if she finally decided to buy a car.

- (a) Based on the information that she chanced upon in her research, deduce if Ann decided to buy a car. Calculate the amount she budgeted monthly for the expenses incurred.

Answer Buys a car: YES / NO (Circle the response)

\$ [1]

In Singapore, a Certificate of Entitlement (COE) is required for vehicle ownership, which represents the right to register and use a vehicle in Singapore for a period of 10 years.

Types of vehicle	Current COE	Previous COE
A Cars up to 1600cc	\$88,200	\$92,700
B Cars above 1600cc	\$100,607	\$105,689
C Goods Vehicle & Bus	\$70,589	\$70,620
D Motorcycle	\$8,989	\$9,649
E Open- All except Motorcycle	\$101,600	\$105,002

Age of vehicle	Certificate of Entitlement (COE)
Brand new vehicle	Based on the table shown above, included in the cost of the brand new vehicle.
Less than 10 years	Buyer takes over the remaining COE, included in the cost of the used vehicle.
More than 10 years	Buyer needs to renew the COE for the used vehicle in 5 or 10 year blocks.

Adapted from: <https://onemotoring.lta.gov.sg/content/onemotoring/home/buying/coe-open-bidding.html>

- 9 Ann approached a car dealer who recommended two different packages for a used and a new car based on Ann's needs and preferences. Ann decided to take a loan for the purchase of the car after placing a downpayment from her savings. She planned to repay the loan monthly.

Details of package	Used car	New car
Engine Capacity	1800 cc	1500 cc
Total Cost	S\$98 000	S\$150 000
Downpayment	40%	30%
Intended loan period	5 years	5 years
Type of interest (per annum)	<u>Years 1 and 2:</u> Simple interest 2.28% <u>Year 3 onwards:</u> Simple interest 3%	Simple interest at 2.88%

She collated all the other expenses that come with owning a car:

Other expenses	Used car	New car
Monthly parking fees	\$110	\$110
Monthly petrol expenditure	\$340	\$270
Annual Electronic Road Pricing (ERP)	\$360	\$360
Annual Insurance	\$1800	\$1600
Car maintenance (every 6 months)	\$650 per maintenance	\$500 per maintenance

The road tax of the car is calculated in dollars based on its engine capacity:

Engine Capacity (EC) (in cc)	Road Tax Formula (per annum)
$EC \leq 600$	$\$200 \times 0.782$
$600 < EC \leq 1000$	$[\$200 + \$0.125 \times (EC - 600)] \times 0.782$
$1000 < EC \leq 1600$	$[\$250 + \$0.375 \times (EC - 1000)] \times 0.782$
$1600 < EC \leq 3000$	$[\$475 + \$0.75 \times (EC - 1600)] \times 0.782$
$EC > 3000$	$[\$1525 + \$1.00 \times (EC - 3000)] \times 0.782$

- (b) Calculate the annual road tax for the
 (i) used car,

Answer \$ [1]

- (ii) new car.

Answer \$ [1]

- 9 (c) Determine which car Ann should purchase based on her monthly budget for all the expenses incurred in owning a car (including road tax).
Justify the decision you make and show your calculations clearly.

.....

..... [7]



Geylang Methodist School (Secondary) Preliminary Examination 2024

Candidate
Name

WORKED SOLUTIONS & SUGGESTED ANSWER SCHEME

Class

Index
Number

MATHEMATICS

4052/02

Paper 2

4 Express
5 Normal (Academic)

Candidates answer on the Question Paper.

2 hours 15 minutes

Setter: Ms Nainee Ismail and Mr Kenneth Tan

Thursday, 15 August 2024

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.

For Examiner's Use

90

This document consists of **20** printed pages including the cover page.

[Turn Over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

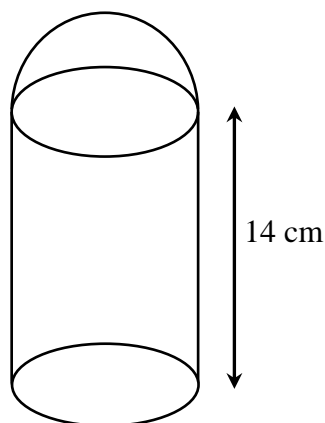
$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f} \right)^2}$$

- 1 A model of a thermos flask consists of a solid hemisphere attached to a solid cylinder as shown. The height of the cylinder is 14 cm.



The curved surface area of the cylinder is twice the curved surface area of the hemisphere.

- (a) Calculate the radius of the base of the cylinder.

Curved Surface area of cylinder = $2 \times$ Curved Surface area of hemisphere

$$2\pi rh = 2(2\pi r^2)$$

$$2\pi r(14) = 4\pi r^2$$

$$28(\pi r) = 4r(\pi r)$$

$$\frac{28}{4} = r$$

$$r = 7 \text{ cm}$$

Answer cm [2]

- (b) Calculate the height of the model of the thermos flask.

$$\text{Height of the model} = 14 + 7 = 21 \text{ cm}$$

Answer cm [1]

- (c) Find the ratio volume of hemisphere : volume of cylinder.

volume of hemisphere : volume of cylinder

$$\frac{2}{3}\pi r^3 : \pi r^2 h$$

$$\frac{2}{3}\pi(7)^3 : \pi(7)^2(14)$$

$$\frac{2}{3}\pi(7)^3 : \pi(7)^3(2)$$

$$\frac{2}{3} : (2)$$

$$2 : 6$$

$$1 : 3$$

Answer : [2]

- 2 (a) (i) Solve the inequalities $-9 < 5 - 4x \leq 6$.

$$\begin{aligned} -9 < 5 - 4x \leq 6 \\ -9 - 5 < -4x \leq 6 - 5 \\ -14 < -4x \leq 1 \\ \frac{-14}{-4} > x \geq \frac{1}{-4} \\ -\frac{1}{4} \leq x < 3\frac{1}{2} \end{aligned}$$

Answer [2]

- (ii) Write down all the integers that satisfy $-9 < 5 - 4x \leq 6$.

$$\begin{aligned} -\frac{1}{4} \leq x < 3\frac{1}{2} \\ \text{Integers } x = 0, 1, 2, 3 \end{aligned}$$

Answer [1]

(b) $y = \frac{5 - x^2}{x^2 + w}$

- (i) Evaluate the value of y when $x = 2.41$ and $w = 1.908$.

Write your answer correct to three decimal places.

$$\begin{aligned} y &= \frac{5 - x^2}{x^2 + w} \\ &= \frac{5 - (2.41)^2}{(2.41)^2 + 1.908} = \frac{-0.8081}{7.7161} \\ y &= -0.104729073 = -0.105 \end{aligned}$$

Answer $y =$ [2]

- (ii) Rearrange the formula to make x the subject.

$$\begin{aligned} y &= \frac{5 - x^2}{x^2 + w} \\ y(x^2 + w) &= 5 - x^2 \\ yx^2 + yw &= 5 - x^2 \\ yx^2 + x^2 &= 5 - yw \\ x^2(y + 1) &= 5 - yw \\ x^2 &= \frac{5 - yw}{y + 1} \\ x &= \pm \sqrt{\frac{5 - yw}{y + 1}} \end{aligned}$$

Answer $x =$ [4]

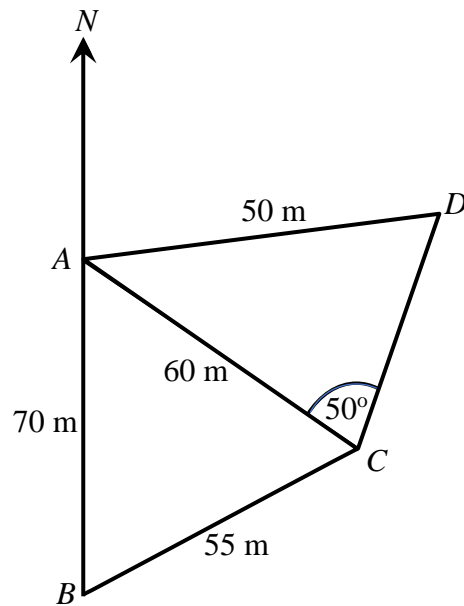
2 (c) Solve $\frac{6}{x-1} - \frac{4-x}{3-x} = 1$.

$$\begin{aligned} \frac{6}{x-1} - \frac{4-x}{3-x} &= 1 \\ \frac{6(3-x)}{(x-1)(3-x)} - \frac{(4-x)(x-1)}{(3-x)(x-1)} &= 1 \\ \frac{18-6x-(4x-4-x^2+x)}{(x-1)(3-x)} &= 1 \\ 18-6x-4x+4+x^2-x &= 1(x-1)(3-x) \\ x^2-11x+22 &= 3x-x^2-3+x \\ x^2+x^2-11x-3x-x+22+3 &= 0 \\ 2x^2-15x+25 &= 0 \\ (2x-5)(x-5) &= 0 \\ x &= 2\frac{1}{2} \text{ or } x = 5 \end{aligned}$$

[Accept working by general formula – correct substitution]

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [4]

3



A , B , C and D are four points on horizontal ground.

$AB = 70$ m, $BC = 55$ m, $CA = 60$ m, $AD = 50$ m, angle $ACD = 50^\circ$ and angle ADC is acute.

(a) Calculate angle BAC .

$$55^2 = 60^2 + 70^2 - 2(60)(70)\cos \angle BAC$$

$$\cos \angle BAC = \frac{60^2 + 70^2 - 55^2}{2(60)(70)}$$

$$\text{Angle } BAC = 49.324^\circ = 49.3^\circ$$

Answer Angle $BAC = \dots\dots\dots 49.3^\circ \dots\dots\dots [3]$

(b) Calculate angle DAC .

$$\frac{\sin \angle ADC}{60} = \frac{\sin 50^\circ}{50}$$

$$\sin \angle ADC = \frac{\sin 50^\circ}{50} \times 60$$

$$= 0.9192533317$$

$$\text{Angle } ADC = 66.817^\circ$$

$$\text{Angle } DAC = 180^\circ - 66.817^\circ - 50^\circ$$

$$= 63.183^\circ$$

$$\text{Angle } DAC = 63.2^\circ$$

Answer Angle $DAC = \dots\dots\dots 63.2^\circ \dots\dots\dots [3]$

- 3 (c) Calculate the bearing of A from D.

$$\begin{aligned}
 &N_1 : \text{North is drawn at } D \\
 &\angle ADN_1 = 63.2^\circ + 49.3^\circ \\
 &\quad = 112.5^\circ \\
 &(\text{Alternate Angles, } AN // DN_1) \\
 &\text{Bearing of A from } D = 360^\circ - 112.5^\circ \\
 &\quad = 247.5^\circ \\
 &(\text{Sum of angles at a point})
 \end{aligned}$$

Answer 247.5° [2]

- (d) A man walks northwards from point A. A vertical pole is placed at point D. The greatest angle of elevation of the pole when viewed along this path is 7° . Calculate the height of the pole.

$\text{Angle } XAD = 180^\circ - 63.2^\circ - 49.30^\circ$
 $\quad = 67.5^\circ$ (angles on a straight line)

$$\sin 67.5^\circ = \frac{XD}{50}$$

$$XD = 50 \sin 67.5^\circ = 46.193 \text{ m}$$

$$\tan 7^\circ = \frac{h}{46.193}$$

$$h = 46.193 \times \tan 7^\circ = 5.67 \text{ m}$$

Answer 5.67 m m [2]

- 4 Mr Tan planned to travel to Kuala Lumpur for a family trip.
Before the trip, he exchanged Singapore Dollars (S\$) for Malaysian Ringgit (RM).
The exchange rate is S\$ 1 = RM x .

- (a) Write down an expression, in terms of x , for the Singapore Dollars he exchanged if he received RM 2000.

$$\begin{aligned} \text{RM } x &= \text{S\$ } 1 \\ \text{RM } 1 &= \text{S\$ } \left(\frac{1}{x}\right) \quad \Rightarrow \quad \text{RM } 2000 = \text{S\$ } \left(\frac{2000}{x}\right) \end{aligned}$$

$$\text{Answer S\$ } \dots\dots\dots \frac{2000}{x} \dots\dots\dots [1]$$

- (b) After the trip, Mr Tan found out that he still had RM 300.
He exchanged the Malaysian Ringgit back to Singapore Dollars.
The exchange rate is now S\$ 1 = RM $(x + 0.05)$.
Write down an expression, in terms of x , for the Singapore Dollars he exchanged for RM 300.

$$\begin{aligned} \text{RM } (x + 0.05)x &= \text{S\$ } 1 \\ \text{RM } 1 &= \text{S\$ } \left(\frac{1}{x + 0.05}\right) \quad \Rightarrow \quad \text{RM } 300 = \text{S\$ } \left(\frac{300}{x + 0.05}\right) \end{aligned}$$

$$\text{Answer S\$ } \dots\dots\dots \frac{300}{x + 0.05} \dots\dots\dots [1]$$

- (c) Mr Tan used up S\$ 485 for the trip.
Form an equation in x and show that it reduces to $1940x^2 - 6703x - 400 = 0$.

Answer

$$\begin{aligned} \text{RM } 2000 - \text{RM } 300 &= \text{S\$ } 485 \\ \frac{2000}{x} - \frac{300}{x + 0.05} &= 485 \\ \frac{2000(x + 0.05) - 300x}{x(x + 0.05)} &= 485 \\ 2000x + 100 - 300x &= 485x(x + 0.05) \\ 0 &= 485x^2 + 24.25x - 2000x - 100 + 300x \\ 0 &= 485x^2 - \frac{6703}{4}x - 100 \\ 1940x^2 - 6703x - 400 &= 0 \quad (\text{Shown}) \end{aligned}$$

[3]

- 4 (d) Solve the equation $1940x^2 - 6703x - 400 = 0$,
giving your solutions correct to two decimal places.

$$\begin{aligned}
 x &= \frac{-(-6703) \pm \sqrt{(-6703)^2 - 4(1940)(-400)}}{2(1940)} \\
 &= \frac{6703 \pm \sqrt{48034209}}{3880} \\
 &= 3.51383289 \quad \text{or} \quad -0.05867825064 \\
 x &= 3.51 \quad \text{or} \quad -0.06 \quad (2 \text{ dp})
 \end{aligned}$$

Answer $x = \dots\dots\dots 3.51 \dots\dots\dots$ or $\dots\dots\dots -0.06 \dots\dots\dots$ [3]

- (e) From your solution to **part (d)**, state the exchange rate in the form is S\$ 1 = RM x , to two decimal places, and hence, find the total amount of Singapore Dollars (S\$) he initially converted to Malaysian Ringgit (RM).

$$\begin{aligned}
 \text{S\$ } 1 &= \text{RM } 3.51 \\
 \text{RM } 2000 &= \text{S\$ } \left(\frac{2000}{x} \right) \\
 &= \text{S\$ } \left(\frac{2000}{3.51} \right) \\
 &= \text{S\$ } 569.8005698 \\
 \text{Amount of S\$} &= \frac{2000}{3.51} = \text{S\$ } 569.80 \quad (2\text{dp})
 \end{aligned}$$

Answer S\$ 1 = RM $\dots\dots\dots 3.51 \dots\dots\dots$

S\$ $\dots\dots\dots 569.80 \dots\dots\dots$ [2]

- 5 A shop sells two flavours of ice-cream, Raisins (R) and Chunkies (C) in three different sizes, small (S), medium (M) and large (L). The sales on two successive days are given in the table below.

Size	Saturday			Sunday		
	small	medium	large	small	medium	large
Number of cups sold (Raisins)	12	17	8	14	12	10
Number of cups sold (Chunkies)	18	15	11	13	21	16

The information for Saturday's sales can be represented by the matrix

$$\mathbf{M} = \begin{matrix} & \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \\ \begin{pmatrix} 12 & 17 & 8 \\ 18 & 15 & 11 \end{pmatrix} & \begin{matrix} \text{R} \\ \text{C} \end{matrix} \end{matrix}.$$

- (a) Represent the information for Sunday's sales in a 2×3 matrix \mathbf{N} .

$$\mathbf{N} = \begin{matrix} & \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \\ \begin{pmatrix} 14 & 12 & 10 \\ 13 & 21 & 16 \end{pmatrix} & \begin{matrix} \text{R} \\ \text{C} \end{matrix} \end{matrix} \quad \text{Answer} \quad \mathbf{N} = \begin{matrix} & \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \\ \left(\begin{matrix} & & \\ & & \\ & & \end{matrix} \right) & \begin{matrix} \text{R} \\ \text{C} \end{matrix} \end{matrix} \quad [1]$$

- (b) The cost of each cup of ice-cream is \$2.50 for small (S), \$3.20 for medium (M) and \$4.50 for large (L).

Represent the cost of ice cream in a 3×1 matrix \mathbf{C} .

$$\mathbf{C} = \begin{matrix} \$ \\ \begin{pmatrix} 2.5 \\ 3.2 \\ 4.5 \end{pmatrix} & \begin{matrix} \text{S} \\ \text{M} \\ \text{L} \end{matrix} \end{matrix} \quad \text{Answer} \quad \mathbf{C} = \begin{matrix} \$ \\ \left(\begin{matrix} & \\ & \\ & \end{matrix} \right) & \begin{matrix} \text{S} \\ \text{M} \\ \text{L} \end{matrix} \end{matrix} \quad [1]$$

- (c) Evaluate the matrix $\mathbf{P} = \mathbf{M} + \mathbf{N}$.

$$\mathbf{P} = \mathbf{M} + \mathbf{N} = \begin{matrix} & \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \\ \begin{pmatrix} 12 & 17 & 8 \\ 18 & 15 & 11 \end{pmatrix} & \begin{matrix} \text{R} \\ \text{C} \end{matrix} \end{matrix} + \begin{matrix} & \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \\ \begin{pmatrix} 14 & 12 & 10 \\ 13 & 21 & 16 \end{pmatrix} & \begin{matrix} \text{R} \\ \text{C} \end{matrix} \end{matrix}$$

$$\mathbf{P} = \begin{matrix} & \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \\ \begin{pmatrix} 26 & 29 & 18 \\ 31 & 36 & 27 \end{pmatrix} & \begin{matrix} \text{R} \\ \text{C} \end{matrix} \end{matrix} \quad \text{Answer} \quad \mathbf{P} = \quad [1]$$

- (d) State what the elements of \mathbf{P} represent.

The elements represent the **total number** of **different sizes** of cups (S, M, L) of

Raisins and Chunkies flavoured ice-cream sold on **Saturday and Sunday**. [1]

- 5 (e) Evaluate the matrix $\mathbf{Q} = \frac{1}{2}\mathbf{PC}$.

$$\begin{matrix} & \text{S} & \text{M} & \text{L} & \text{\$} \\ \mathbf{Q} = \frac{1}{2}\mathbf{PC} = \frac{1}{2} \begin{matrix} \text{R} \\ \text{C} \end{matrix} \begin{pmatrix} 26 & 29 & 18 \\ 31 & 36 & 27 \end{pmatrix} & \begin{pmatrix} 2.5 \\ 3.2 \\ 4.5 \end{pmatrix} & \begin{matrix} \text{S} \\ \text{M} \\ \text{L} \end{matrix} \\ & \text{\$} & & & \end{matrix}$$

$$\mathbf{Q} = \frac{1}{2}\mathbf{PC} = \frac{1}{2} \begin{matrix} \text{R} \\ \text{C} \end{matrix} \begin{pmatrix} 238.8 \\ 314.2 \end{pmatrix}$$

$$\mathbf{Q} = \begin{matrix} \text{R} \\ \text{C} \end{matrix} \begin{pmatrix} 119.4 \\ 157.1 \end{pmatrix} \quad \text{Answer} \quad \mathbf{Q} = \quad [2]$$

- (f) State what the elements of \mathbf{Q} represent.

The **average amount** received from the sales on Saturday and Sunday of all

 sizes (S, M, L) of **Raisins and Chunkies** flavoured ice-cream **respectively**.
 [1]

- (g) Evaluate the matrix $\mathbf{R} = (1 \ 1)\mathbf{PC}$.

$$\begin{matrix} \text{\$} \\ \mathbf{PC} = \begin{matrix} \text{R} \\ \text{C} \end{matrix} \begin{pmatrix} 238.8 \\ 314.2 \end{pmatrix} \end{matrix}$$

$$\mathbf{R} = (1 \ 1)\mathbf{PC}$$

$$\begin{matrix} \text{\$} \\ \mathbf{R} = (1 \ 1) \begin{pmatrix} 238.8 \\ 314.2 \end{pmatrix} \begin{matrix} \text{R} \\ \text{C} \end{matrix} \end{matrix}$$

$$\mathbf{R} = (238.8 + 314.2)$$

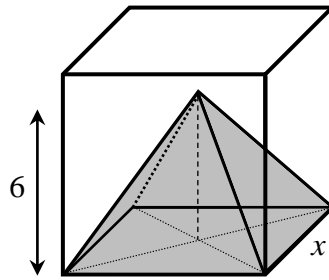
$$\mathbf{R} = (553) \quad \text{Answer} \quad \mathbf{R} = \quad [2]$$

- (h) State what the elements of \mathbf{R} represent.

The **total amount** received from the sales on Saturday and Sunday of all

 sizes (S, M, L) of **Raisins and Chunkies** flavoured ice-cream.
 [1]

- 6 The following diagram shows a solid which is made up of a cube whose edge is x cm with a right square pyramid removed from the base of the cube. The pyramid has the same square base as the cube and its vertical height is 6 cm.



- (a) Show that the volume of the remaining solid, y , is $x^3 - 2x^2$.

Answer

Volume of solid = Volume of (cube - pyramid)

$$y = (L \times B \times H) - \left(\frac{1}{3} \times \text{base area} \times h\right) = (x \times x \times x) - \left(\frac{1}{3} \times x \times x \times 6\right)$$

$$y = x^3 - 2x^2$$

[1]

- (b) Complete the table of values for $y = x^3 - 2x^2$. Values are given to one decimal place where appropriate.

x	0	1	1.5	2	3	4	5	6	6.5
y	0	-1	-1.1	0	9	32	75	144	190.1

[1]

- (c) On the grid opposite, draw the graph of $y = x^3 - 2x^2$ for $0 \leq x \leq 6.5$. [3]

- (d) Use your graph to estimate the volume of the remaining solid when the pyramid has a base area of 24 cm^2 .

Base Area = 24 cm^2
 $x^2 = 24$
 $x = \sqrt{24} = 4.898979... = 4.9$

From graph, $x = 4.9$
 $y = (70 \pm 2) \text{ cm}^3$
 [Actual value of $y = 69.6$]

Answer Volume = cm^3 [2]

- (e) (i) Use your graph, for $0 < x < 2$, to give a possible deduction on the volume of the remaining solid, y .

For $0 < x < 2$, graph is below y -axis, to indicate volume is negative, it shows that the volume of pyramid $>$ volume of cube and hence removing the pyramid is not even possible. [1]

- (ii) Suggest a suitable range of x such that it is possible to remove a full right square pyramid from the cube.

$x \geq 6$

Answer [1]

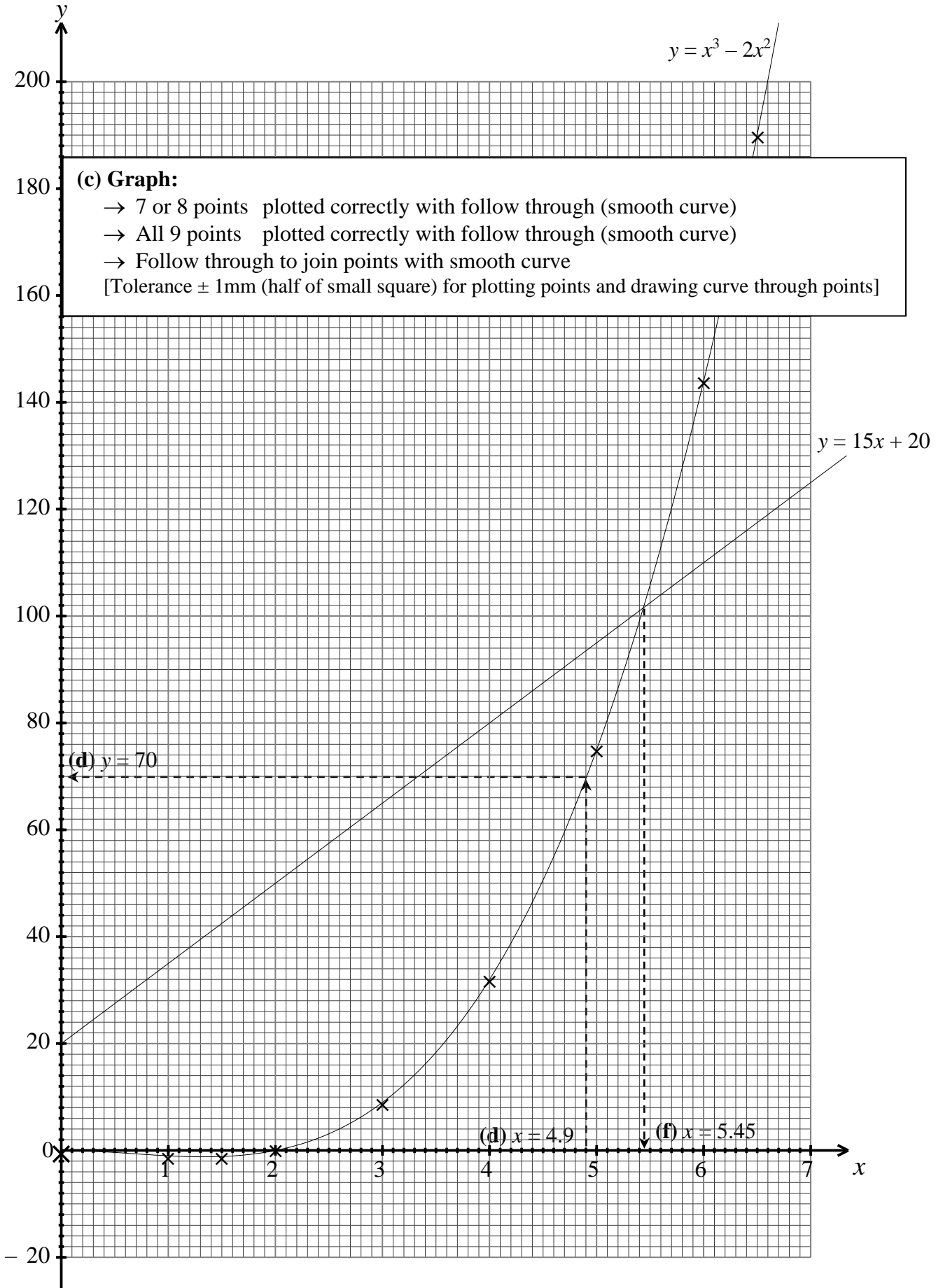
- (f) By drawing the line $y = ax + b$, solve the equation $-x^3 + 2x^2 + 15x + 20 = 0$.

$-x^3 + 2x^2 + 15x + 20 = 0$
 $15x + 20 = x^3 - 2x^2$
 $15x + 20 = y$

Draw $y = 15x + 20$
 From graph, point of intersection: $x = (5.45 \pm 0.1)$
 [Actual value of $x = 5.45$]

Answer $x =$ [2]

6 (c)



- 7 (a) The point P is $(5, -8)$ and the point Q is $(-4, 7)$.

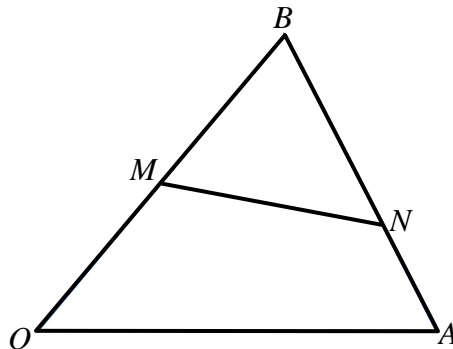
The point L is such that $\overrightarrow{QP} = \frac{1}{2}\overrightarrow{PL}$ and O is the origin.

Find $|\overrightarrow{OL}|$.

$$\begin{aligned}\overrightarrow{OP} &= \begin{pmatrix} 5 \\ -8 \end{pmatrix}, \quad \overrightarrow{OQ} = \begin{pmatrix} -4 \\ 7 \end{pmatrix}, \quad \overrightarrow{QP} = \frac{1}{2}\overrightarrow{PL} \\ \overrightarrow{QP} &= \overrightarrow{QO} + \overrightarrow{OP} = -\begin{pmatrix} -4 \\ 7 \end{pmatrix} + \begin{pmatrix} 5 \\ -8 \end{pmatrix} = \begin{pmatrix} 4 \\ -7 \end{pmatrix} + \begin{pmatrix} 5 \\ -8 \end{pmatrix} = \begin{pmatrix} 9 \\ -15 \end{pmatrix} \\ \frac{1}{2}\overrightarrow{PL} &= \frac{1}{2}(\overrightarrow{PO} + \overrightarrow{OL}) \quad , \quad \overrightarrow{QP} = \frac{1}{2}(\overrightarrow{PO} + \overrightarrow{OL}) \\ \begin{pmatrix} 9 \\ -15 \end{pmatrix} &= \frac{1}{2}\left[-\begin{pmatrix} 5 \\ -8 \end{pmatrix} + \overrightarrow{OL}\right] \\ \begin{pmatrix} 18 \\ -30 \end{pmatrix} &= \begin{pmatrix} -5 \\ 8 \end{pmatrix} + \overrightarrow{OL} \\ \overrightarrow{OL} &= \begin{pmatrix} 18 \\ -30 \end{pmatrix} - \begin{pmatrix} -5 \\ 8 \end{pmatrix} = \begin{pmatrix} 23 \\ -38 \end{pmatrix} \\ |\overrightarrow{OL}| &= \sqrt{23^2 + (-38)^2} = \sqrt{1973} = 44.418 \approx 44.4 \text{ units (3sf)}\end{aligned}$$

Answer [4]

- (b) In the diagram $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
 M is a point on OB where $OB = 2MB$ and the point N lies on AB
such that $3BN = 2BA$.



- (i) Express \overrightarrow{BN} , as simply as possible, in terms of \mathbf{a} and \mathbf{b} .

$$\begin{aligned}\overrightarrow{BA} &= \overrightarrow{BO} + \overrightarrow{OA} = -\mathbf{b} + \mathbf{a} \\ \overrightarrow{BN} &= \frac{2}{3}\overrightarrow{BA} = \frac{2}{3}(-\mathbf{b} + \mathbf{a}) = \frac{2}{3}\mathbf{a} - \frac{2}{3}\mathbf{b}\end{aligned}$$

Answer $\overrightarrow{BN} = \dots\dots\dots$ [1]

7 (b) (ii) Express \overrightarrow{MN} , as simply as possible, in terms of \mathbf{a} and \mathbf{b} .

$$\begin{aligned} \overrightarrow{MN} &= \overrightarrow{MB} + \overrightarrow{BN} \\ &= \frac{1}{2}\mathbf{b} + \frac{2}{3}\mathbf{a} - \frac{2}{3}\mathbf{b} \\ \overrightarrow{MN} &= \frac{2}{3}\mathbf{a} - \frac{1}{6}\mathbf{b} \end{aligned}$$

Answer $\overrightarrow{MN} = \dots\dots\dots$ [2]

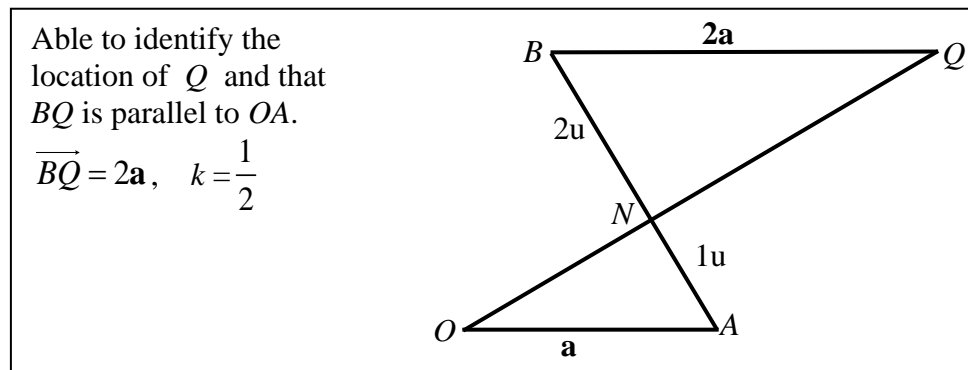
(iii) The point P lies on OA produced such that $OA : OP = 1 : 2$. Determine whether the points M, N and P are collinear. Justify your answer.

$$\overrightarrow{MP} = \overrightarrow{MO} + \overrightarrow{OP} = -\frac{1}{2}\mathbf{b} + 2\mathbf{a}, \quad \overrightarrow{MN} = \frac{2}{3}\mathbf{a} - \frac{1}{6}\mathbf{b} = \frac{1}{3}\left(2\mathbf{a} - \frac{1}{2}\mathbf{b}\right) = \frac{1}{3}\overrightarrow{MP}$$

Since both vectors \overrightarrow{MP} and \overrightarrow{MN} have the same direction components of $2\mathbf{a} - \frac{1}{2}\mathbf{b}$, and N is connecting point, they are collinear.

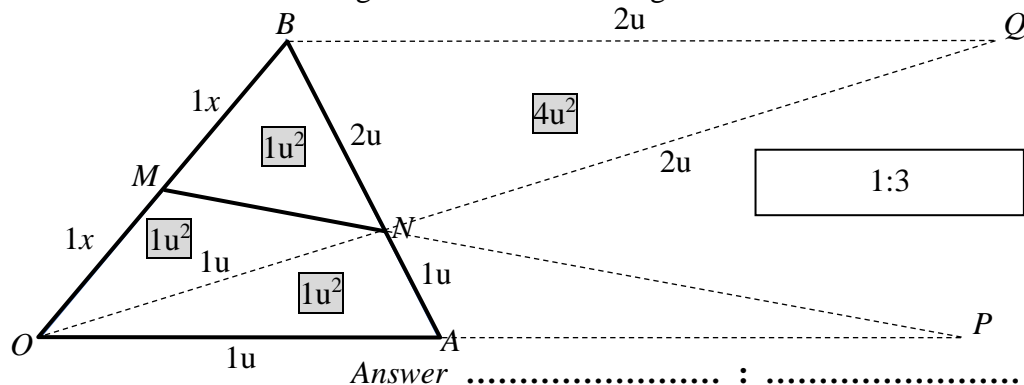
 [includes explanation on the same direction component which makes the line collinear]
 [2]

(iv) The point Q is such that $k\overrightarrow{BQ} = \mathbf{a}$ and ONQ is a straight line. Write down the value of k .

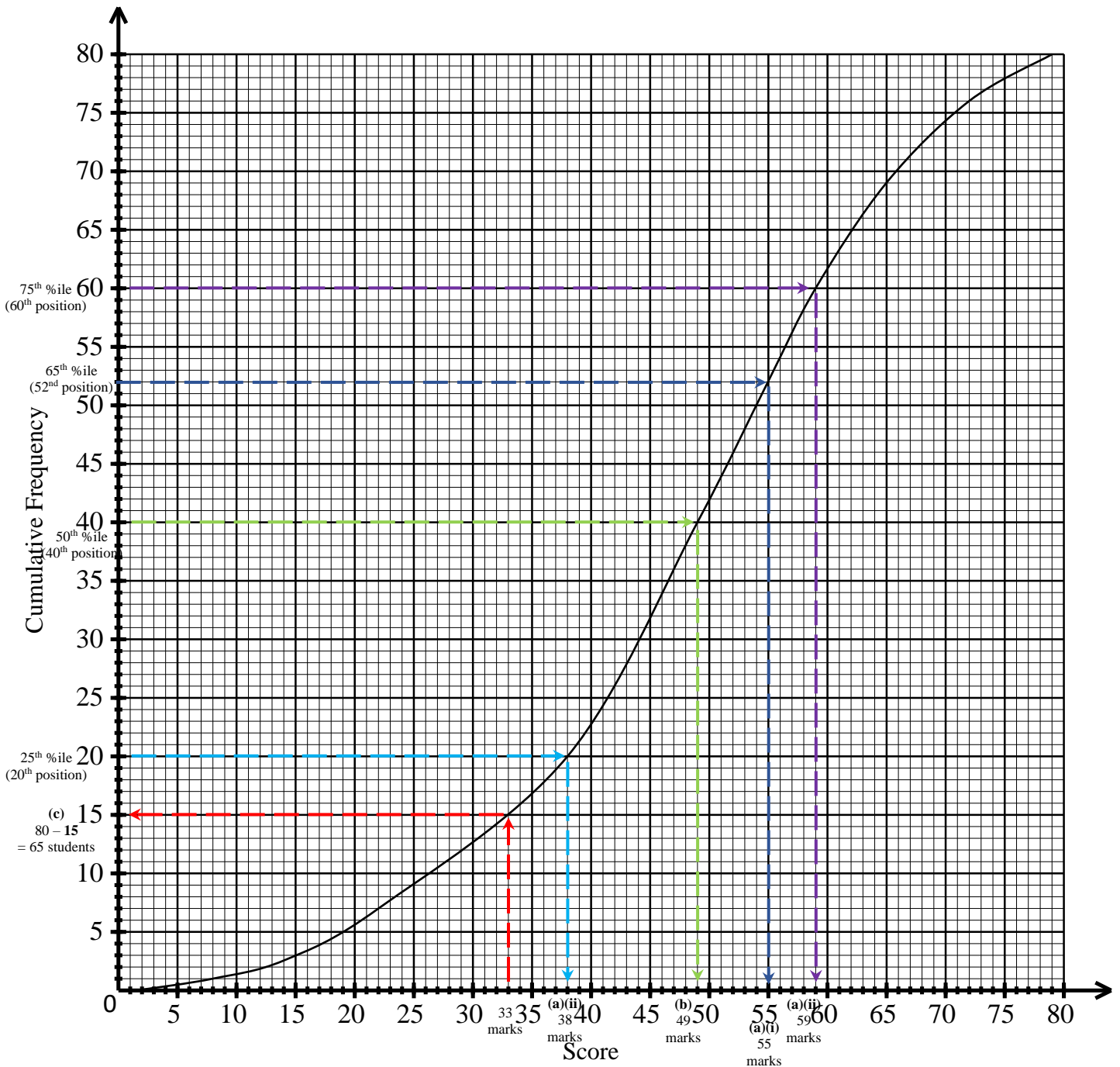


Answer $k = \dots\dots\dots$ [2]

(v) Find the ratio area of triangle BMN : area of triangle OBA .



8 The cumulative frequency curve shows the distribution of the test scores of a group of 80 students.



(a) Use the curve to estimate

(i) the 65th percentile,

$$\frac{65}{100} \times 80 = 52^{\text{nd}} \text{ student}$$

∴ From graph: 65th percentile → 55 marks

Answer 55 [1]

(ii) the interquartile range.

$$\frac{25}{100} \times 80 = 20^{\text{th}} \text{ student}$$

$$\frac{75}{100} \times 80 = 60^{\text{th}} \text{ student}$$

From graph:

75th %ile (UQ) = 59.0 marks

25th %ile (LQ) = 38.0 marks

$$\text{IQ Range} = \text{UQ} - \text{LQ}$$

$$= 59.0 - 38.0$$

$$= 21.0$$

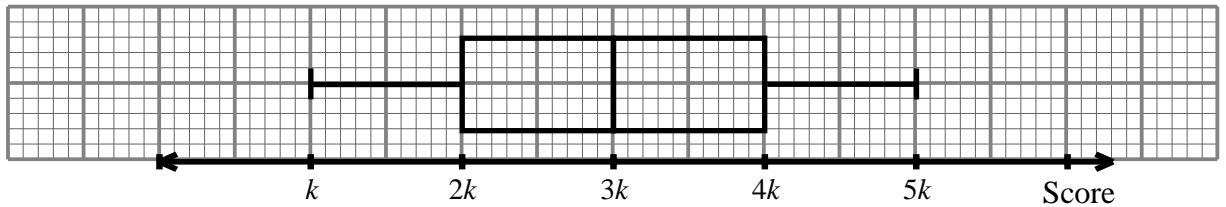
Answer 21 [2]

8 (b) (i) Complete the table by using the estimations from the curve.

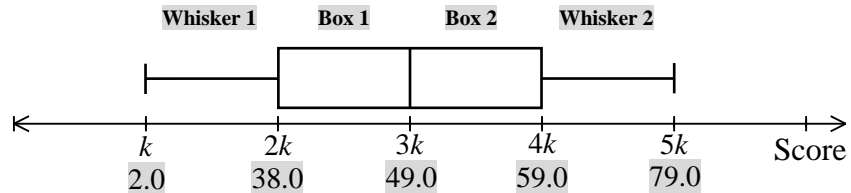
Lowest Score	2.0 (3.0)
Lower Quartile	38.0
Median	49.0
Upper Quartile	59.0
Highest Score	79.0

[2]

(ii) The distribution of the scores of the 80 students is represented in a box-and-whisker plot below, where k is a constant.
Do you agree with the representation?
Provide justifications by using your answer from **part (b)(i)**.



Answer



Disagree. Each box and whisker **should not be equally spaced.**

The box-and-whisker plot shows that $k, 2k, 3k, 4k, 5k$ are all equally spaced (Increase by k each time from Lowest, LQ, Median, UQ, Highest).

From the actual values, Length of: Whisker 1 > Whisker 2 > Box 1 > Box 2.
(36 > 20 > 11 > 10)

For any comment substantiated with logical reasons. [2]

(c) Students who scored 33 marks and below are required to attend remedial.
A student is selected at random from the group.
Find the probability that the student selected is not required to attend remedial.

From graph: 33 marks \rightarrow 15th student

Number of the students who are surfaced for remedial
= 15 students

Number of the students who are not surfaced for remedial
= 80 - 15 = 65 students

Probability that student selected is not required to attend remedial
= $\frac{65}{80} = \frac{13}{16}$ students

Answer $\frac{13}{16}$ [2]

- 9 Ann was planning to buy a car and chanced upon the following while doing her research.



Adapted from: <https://blog.seedly.sg/factors-cost-car-price-singapore/>

Her annual salary is \$108 000. When calculating her budget, she planned to set aside 27% of her monthly salary for the expenses incurred if she finally decided to buy a car.

- (a) Based on the information that she chanced upon in her research, deduce if Ann decided to buy a car. Calculate the amount she budgeted monthly for the expenses incurred.

(a) Ann's monthly salary = $\frac{108000}{12} = \$9000 (> \$8729, \text{ from research})$

27% of Ann's monthly salary = $\frac{27}{100} \times \$9000 = \boxed{\$2430}$

Answer Buys a car: YES / NO (Circle the response)

\$ [1]

In Singapore, a Certificate of Entitlement (COE) is required for vehicle ownership, which represents the right to register and use a vehicle in Singapore for a period of 10 years.

Types of vehicle	Current COE	Previous COE
A Cars up to 1600cc	\$88,200	\$92,700
B Cars above 1600cc	\$100,607	\$105,689
C Goods Vehicle & Bus	\$70,589	\$70,620
D Motorcycle	\$8,989	\$9,649
E Open- All except Motorcycle	\$101,600	\$105,002

Age of vehicle	Certificate of Entitlement (COE)
Brand new vehicle	Based on the table shown above, included in the cost of the brand new vehicle.
Less than 10 years	Buyer takes over the remaining COE, included in the cost of the used vehicle.
More than 10 years	Buyer needs to renew the COE for the used vehicle in 5 or 10 year blocks.

Adapted from: <https://onemotoring.lta.gov.sg/content/onemotoring/home/buying/coe-open-bidding.html>

- 9 Ann approached a car dealer who recommended two different packages for a used and a new car based on Ann's needs and preferences. Ann decided to take a loan for the purchase of the car after placing a downpayment from her savings. She planned to repay the loan monthly.

Details of package	Used car	New car
Engine Capacity	1800 cc	1500 cc
Total Cost	S\$98 000	S\$150 000
Downpayment	40%	30%
Intended loan period	5 years	5 years
Type of interest (per annum)	Years 1 and 2: Simple interest 2.28% Year 3 onwards: Simple interest 3%	Simple interest at 2.88%

She collated all the other expenses that come with owning a car:

Other expenses	Used car	New car
Monthly parking fees	\$110	\$110
Monthly petrol expenditure	\$340	\$270
Annual Electronic Road Pricing (ERP)	\$360	\$360
Annual Insurance	\$1800	\$1600
Car maintenance (every 6 months)	\$650 per maintenance	\$500 per maintenance

The road tax of the car is calculated in dollars based on its engine capacity:

Engine Capacity (EC) (in cc)	Road Tax Formula (per annum)
$EC \leq 600$	$\$200 \times 0.782$
$600 < EC \leq 1000$	$[\$200 + \$0.125 \times (EC - 600)] \times 0.782$
$1000 < EC \leq 1600$	$[\$250 + \$0.375 \times (EC - 1000)] \times 0.782$
$1600 < EC \leq 3000$	$[\$475 + \$0.75 \times (EC - 1600)] \times 0.782$
$EC > 3000$	$[\$1525 + \$1.00 \times (EC - 3000)] \times 0.782$

- (b) Calculate the annual road tax for the

- (i) used car,
1800 cc:

Engine Capacity (EC) (in cc)	Road Tax Formula (per annum)
$1600 < EC \leq 3000$	$[\$475 + \$0.75 \times (EC - 1600)] \times 0.782$

Annual road tax (Used Car)

$$= [\$475 + \$0.75 \times (1800 - 1600)] \times 0.782 = \$488.75$$

Answer \$ [1]

- (ii) new car.
1500 cc:

Engine Capacity (EC) (in cc)	Road Tax Formula (per annum)
$1000 < EC \leq 1600$	$[\$250 + \$0.375 \times (EC - 1000)] \times 0.782$

Annual road tax (New Car)

$$= [\$250 + \$0.375 \times (1500 - 1000)] \times 0.782 = \$342.125 = \$342.13$$

Answer \$ [1]

- 9 (c) Determine which car Ann should purchase based on her monthly budget for all the expenses incurred in owning a car (including road tax).
Justify the decision you make and show your calculations clearly.

[Reasonable and correct working preferably with accuracy of values]

Used car

$$\text{Loan amount (after making downpayment)} = \frac{(100 - 40)}{100} \times 98000 = 60 \times 980 = \$58800$$

$$\text{Simple Interest incurred (Years 1 and 2)} = \frac{P \times R \times T}{100} = \frac{58800 \times 2.28 \times 2}{100} = \$2681.28$$

$$\text{Simple Interest incurred (Years 3, 4 and 5)} = \frac{P \times R \times T}{100} = \frac{58800 \times 3 \times 3}{100} = \$5292.00$$

$$\text{Monthly (Bank Loan Repayment)} = \frac{58800 + 2681.28 + 5292}{(5 \times 12)} = \frac{66773.28}{60} = \$1112.888$$

$$\text{Monthly (other expenses)} = \underset{\text{Parking Fees}}{110} + \underset{\text{Petrol Expense}}{340} + \underset{\text{ERP}}{(360 \div 12)} + \underset{\text{Insurance}}{(1800 \div 12)} + \underset{\text{Maintenance}}{(650 \div 6)} = \$738 \frac{1}{3}$$

$$\text{Monthly (Road Tax)} = \frac{488.75}{12} = \$40 \frac{35}{48}$$

TOTAL Monthly (All Expenses Incurred)

TOTAL Monthly (Bank Loan Repayment + Others + Road Tax)

$$= \$1112.888 + \$738 \frac{1}{3} + \$40 \frac{35}{48} = \$1891.9505$$

$$= \$1891.95 \text{ (USED Car, Total monthly expense)} < \$2430 \text{ (Budget of 27\% monthly salary)}$$

New car

$$\text{Loan amount (after making downpayment)} = \frac{(100 - 30)}{100} \times 150000 = 70 \times 1500 = \$105000$$

$$\text{Simple Interest incurred (Years 1 to 5)} = \frac{P \times R \times T}{100} = \frac{105000 \times 2.88 \times 5}{100} = \$15120$$

$$\text{Monthly (Bank Loan Repayment)} = \frac{105000 + 15120}{(5 \times 12)} = \frac{120120}{60} = \$2002$$

$$\text{Monthly (other expenses)} = \underset{\text{Parking Fees}}{110} + \underset{\text{Petrol Expense}}{270} + \underset{\text{ERP}}{(360 \div 12)} + \underset{\text{Insurance}}{(1600 \div 12)} + \underset{\text{Maintenance}}{(500 \div 6)} = \$626 \frac{2}{3}$$

$$\text{Monthly (Road Tax)} = \frac{342.125}{12} = \$28 \frac{49}{96}$$

TOTAL Monthly (All Expenses Incurred)

TOTAL Monthly (Bank Loan Repayment + Others + Road Tax)

$$= \$2002 + \$626 \frac{2}{3} + \$28 \frac{49}{96} = \$2657.177083$$

$$= \$2657.18 \text{ (NEW Car, Total monthly expense)} > \$2430 \text{ (Budget of 27\% monthly salary)}$$

Ann should purchase the **Used car**, as the total expenses incurred is **within her budget of 27%**
.....
of monthly salary earned. Purchase of new car will incur higher monthly expenses.
..... [7]